Substitute $x = \tan y$.

$x = 0 \Rightarrow y = 0$, $x = 1 \Rightarrow y = \tan^{-1} 1 = \pi/4$

$\frac{dx}{dy} = \frac{d}{dy}(\tan y) = \sec^2 y \Rightarrow dx = \sec^2 y\, dy$

$$\int_0^1 \frac{dx}{(1+x^2)} = \int_0^{\pi/4} \frac{\sec^2 y\, dy}{(1+\tan^2 y)} = \int_0^{\frac{\pi}{4}} \frac{\sec^2 y}{\sec^2 y} dy = \int_0^{\frac{\pi}{4}} dy = \frac{\pi}{4}$$

$\therefore I_1 = \pi/4$

$$I_3 = \frac{1}{4} + \frac{3}{8} I_1 = \frac{1}{4} + \frac{3}{8} \times \frac{\pi}{4} = \frac{1}{4} + \frac{3\pi}{32}$$

3/3

$\frac{24}{25}$

3) a) $f(x) = (1+x)^{1/2}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$

$f''(x) = \frac{1}{2} \times -\frac{1}{2}(1+x)^{-3/2} = -\frac{1}{4}(1+x)^{-3/2}$

$f'''(x) = -\frac{1}{4} \times -\frac{3}{2}(1+x)^{-5/2} = \frac{3}{8}(1+x)^{-5/2}$

$f''''(x) = \frac{3}{8} \times -\frac{5}{2}(1+x)^{-5/2} = \frac{-15}{16}(1+x)^{-7/2}$

$$T_n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^n(a)}{n!}(x-a)^n$$

with $n = 3$, $a = 1$

$$T_3 = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \frac{f'''(1)}{6}(x-1)^3$$

$$= (1+1)^{1/2} + \frac{(x-1)}{2(1+1)^{1/2}} - \frac{(x-1)^2}{4(1+1)^{3/2} \times 2} + \frac{3(x-1)^3}{8(1+1)^{5/2} \times 6}$$

$$= \sqrt{2} + \frac{(x-1)}{2\sqrt{2}} - \frac{(x-1)^2}{16\sqrt{2}} + \frac{(x-1)^3}{64\sqrt{2}}$$

$f(x) = T_n(x) + R_n(x)$

where $R_n(x) \leq \frac{M}{(n+1)!}(x-a)^{n+1}$ and $M \geq |f^{n+1}(c)|$

for $n = 3$

$R_3(x) \leq \frac{M}{4!}(x-1)^4$

$f''''(x) = -\frac{15}{16}(1+x)^{-7/2}$