(b) This part of the question concerns the two vectors in $\mathbb{R}^3$, $\mathbf{p} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{q} = 6\mathbf{i} - 6\mathbf{j} + \mathbf{k}$.

(i) Find the angle, θ, between $\mathbf{p}$ and $\mathbf{q}$, giving your answer in radians to two decimal places. [3]

(ii) Find a vector $\mathbf{n}$ of length 7 perpendicular to both $\mathbf{p}$ and $\mathbf{q}$. [6]

(iii) Find the equations of the planes which are parallel to the plane (through the origin) containing $\mathbf{p}$ and $\mathbf{q}$ and distance 7 from that plane. [*Hint:* Use your answer to part (ii) to find the normal to the plane and a point in the plane.] [3]

11 θ -7

$\sin^{-1} \frac{11}{\sqrt{170}}$

$\cos^{-1}$

Question 3 (Unit 3) – 10 marks

In this question z_1 and z_2 are the complex numbers

$$z_1 = 1 + 2i, \quad z_2 = 3 - 5i.$$

(i) Write down $z_2 - z_1$, and $\overline{z}_1 z_2$. [2]

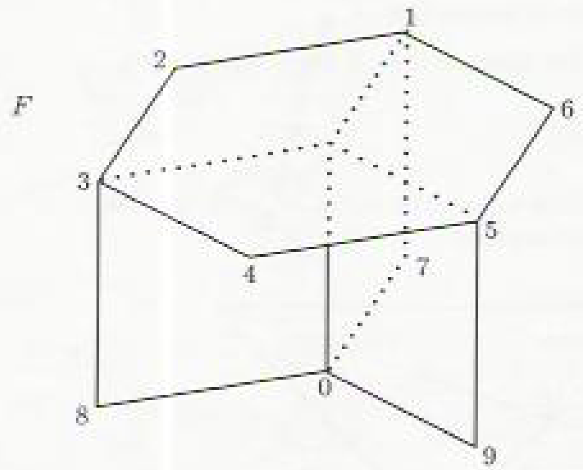
(ii) Find a complex number w, in the form $a + ib\,(a, b \in \mathbb{R})$, such that

$$w\left(\frac{z_2 - z_1}{1 - \overline{z}_1 z_2}\right) = i. \qquad [5]$$

(iii) Express z_1, z_2 and z_1/z_2 in polar form, using the principal value of the arguments in radians correct to 3 decimal places. [3]

Question 4 (Unit 4) – 25 marks

A low table, F, consists of a regular hexagonal top, with vertices labelled 1, 2, 3, 4, 5, 6 supported by three vertical rectangular 'legs' whose inner edges meet at the centre of the hexagon and whose outer edges meet the hexagon at vertices 1, 3 and 5. The outer edges of the legs are labelled 7, 8 and 9 at the base corners and the centre of the base is labelled 0.



(a) Write down, using two-line notation, the elements of the symmetry group $S(F)$, and describe each symmetry geometrically. [9]

(b) Identify each of your symmetries by a single letter, and use these letter symbols to construct a Cayley table for $S(F)$. [8]

(c) The group $S(F)$ is isomorphic to one of the following groups: $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{Z}_6$, $S(\triangle)$, $S(\square)$, $S(\bigcirc)$. Decide to which group $S(F)$ is isomorphic, and write down an isomorphism between $S(F)$ and that group. [8]