ii) Let P_n be the standard partition of $[0,1]$

$\{[0, 1/n], [1/n, 2/n], [3/n, 3/n], \ldots\ldots [\frac{n-1}{n}, 1]\}$

f is increasing, therefore

$m_i = f(\frac{i-1}{n}), M_i = f(1/n)$

Also $f(1) = 1$.

$$U(f, P_n) = \sum_{i=1}^{n} f(i/n)\,\delta x_i$$

$$= \sum_{i=1}^{n}(1 + \frac{2i}{n})\delta x_i = \frac{1}{n}\sum_{i=1}^{n}\cancel{(1+\frac{2i}{n})} \quad (\text{since } \delta x_i = 1/n)$$

$$= \frac{1}{n}\sum_{i=1}^{n} 1 + \frac{1}{n}\sum_{i=1}^{n}\frac{2i}{n} = \frac{1}{n}\times n + \frac{2}{n^2}\sum_{i=1}^{n} i$$

$$= 1 + \frac{2}{n^2}(1+2+3+4\ldots\ldots n)$$ You must stop this!!

$$= 1 + \frac{2}{n^2}\times\frac{1}{2}n(n+1) = 1 + \frac{n+1}{n} = \frac{n+n+1}{n} = \frac{2n+1}{n}$$

This is ambiguous: is it $\frac{1+n+1}{n}$ or $1 + \frac{n+1}{n}$???

$$\therefore U(f, P_n) = \frac{2n+1}{n}$$

$$L(f, P_n) = \sum_{i=1}^{n} m_i\,\delta x_i$$

$$= \left(\sum_{i=1}^{n-1} f\left(\frac{i-1}{n}\right)\frac{1}{n}\right) + \frac{1}{n} = \left(\frac{1}{n}\sum_{i=1}^{n-1}\left(1 + \frac{2i-2}{n}\right)\right) + \frac{1}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n-1} 1 + \frac{1}{n}\sum_{i=1}^{n-1}\frac{2i}{n} - \frac{1}{n}\sum_{i=1}^{n-1}\frac{2}{n} + \frac{1}{n}$$

$$= \frac{1}{n}\times(n-1) + \frac{1}{n^2}\times 2(1+2+3\ldots+n-1) - \frac{1}{n^2}\times 2(n-1) + \frac{1}{n}.$$

$$= \frac{n-1}{n} + \frac{2\times\frac{1}{2}n(n-1)}{n^2} - \frac{2(n^2-1)}{n^2} + \frac{1}{n}$$

$$= \frac{n-1}{n} + \frac{n-1}{n} - \frac{2(n^2-1)}{n^2} + \frac{1}{n}$$

$$= \frac{1}{n^2}(n^2 - n + n^2 - n - 2n + 2 + n) = \frac{1}{n^2}(2n^2 - 3n + 2)$$

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iii) $\|P_n\| \to 0$ as $n \to \infty$

$\lim_{n\to\infty} L(f, P_n) = \lim_{n\to\infty} U(f, P_n)$ since