If you continue to write fractions as $\frac{a}{b}$ instead of $\frac{a}{b}$ you will lose marks!!

$$\left(\sum_{n=1}^{\infty} \frac{2n}{n^3-4n+5}\right) - 1 - \frac{4}{5} < \sum_{n=3}^{\infty} \frac{2}{n^2-4}$$

$$\sum_{n=1}^{\infty} \frac{2n}{n^3-4n+5} < \frac{9}{5} + \sum_{n=3}^{\infty} \frac{2}{n^2-4} = \frac{9}{5} + \sum_{n=3}^{\infty} \frac{1}{2(n-2)} - \frac{1}{2(n+2)}$$

$$\sum_{n=3}^{\infty} \frac{1}{2(n-2)} - \frac{1}{2(n+2)} = \sum_{n=3}^{\infty} \frac{1}{2(n-2)} - \sum_{n=3}^{\infty} \frac{1}{2(n+2)} =$$

In the first series put $i = n-2$, in the second put $i = n+2$, then

$$\sum_{n=3}^{\infty} \frac{1}{2(n-2)} - \sum_{n=3}^{\infty} \frac{1}{2(n+2)} = \sum_{i=1}^{\infty} \frac{1}{2i} - \sum_{i=5}^{\infty} \frac{1}{2i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \sum_{i=5}^{\infty} \frac{1}{2i} - \sum_{i=5}^{\infty} \frac{1}{2i}$$

$$= 1/2 + 1/4 + 1/6 + 1/8 < 1.$$

Use Limit Comparison Test with $\sum_{n=1} \frac{1}{n^2}$

Hence series converges.

ii) $\sum_{n=1}^{\infty} \frac{n+\sin n}{4^n}$ $\quad \sin n < n$, so $\sum_{n=1}^{\infty} \frac{n+\sin n}{4^n} < \sum_{n=1}^{\infty} \frac{n+n}{4^n} = 2\sum_{n=1}^{\infty} \frac{n}{4^n}$

The series is now a multiple of a basic convergent series of the form n^p/c^n, $c>1$. A multiple of a convergent series is also a convergent series, hence the series converges by the comparison test

4/5

5/10