Question 5 (Unit 4) – 25 marks

(a) Determine which of the following groups are cyclic, justifying your answer in each case, and give all the generators of those that are cyclic. (You are NOT asked to prove that any of the following are groups.)

(i) $(\{1, 11, 19, 29\}, \times_{30})$

(ii) $(\{1, 3, 7, 9\}, \times_{10})$

(iii) $(\{1, 5, 7, 11, 13, 17\}, \times_{18})$

(iv) $(\{0, 6, 12, 18\}, +_{24})$ *[12]*

(b) Show that two of the groups listed in part (a) are isomorphic and construct an isomorphism between these two groups. *[6]*

(c) Find all the subgroups of the group

$$(\{1, 5, 7, 11, 13, 17\}, \times_{18}).$$ *[7]*

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(Linear Algebra Block)

Question 1 (Unit 1) – 25 marks

This question concerns the following two subsets of $\mathbb{R}^3$:

$$S = \{(1, -1, -4), (3, 2, 3)\},$$
$$T = \{(a, b, 3b - a) : a, b \in \mathbb{R}\}.$$

(a) Show that $S \subseteq T$, and write down a vector in $\mathbb{R}^3$ which does not belong to T. *[3]*

(b) Show that T is a subspace of $\mathbb{R}^3$. *[6]*

(c) Show that S is a basis for T. Write down the dimension of T and describe T geometrically. *[6]*

(d) Find an orthogonal basis for T that includes the vector $(1, -1, -4)$. *[5]*

(e) Express the vector $(a, b, 3b - a)$ of T as a linear combination of the vectors in your orthogonal basis for T. *[5]*

Question 2 (Unit 2) – 20 marks

This question concerns the linear transformation

$$t : \mathbb{R}^2 \longrightarrow \mathbb{R}^4$$
$$(a, b) \longmapsto (a, b - a, 2a + b, a + 3b).$$

(a) Write down the matrix for t with respect to the standard basis in both the domain and the codomain. *[4]*

The set $X = \{(1, 1), (1, -2)\}$ is a basis for $\mathbb{R}^2$ and the set $Y = \{(1, 0, 0, 0), (1, 1, 0, 0), (1, 0, 1, 0), (1, 0, 0, 1)\}$ is a basis for $\mathbb{R}^4$. (You are NOT asked to prove these facts.)

(b) (i) Determine the matrix of t with respect to the basis X in the domain and the standard basis in the codomain. *[5]*

(ii) Determine the matrix of t with respect to the basis X in the domain and the basis Y in the codomain. *[6]*

(c) Let $r : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the rotation of the plane through $\frac{5\pi}{4}$ anticlockwise about the origin.

Determine the matrices of the transformations r and $t \circ r$ with respect to the standard basis in both the domain and the codomain. *[5]*