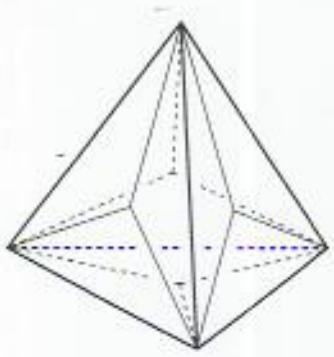
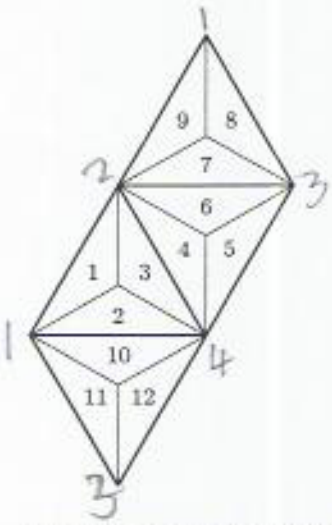
Question 3 (*Unit GE1*)

This question concerns the regular tetrahedron T, each of whose faces is divided into three isosceles triangles.



A paper pattern for making T is shown below to indicate a numbering system for these triangles. (You may wish to make a model to help you to think about the question.)



Let G be the group of direct symmetries of T.

(a) Write down the elements of G as permutations of the positions of the isosceles triangles, using the numbering system given above. [8]

(b) Find the cycle index of G considered as acting on these triangle positions. [4]

(c) Hence calculate the number of equivalence classes of colourings of the isosceles triangles when m colours are available. [2]

(d) Write down an expression for the pattern inventory of all colourings using just the two colours: R (red) and B (blue). There is no need to expand this expression. [3]

(e) By finding the coefficients of a subset of the terms in the above inventory, calculate the number of equivalence classes of colourings in which:

(i) exactly two isosceles triangles are coloured red; [4]

(ii) exactly three isosceles triangles are coloured red; [4]

all the others being coloured blue.