

Question 4 (Unit 2) - 30 marks

For each of the following sets, with the binary operation given, decide whether it forms a group, justifying your answer.

- (a) $(\{1, 3, 5, 9, 11, 13\}, \times_{14})$ [12]
 (b) $(\mathbb{Z}, *)$ where $x * y = x - y + 1$ [4]
 (c) $(\mathbb{Z}, *)$ where $x * y = x + y - 1$ [14]

TMA M203 02

Cut-off date 14 March 1996

(Introduction Block)**Question 1 (Unit 3) - 20 marks**

- (a) (i) Show that the curve given by the parametric equations

$$x = a + r \cos t, \quad y = b + r \sin t \quad (t \in \mathbb{R}),$$

is a circle with centre (a, b) and radius r . [2]

- (ii) Show that the gradient of the tangent to the circle in part (i) at the point (x_1, y_1) is $-\frac{(x_1 - a)}{y_1 - b}$. [3]

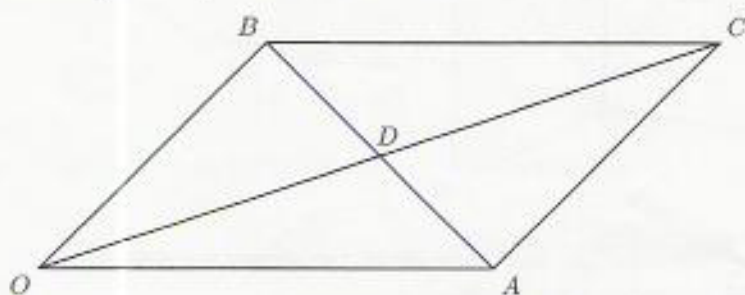
- (b) This part of the question concerns the circle C given by

$$C = \{(x, y) : x^2 + y^2 - 5x - 2y - 9 = 0\}.$$

- (i) Find the centre and radius of the circle C . [2]
 (ii) Verify that the points $P(-1, 3)$ and $Q(2, 5)$ lie on the circle C and find the equation of the line through P and Q . [3]
 (iii) Find the equations of the tangents to the circle C at the points P and Q and the coordinates of their point of intersection, R . [7]
 (iv) Verify that $RP = RQ$, i.e. that the tangents from R are of equal length. [3]

Question 2 (Unit 3) - 20 marks

- (a) In this part of the question you are asked to prove by vector methods that the diagonals of a parallelogram bisect one another.



The parallelogram $OACB$ has vertices at O (the origin) and at points A , B and C with position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively. The diagonals OC and AB intersect at D with position vector \mathbf{d} .

- (i) Write down \mathbf{c} in terms of \mathbf{a} and \mathbf{b} . [1]
 (ii) Write down the position vector of a general point on the line OC . [1]
 (iii) Write down the position vector of a general point on the line AB . [1]
 (iv) Show that D is the midpoint of OC and of AB , i.e. that the diagonals bisect one another. [5]