

Question 9 – 10 marks

Let f be the function with domain $[0, 1]$ given by

$$f(x) = \begin{cases} 2, & x = 0, \\ 1 + 2x, & 0 < x \leq 1, \end{cases}$$

and let P_n denote the standard partition of $[0, 1]$, where $n > 1$.

(a) Sketch the graph of f .

[1]

(b) Evaluate $L(f, P_3)$ and $U(f, P_3)$.

[6]

You may assume now that, for any $n > 1$,

$$L(f, P_n) = \frac{2n-1}{n} \quad \text{and} \quad U(f, P_n) = \frac{2(n^2+n-1)}{n^2}.$$

(c) Show that f is integrable on $[0, 1]$, and evaluate $\int_0^1 f$.

[3]

Question 10 – 10 marks

Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3}{n} \left(\frac{4}{5}\right)^n (x+2)^n.$$

[10]