Question 4 (*Unit IB4*)

This question concerns the group G of order 27 given by the presentation

$$G = \langle a, b : a^9 = e,\ b^3 = e,\ ba = a^4 b \rangle,$$

as discussed in Question 2. As before, the elements of G are written in standard form as

$$a^i b^j, \quad i = 0, \ldots, 8, \quad j = 0, 1, 2.$$

(a) (i) Show that a^3 commutes with b. For which value(s) of i other than 3 (if any) does the element a^i commute with b? [4]

(ii) Hence, or otherwise, find the set H of those elements of G which commute with all elements of G. List the elements of H in standard form. [5]

(b) Find all the left cosets of H; for each left coset xH, you should give the element x in standard form. (You may *assume* that H is a subgroup of G.) [5]

(c) Show that H is a normal subgroup of H. [4]

(d) Is the quotient group G/H cyclic? If so, find a generator; if not, explain why not. [5]

(e) To which Abelian group is the quotient group G/H isomorphic? Give brief reasons for your answer. [2]