Both limits exist, and $\lim_{h \to 0^-} Q(h) = \lim_{h \to 0^+} Q(h)$

hence $f(x)$ is differentiable at 1, and its derivative is given by

$$\lim_{h \to 0^-} Q(h) = \lim_{h \to 0^+} (h) = \frac{f(c+h) - f(c)}{h} = 3 \text{ for } c = 1.$$

The derivative of $f(x)$ at $x = 1$ is $f'(x) = 3$

b) i) $g(x) = x^7 + x^3 + x + 1.$

$g'(x) = 7x^6 + 3x^2 + 1 \geq 1$

hence $g(x)$ is monotonically increasing on it's domain $\mathbb{R}$. $g(x)$ is a polynomial so it is continuous. Since $g(x)$ is differentiable and $g'(x) \geq 0$ for $x \in \mathbb{R}$, by the inverse function rule, $g(x)$ has an inverse function $g^{-1}(x)$, which is strictly increasing on $\mathbb{R}$ and is continuous on $\mathbb{R}$, and $g^{-1}(x)$ is differentiable on $\mathbb{R}$ (since $g(x)$ is differentiable on $\mathbb{R}$ and $g'(x) \neq 0$ for $x \in \mathbb{R}$).

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ii) $g(1) = 4$

$g^{-1}(g(1)) = g^{-1}(4)$

$1 = g^{-1}(4)$

By the inverse function rule

$(g^{-1})'(d) = \frac{1}{g'(c)}$ if the conditions are satisfied, where $d = g(c)$

$g(1) = 4$ so $c = 1$, $d = 4$

$(g^{-1})'(4) = \frac{1}{g'(1)}$

$g'(x) = \frac{d}{dx}(x^7 + x^3 + x + 1) = 7x^6 + 3x^2 + 1$

$g'(1) = 7 \times 1^6 + 3 \times 1^2 + 1 = 11$

$\therefore (g^{-1})(4) = \frac{1}{g'(1)} = \frac{1}{11}$

$\frac{1\frac{1}{2}}{2}$