M203 TMA 08

1) a) If S is a linearly dependent set there is a solution to the problem (such that a, b, c are not all equal to zero)

$$a\begin{pmatrix}0\\2\\1\end{pmatrix}+b\begin{pmatrix}1\\1\\1\end{pmatrix}+c\begin{pmatrix}3\\-1\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

ie
$$b+3c=0 \quad (1)$$
$$2a+b-c=0 \quad (2)$$
$$a+b+c=0 \quad (3)$$

Equation (1) implies $b=-3c$

sub $b=-3c$ into (2) and (3) giving

$2a-4c=0$ and $a-2c=0$

$a=2c$, $b=-3c$, $c=c$

c can take any value, so if $c=1$, $b=-3$, $a=2$ then the equation has non-zero solution, and is therefore linearly dependent.

If $\{(0,2,1),(1,1,1)\}$ is linearly independent, there is no solution to (for which a, b are not both zero)

$$a\begin{pmatrix}0\\2\\1\end{pmatrix}+b\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$b=0 \quad (4)$$
$$2a+b=0 \quad (5)$$
$$a+b=0 \quad (6)$$

(4) implies $b=0$

sub $b=0$ into (5) and (6)

$2a=0$ $\quad a=0$

There're no solutions to the equations for which a, b are not both zero, hence $\{(0,2,1)(1,1,1)\}$ is a linearly independent set.

b) $T=\{(x,y,z)\in\mathbb{R}^3 : x+y-2z=0\}$

Any vector in T can be expressed as