or equivalently

$$T(\vec{x}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

for $A_1' = \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$

$3 = b + e \quad 4 = d + f$

for B $\begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -1 \\ 0 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$

$-1 = -a + e \quad 5 = -c + f$

for C $\begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$

$6 = a + e \quad -2 = c + f$

We have six simultaneous equations

① $3 = b + e$ ② $4 = d + f$

③ $-1 = -a + e$ ④ $5 = -c + f$

⑤ $6 = a + e$ ⑥ $-2 = c + f$

③ + ⑤ $5 = 2e \Rightarrow e = 5/2$

④ + ⑥ $3 = 2f \Rightarrow f = 3/2$

Put $e = 5/2$ into ⑤

$6 = a + 5/2 \Rightarrow a = 6 - 5/2 = 7/2$.

Put $e = 5/2$ into ①

$3 = b + 5/2 \Rightarrow b = 3 - 5/2 = 1/2$.

Put $f = 3/2$ into ②

$4 = d + 3/2 \Rightarrow d = 4 - 3/2 = 5/2$.

Put $f = 3/2$ into ⑥

$-2 = c + 3/2 \Rightarrow c = -2 - 3/2 = -7/2$.

5/5

Then $t(\vec{x}) = \begin{pmatrix} 7/2 & 1/2 \\ -7/2 & 5/2 \end{pmatrix}\vec{x} + \begin{pmatrix} 5/2 \\ 3/2 \end{pmatrix}$

For check P.T.O.

d) $X = (0, 1/3)$

$X' = t(X) = \begin{pmatrix} 7/2 & 1/2 \\ -7/2 & 5/2 \end{pmatrix}\begin{pmatrix} 0 \\ 1/3 \end{pmatrix} + \begin{pmatrix} 5/2 \\ 3/2 \end{pmatrix}$