b) $Q = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$

The matrix of f w.r.t. the standard basis in the domain and the basis Y in the codomain is given by

$C = Q^{-1}A$

$= \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \end{pmatrix}$

$= \begin{pmatrix} 3/2 & -5/2 & 2 \\ -5/2 & 13/2 & -5 \end{pmatrix}$

c) The matrix of f wrt the new bases in domain and codomain is given by

$D = Q^{-1}AP$

$= \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ 1 & 3 & -3 \end{pmatrix}\begin{pmatrix} 1 & -2 & 3 \\ 1 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 3 & -5 & 7 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -11/2 & 6 \\ -6 & 23/2 & -11 \end{pmatrix}$

$\frac{10}{10}$

3) i) $a_n = \dfrac{2n^4 + 3(n!) + 5^n}{4n^4 - 2(n!) + 3^n}$ ← NB

Divide through by $(n!)$

$a_n = \dfrac{2n^4/(n!) + 3 + 5^n/(n!)}{4n^4/(n!) - 2 + 3^n/(n!)} = \dfrac{2n^4/(n!) + 5^n/(n!) + 3}{4n^4/(n!) + 3^n/(n!) - 2}$

$n^p/(n!)$ and $5^n/n!$ are basic null sequences and a constant multiple of a basic null sequence is also a ~~basic~~ null sequence (multiple rule). The sum of two basic null sequences is also a null sequence (sum rule). If we treat the sequence as the quotient of two sequences, the top tends to 3 by the combination rules, and the bottom tends to -2, hence