$(x, y, (x+y)/2)$. We therefore have to show that any vector in T can be expressed as a linear combination of $(0,2,1), (1,1,1)$ (we have already shown that these vectors are linearly independent)

$$a\begin{pmatrix}0\\2\\1\end{pmatrix} + b\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}x\\y\\(x+y)/2\end{pmatrix}$$

$$b = x \quad ①$$
$$2a + b = y \quad ②$$
$$a + b = \tfrac{1}{2}(x+y) \quad ③$$

① implies $b = x$. Sub into ② and ③

$2a + x = y \qquad a + x = \frac{1}{2}(x+y)$

$a = (y-x)/2 \qquad a = \frac{1}{2}(y-x)$

These are equivalent. Any vector in T can be expressed as a linear combination of $(0,2,1), (1,1,1)$ in only one way. The set is also linearly independent, hence the set $(0,2,1), (1,1,1)$ is a basis for T.

c) Any vector in T can be expressed as a linear combination of $\{(0,2,1), (1,1,1)\}$, which is a linearly independent set. The set is a basis for T, hence the set spans T. Also from (a) $(3,-1,1) \in$ span of $(0,2,1) + (1,1,1)$. Hence $\langle S \rangle = T$.

$\frac{9}{10}$

2)a)

$$P = \begin{pmatrix}1 & -2 & 3\\1 & 1 & 1\\2 & 0 & 2\end{pmatrix}$$

$B = AP$

$$= \begin{pmatrix}2 & -1 & 1\\1 & 3 & -2\end{pmatrix}\begin{pmatrix}1 & -2 & 3\\1 & 1 & 1\\2 & 0 & 2\end{pmatrix} = \begin{pmatrix}3 & -5 & 7\\0 & 1 & 2\end{pmatrix}$$