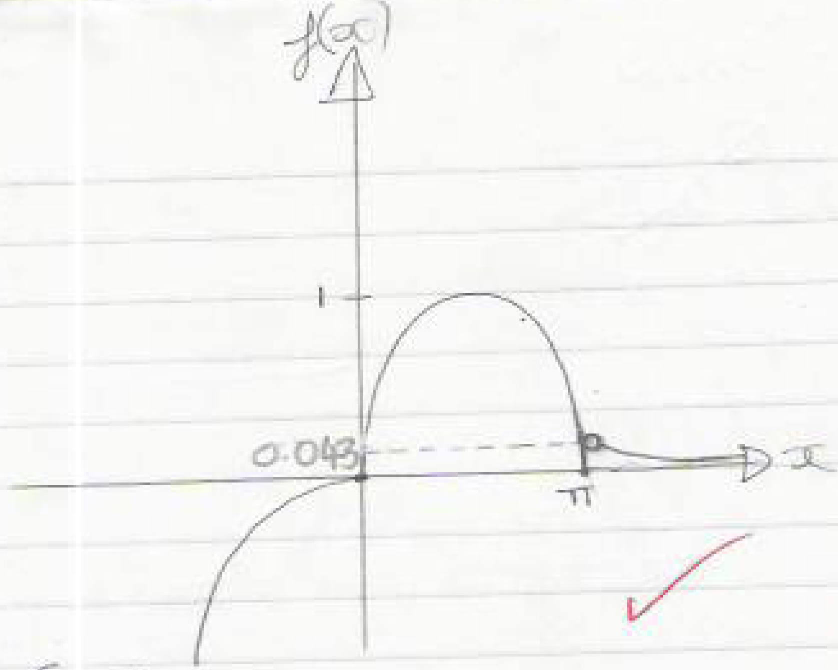
4)



✓ 2½

b) $f(x) = \begin{cases} x^3 & x<0 \\ \sin 2x & 0 \leq x \leq \pi \\ e^{-x} & x>\pi \end{cases}$

A function is continuous at a if for each sequence $\{x_n\}$ on the interval over which $f(x)$ is defined, $f(x_n) \to f(a)$ ✓

Consider

$b_n = \pi + 1/n$

$f(\pi) = \sin 2\pi$ ✓
$= 0$

Consider $b_n = \pi + 1/n$ ✓

$f(b_n) = f(\pi + 1/n) = e^{-(\pi + 1/n)} = e^{-\pi} e^{-1/n}$ ✓ 2½

as $n \to \infty$, $e^{-1/n} \to 1$, $f(\pi + 1/n) \to e^{-\pi}$, 0 ✓

Hence $f(\pi + 1/n) \not\to f(\pi)$, so the function is discontinuous at $x = \pi$. ✓

c) $f(x) = \begin{cases} x^3 & x<0 \\ \sin 2x & 0 \leq x \leq \pi \\ e^{-x} & x>\pi \end{cases}$

we can use Local Rule

$f(x)$ is continuous of $]-\infty, 0[$, since on this interval $f(x) = x^3$, a polynomial, and a basic continuous function on $[0, \pi]$, $f(x) = \sin 2x$, a trigonometric function, which are all contin-