TMA M203 08 **Cut-off date** 26 September 1996

(Revision Assignment)

Your attention is drawn to Section 7.4.6 of the *Student Handbook*, which reads as follows: 'A TMA with a cut-off date *before 1 October* will not be marked, in any circumstances, *if it is submitted later than 1 October*.'

Question 1 — 10 marks

Let $S = \{(0,2,1),(1,1,1),(3,-1,1)\}$ be a set of three vectors in $\mathbb{R}^3$ and let

$$T = \{(x,y,z) \in \mathbb{R}^3 : x + y - 2z = 0\}.$$

You may assume that T is a subspace of $\mathbb{R}^3$.

(a) Show that S is a linearly dependent set, but that $\{(0,2,1),(1,1,1)\}$ is a linearly independent set. [3]

(b) Prove that $\{(0,2,1),(1,1,1)\}$ is a basis for T. [5]

(c) Hence or otherwise show that $< S > = T$. [2]

Question 2 — 10 marks

This question concerns the linear transformation $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ which is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \end{pmatrix}$$

with respect to the standard bases in the domain and codomain.

(a) Find the transition matrix, $\mathbf{P}$, from the basis

$$X = \{(1,1,2),(-2,1,0),(3,1,2)\}$$

to the standard basis for $\mathbb{R}^3$.

Hence find the matrix of f with respect to the basis X in the domain and the standard basis in the codomain. [4]

(b) Find the transition matrix, $\mathbf{Q}$, from the basis

$$Y = \{(3,4),(1,2)\}$$

to the standard basis for $\mathbb{R}^2$.

Hence find the matrix of f with respect to the standard basis in the domain and the basis Y in the codomain. [4]

(c) Use the results of parts (a) and (b) to find the matrix of f with respect to the basis X in the domain and the basis Y in the codomain. [2]