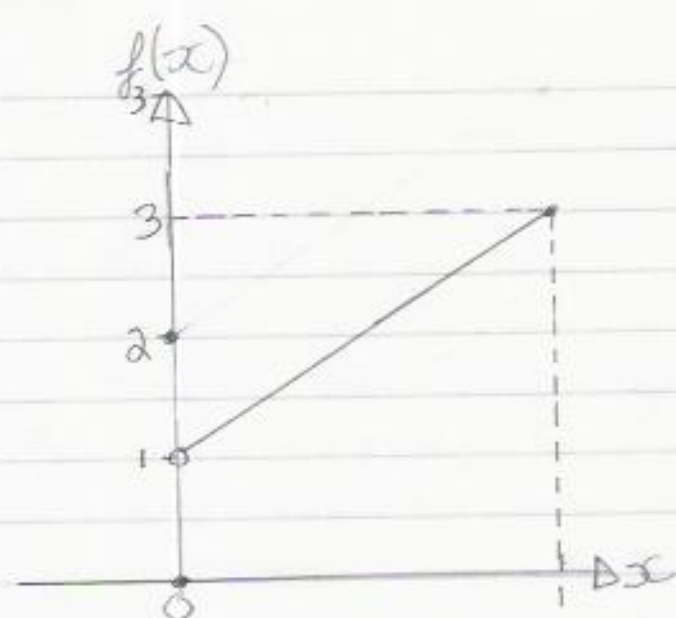


9) a)



$$b) L(f, P_n) = \sum_{i=1}^n m_i \Delta x_i$$

$f(x)$ is increasing on $[0, 1]$, hence $m_i = f(x_{i-1})$

$$\begin{aligned} L(f, P_3) &= \sum_{i=1}^3 f\left(\frac{i-1}{3}\right) \Delta x \\ &= \frac{1}{3} f(0) + \frac{1}{3} f\left(\frac{1}{3}\right) + \frac{1}{3} f\left(\frac{2}{3}\right) \\ &= \frac{1}{3} + \frac{1}{3} \times \left(1 + \frac{2}{3}\right) + \frac{1}{3} \left(1 + \frac{4}{3}\right) \\ &= \frac{1}{3} + \frac{5}{9} + \frac{7}{9} = \frac{5}{3} \end{aligned}$$

$$U(f, P_n) = \sum_{i=1}^n M_i \Delta x_i$$

$f(x)$ is increasing on $[0, 1]$ except for $x=0$ where $f(x) = 2$, so $M_i = f(x_i)$

$$\begin{aligned} U(f, P_3) &= \frac{1}{3} f\left(\frac{1}{3}\right) + \frac{1}{3} f\left(\frac{2}{3}\right) + \frac{1}{3} f(1) \\ &= \frac{1}{3} \times \left(1 + \frac{2}{3}\right) + \frac{1}{3} \times \left(1 + \frac{4}{3}\right) + \frac{1}{3} \times 3 \\ &= \frac{2}{3} + \frac{7}{9} + 1 = \frac{20}{9} \end{aligned}$$

$$c) L(f, P_n) = 2n - 1 = 2 - \frac{1}{n}$$

$$\text{as } n \rightarrow \infty, L(f, P_n) \rightarrow 2$$