Question 3 (Unit 3) – 25 marks

(a) Determine the Taylor polynomial $T_3(x)$ for the function $f(x) = (1+x)^{\frac{1}{2}}$ at 1.

Show that $T_3(x)$ approximates $f(x)$ to within 0.005 on the interval $[1, 2]$. [9]

(b) Determine the interval of convergence of the power series

$$\sum_{n=0}^{\infty}(-1)^n \frac{2^n}{3^n(n+1)}(x-1)^n.$$

State the radius of convergence of the series

$$\sum_{n=0}^{\infty}(-1)^n \frac{2^n}{3^n(n+1)^2}(x-1)^{n+1},$$

explaining your answer briefly. [10]

(c) Use the General Binomial Theorem to determine the first four non-zero terms in the power series for the function defined by $f(x) = (1+3x^5)^{7/11}$ at 0.

State the radius of convergence of this power series. [6]

Question 4 (Unit 4) – 25 marks

This question concerns the flow with velocity function

$$V(x,y) = (5x - 2y, 7x - 4y) \quad ((x,y) \in \mathbb{R}^2).$$

(a) Write down

(i) the matrix $\mathbf{A}$ of the flow, [1]

(ii) a second-order differential equation satisfied by the coordinate functions of any flow function for the flow. [1]

(b) (i) Determine the velocity vector at each of the points

$$(1,0),\ (1,1),\ (1,2),\ (1,3),\ (1,4).$$ [2]

(ii) On a diagram sketch the direction of the flow at each of the points in part (b)(i). [2]

(c) (i) Find the barrier lines for the flow and the direction of the flow on each. [6]

(ii) On another diagram, sketch the barrier lines, indicating the direction of the flow on each. [1]

(d) On a third diagram, sketch a sufficient number of flow lines, indicating the direction of flow on each, to illustrate how the flow behaves. [2]

(e) Find the general solution to the differential equation in part (a)(ii). [4]

(f) Determine the flow function α corresponding to V which satisfies $\alpha(0) = (1,2)$. [6]