

$$7) \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 3 & 3 & 2 \\ 4 & 0 & -1 \end{pmatrix} t_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow{t_2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 1 & -1 & 2 \\ 2 & -2 & 1 \end{pmatrix}$$

$$t_1: \begin{pmatrix} 2a & b & 3c \\ -a & 2b & 3c \\ a & 4b & 2c \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \begin{cases} 2a+b+3c=4 & (1) \\ -a+2b+3c=0 & (2) \\ a+4b+2c=-1 & (3) \end{cases}$$

$$(2)+(3)$$

$$6b+5c=-1$$

$$b=(-1-5c)/6$$

$$(1)+2 \times (2)$$

$$5b+9c=4$$

$$\text{sub } b=(-1-5c)/6 \text{ into } 5b+9c=4$$

$$5(-1-5c)/6+9c=4$$

$$-5-25c+54c=24 \Rightarrow c=1$$

$$\text{and } b=(-1-5c)/6=(-1-5)/6=-1$$

$$\text{from (2) } a=2b+3c=-2+3=1$$

$$t_1 = \begin{pmatrix} 2 & -1 & 3 \\ -1 & -2 & 3 \\ 1 & -4 & 2 \end{pmatrix}$$

To find t_1^{-1}

$$\begin{pmatrix} 2 & -1 & 3 & 1 & 0 & 0 \\ -1 & -2 & 3 & 0 & 1 & 0 \\ 1 & -4 & 2 & 0 & 0 & 1 \end{pmatrix} \equiv \begin{pmatrix} 1 & -1/2 & 3/2 & 1/2 & 0 & 0 \\ -1 & -2 & 3 & 0 & 1 & 0 \\ 1 & -4 & 2 & 0 & 0 & 1 \end{pmatrix} \equiv \begin{pmatrix} 1 & -1/2 & 3/2 & 1/2 & 0 & 0 \\ 0 & -5/2 & 9/2 & 1/2 & 1 & 0 \\ 0 & -7/2 & 1/2 & -1/2 & 0 & 1 \end{pmatrix}$$

$$\equiv \begin{pmatrix} 1 & -1/2 & 3/2 & 1/2 & 0 & 0 \\ 0 & 1 & -9/5 & -1/5 & -2/5 & 0 \\ 0 & 7/2 & -1/2 & 1/2 & 0 & -1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 3/5 & 2/5 & -1/5 & 0 \\ 0 & 1 & -9/5 & -1/5 & -2/5 & 0 \\ 0 & 0 & 29/5 & 6/5 & 7/5 & -1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 3/5 & 2/5 & -1/5 & 0 \\ 0 & 1 & -9/5 & -1/5 & -2/5 & 0 \\ 0 & 0 & 1 & 6/29 & 7/29 & 5/29 \end{pmatrix}$$

$$\equiv \begin{pmatrix} 1 & 0 & 0 & 8/29 & -10/29 & 3/29 \\ 0 & 1 & 0 & 5/29 & 1/29 & 1/29 \\ 0 & 0 & 1 & 6/29 & 7/29 & 5/29 \end{pmatrix}$$

$$t_1^{-1} = \frac{1}{29} \begin{pmatrix} 8 & -10 & 3 \\ 5 & 1 & 1 \\ 6 & 7 & 5 \end{pmatrix}$$