$g'(t) = \frac{d}{dt}(c_2 e^{3t} + d_2 e^{-2t}) = 3c_2 e^{3t} - 2d_2 e^{-2t}$

Sub into ① and ②

$3c_1 e^{3t} - 2d_1 e^{-2t} = 5c_1 e^{3t} + 5d_1 e^{-2t} - 2c_2 e^{3t} - 2d_2 e^{-2t}$

equating for powers of e

$e^{3t}(3c_1 - 5c_1 + 2c_2) = 0 \Rightarrow c_1 = c_2$

$e^{-2t}(-2d_1 - 5d_1 + 2d_2) = 0 \Rightarrow d_2 = 7d_1/2$

Hence $f(t) = c_1 e^{3t} + d_1 e^{-2t}$

$g(t) = c_1 e^{3t} + \frac{7d_1}{2} e^{-2t}$

f) $\alpha(0) = (1, 2)$

$1 = c_1 + d_1$ ①

$2 = c_1 + \frac{7d_1}{2}$ ②

② - ① $\Rightarrow$ $1 = \frac{5d_1}{2} \Rightarrow d_1 = \frac{2}{5}$

6/6.

$c_1 = 1 - d_1 = 1 - 2/5 = 3/5$

$\alpha : \mathbb{R} \to \mathbb{R}$

$t \to \left(\frac{3}{5} e^{3t} + \frac{2}{5} e^{-2t}, \frac{3}{5} e^{3t} + \frac{7}{5} e^{-2t}\right)$

25/25