$$(ABCU) = \frac{\beta}{\alpha} \Big/ \frac{\delta}{\gamma}$$

$$= \left(\frac{1/a}{(a^2-2a+1)/a}\right) \Big/ \left(\frac{1/a}{(1-a)/a}\right)$$

$$= \frac{1}{(a^2-2a+1)} \Big/ \frac{1}{(1-a)}$$

$$= \frac{1}{(a-1)(a-1)} \Big/ \frac{1}{(1-a)}$$

$$= \frac{-1}{(a-1)} = \frac{1}{(1-a)}$$

6/6

19/20

5) a) i) With $(x_1, y_1) = (-1, 0)$ $(x_2, y_2) = (3, 4)$.
the ratios are the values of k for which

$$S_{22}k^2 + 2S_{12}k + S_{11} = 0.$$

where $S_{11} = Ax_1^2 + Bx_1y_1 + Cy_1^2 + Fx_1 + Gy_1 + H$

$$= 3(-1)^2 + (-2)(-1\times 0) + 3(0)^2 + 2\times -1 + (-6)\times 0 + 2$$

$$= 3 + 0 + 0 - 2 + 0 + 2 = 3.$$

$$S_{12} = 3(-1\times 3) + (-2)(-1\times 4 + 0\times 3)/2 + 3(0\times 4) + 2(-1+3)/2 - 6(0+4)/2 + 2$$

$$= -9 + 4 + 2 - 12 + 2 = -13$$

$$S_{22} = 3(3)^2 + (-2)(3\times 4) + 3(4)^2 + 2\times 3 - 6\times 4 + 2$$

$$= 27 - 24 + 48 + 6 - 24 + 2 = 35$$

$$S_{22}k^2 + 2S_{12}k + S_{11} = 35k^2 - 26k + 3 = 0.$$

$$(7k-1)(5k-3) = 0$$

$$k = 1/7,\ k = 3/5$$

Hence conic divides line segment in ratios

$1/7 : 1$ and $3/5 : 1$

or $1:7$ and $3:5$

3/3

ii) The point nearest to $(-1,0)$ is given by $(-1,0) + \frac{1}{8}((3,4) - (-1,0))$

$$= (-1,0) + \frac{1}{8}(4,4) = (-1,0) + (1/2, 1/2)$$

$$= (-1/2, 1/2)$$

3/3