On $[1,2]$, $|f''''(x)| = \left|\frac{-15}{16(1+x)^{7/2}}\right| \leq \left|\frac{15}{16\sqrt{(1+1)^7}}\right| = \frac{15}{128\sqrt{2}}$

so $M = \frac{15}{128\sqrt{2}}$, and $R_3(x) \leq \frac{15}{128\sqrt{2} \times 4!}(x-1)^4 =$

On $[1,2]$ the RHS is maximised when $x=2$

$R_3(x) \leq \frac{15}{128\sqrt{2}\times 4!} \times 1^4 \approx 3.45E-3 < 0.005$ 9/9

(b) $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n(n+1)}(x-1)^n$

Use the Ratio Test for Radius of Convergence

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{n+1}2^{n+1}}{3^{n+1}(n+2)} \Big/ \frac{(-1)^n 2^n}{3^n(n+1)}\right|$

$= \left|(-1)^{n+1}\left(\frac{2}{3}\right)^{n+1} \frac{1}{(n+2)} \times \left(\frac{3}{2}\right)^n \frac{(n+1)}{(-1)^n}\right|$

$= \left|-\left(\frac{2}{3}\right)\frac{(n+1)}{(n+2)}\right| \to \frac{2}{3}$ as $n \to \infty$

To find the interval of convergence, we have to find the behaviour of the series at the endpoints. The radius of convergence is $1/(2/3) = 3/2$. The series converges for $|x-1| < 3/2$

ie $-3/2 < x-1 < 3/2$

$-1/2 < x < 5/2$

The endpoints are $x = -1/2$ and $x = 5/2$.

At $x = -1/2$

$\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n} \frac{(x-1)^n}{(n+1)} = \sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{3}\right)^n \times \left(\frac{-3}{2}\right)^n \times \frac{1}{(n+1)}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \dots + \frac{1}{n+1}$

$= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7}\right) + \left(\frac{1}{8} + \frac{1}{9} + \dots\right) + \dots$

$+ \left(\frac{1}{2^{k+1}} + \frac{1}{2^k+2} + \dots + \frac{1}{2^k+2^k}\right) + \dots$