$$I_{n+1} - I_n = -\left([uv]_0^1 - \int_0^1 v\,du \right)$$

$$= -\left(\left[\frac{-x^2(1+x^2)^{-n}}{2nx} \right]_0^1 - \int_0^1 \frac{-2x(1+x^2)^{-n}}{2nx} \right)$$

$$= \left[\frac{x(1+x^2)^{-n}}{2n} \right]_0^1 - \int_0^1 \frac{(1+x^2)^{-n}}{n}\,dx$$

$$= \left[\frac{1 \times (2)^{-n}}{2n} - 0 \right] - \frac{1}{n} I_n$$

$$= \frac{1}{n2^{n+1}} - \frac{1}{n} I_n$$ Multiply through by $2n$

$$2nI_{n+1} - 2nI_n = \frac{1}{2^n} - 2I_n$$

$$2nI_{n+1} = \frac{1}{2^n} + (2n-2)I_n$$ ✓ 4/6

ii) $2nI_{n+1} = \frac{1}{2^n} + (2n-1)I_n$

If $n=2$

$$2 \times 2 \times I_3 = \frac{1}{2^2} + (2 \times 2 - 1)I_2$$

$$I_3 = \frac{1}{4}\left(\frac{1}{4} + 3I_2\right) = \frac{1}{16} + \frac{3}{4}I_2 \quad (1)$$

If $n=1$

$$2 \times 1 \times I_2 = \frac{1}{2^1} + (2 \times 1 - 1)I_1$$

$$I_2 = \frac{1}{2}\left(\frac{1}{2} + I_1\right) = \frac{1}{4} + \frac{I_1}{2} \quad (2)$$

Sub (2) into (1)

$$I_3 = \frac{1}{16} + \frac{3}{4}I_2$$

$$= \frac{1}{16} + \frac{3}{4}\left(\frac{1}{4} + \frac{I_1}{2}\right) = \frac{1}{16} + \frac{3}{16} + \frac{3I_1}{8} = \frac{1}{4} + \frac{3I_1}{8}$$ ✓

$$I_1 = \int_0^1 \frac{dx}{(1+x^2)}$$