Let h be the plane isometry, other than the identity, that maps T_1 to itself.

(i) Find f, g and h in standard form. [5]

(ii) Find the fixed points and fixed lines (if any) of f, g and h, and classify each of these isometries in terms of the geometric types listed in Theorem 5.1 of *Unit IB1*. [3]

(iii) Find fg and express it in standard form. Describe fg geometrically. [2]

(c) The tiling $\mathcal{T}_3$ can be transformed to one of the Laves tilings by an affine transformation.

(i) Describe such a transformation, in terms of the coordinate system in the figure for part (b). [2]

(ii) Specify the resulting Laves tiling in terms of its tile type. [1]

Question 2 (*Unit IB2*)

A group G of order 27 is given by the presentation

$$G = \langle a, b : a^9 = e,\ b^3 = e,\ ba = a^4 b \rangle.$$

The elements of G are written in standard form as

$$a^i b^j, \quad i = 0, \ldots, 8, \quad j = 0, 1, 2.$$

(a) Write each of the following products in this standard form:

(i) aba; [1]

(ii) bab; [1]

(iii) ba^3b^{-1}; [3]

(iv) aba^{-1}. [3]

(b) Let H be the subgroup of G generated by a^3 and b:

$$H = \langle a^3, b \rangle.$$

(i) Find the orders of the generators of H. [3]

(ii) Explain why H is not cyclic. [4]

(iii) List the elements of H in standard form. [3]

(c) Define a group action of G on G by

$$g \wedge x = gxg^{-1}, \qquad g \in G, x \in G.$$

(You *need not check* that this is a group action.)

Find

(i) $\mathrm{Orb}(a)$; [4]

(ii) $\mathrm{Stab}(a)$; [3]

giving the elements in standard form.