$$U(f,P_n) = \frac{2n^2+2n-2}{n^2} = 2 + \frac{2}{n} - \frac{2}{n^2}$$???

As $n \to \infty$ $\frac{2}{n} \to 0$, $\frac{-2}{n^2} \to 0$

$\therefore U(f,P_n) \to 2$

$U(f,P_n)$ and $L(f,P_n)$ tend to the same limit on $[0,1]$, so f is integrable on $[0,1]$ and $\int_0^1 f = 2$. since $\|P_n\| \to 0$ as $n \to \infty$

(N13)

2/3

$\frac{9}{10}$

10) $\sum_{n=1}^{\infty} \frac{3}{n}\left(\frac{4}{5}\right)^n (x+2)^n$

??

Use the ratio test

$$\frac{a_{n+1}}{a_n} = \frac{3}{n+1}\left(\frac{4}{5}\right)^{n+1} \Big/ \left(\frac{3}{n}\left(\frac{4}{5}\right)^n\right) = \frac{n}{n+1} \times \frac{4}{5} = \frac{4}{5}\left(\frac{n}{n+1}\right)$$

as $n \to \infty$ $\frac{a_{n+1}}{a_n} \to \frac{4}{5}$

$$\frac{5}{4} \leq |x+2| \quad \text{ie} \quad -\frac{5}{4} \leq x+2 \leq \frac{5}{4}$$

$$-\frac{13}{4} \leq x \leq -\frac{3}{4}$$

To find the behaviour of the series at $x = -\frac{13}{4}$

$$\sum_{n=1}^{\infty} \frac{3}{n}\left(\frac{4}{5}\right)^n \left(-\frac{13}{4}+2\right)^n = \sum_{n=1}^{\infty} \frac{3}{n}\left(\frac{4}{5}\right)^n \left(\frac{-5}{4}\right)^n = \sum \frac{3(-1)^n}{n}$$

$3/n$ is a null series, with positive terms tending to zero. $3(-1)^n/n$ is an alternating series with terms tending to zero. The series therefore converges by the alternating test, since the series is null.

Need $3 \times (-1)$ terms known cgt series.

At $x = -3/4$

$$\sum_{n=1}^{\infty} \frac{3}{n}\left(\frac{4}{5}\right)^n \left(-3/4+2\right)^n = \sum_{n=1}^{\infty} \frac{3}{n}\left(\frac{4}{5}\right)^n \left(\frac{5}{4}\right)^n = \sum_{n=1}^{\infty} \frac{3}{n} = 3\sum_{n=1}^{\infty} \frac{1}{n}$$