a_n tends to $-3/2$ as $n \to \infty$ by the quotient rule.

ii) $a_n = \dfrac{4n^3 + 2^n - 3}{2n^3 + 8n^2 + n - 2}$

divide through by 2^n

$a_n = \dfrac{4n^3/(2^n) + 1 - 3/(2^n)}{2n^3/(2^n) + 8n^2/(2^n) + n/(2^n) - 2/(2^n)}$

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Treat the expression as the quotient of two sequences, $b_n = 4n^3/(2^n) + 1 - 3/(2^n)$

and $c_n = 2n^3/(2^n) + 8n^2/(2^n) + n/(2^n) - 2/(2^n)$

Consider b_n. $n^3/2^n$ is a basic null sequence, hence $4n^3/(2^n)$ is a null sequence (multiple rule) $1/2^n$ is a basic null sequence, hence $-3/(2^n)$ is a null sequence (multiple rule). Similarly $2n^3/(2^n)$ and $8n^2/(2^n)$ and $-2/(2^n)$ are null sequences, and $n/2^n$ is a null sequence (since n^p/c^n is a basic null sequence, $|c|>1$). So by the sum rule, numerator tends to 1, denominator tends to 0, $a_n \to \infty$ as $n \to \infty$ by the reciprocal rule.

Reciprocal Rule requires you to look at $\frac{1}{a_n}$ + show $\frac{1}{a_n} \to 0$ & a_n is eventually +ve

b) i) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^3 - 4n + 5}$

$n^3 - 4n + 5 > n^3 - 4n$

$\therefore \dfrac{1}{n^3 - 4n + 5} < \dfrac{1}{n^3 - 4n}$ for $n \geq 3$

Put $\left(\displaystyle\sum_{n=3}^{\infty} \frac{2n}{n^3 - 4n + 5}\right) - a_1 - a_2 < \sum_{n=3}^{\infty} \frac{2n}{n^3 - 4n} = \sum_{n=3}^{\infty} \frac{2}{n^2 - 4}$

a_1 and a_2 are both finite numbers

$a_1 = \dfrac{2 \times 1}{1^3 - 4 + 5} = \dfrac{2}{6-4} = 1$

$a_2 = \dfrac{2 \times 2}{2^3 - 4 \times 2 + 5} = \dfrac{4}{5}$