(a) Consider the case where $A$ is $(0,1)$, $B$ is $(-1,0)$ and $C$ is $(1,0)$. Find the coordinates of $P$ and $Q$ and hence find the ratios $\dfrac{BX}{XQ}$ and $\dfrac{AX}{XP}$. [3]

(b) Deduce from your result to part (a) that the point, $X$, where the medians of *any* triangle $ABC$ meet, lies $\frac{2}{3}$ of the way along the line segments from each of the points $A$, $B$ and $C$ to the midpoints ($P$, $Q$ and $R$) of the opposite sides. You should state any results or theorems that you use in reaching this conclusion. [5]

(c) Find the affine transformation that maps the points $A$, $B$ and $C$ from part (a) to the points $A'(3,4)$, $B'(-1,5)$, and $C'(6,-2)$. [5]

(d) Find the image, $X'$, of the centroid of the triangle $ABC$ under this transformation. [2]

(e) Find the midpoint, $P'$, of the line $B'C'$ and verify that $X'$ does divide the line $A'P'$ in the ratio $2:1$. [5]

**Question 2** (Unit 2) – 25 marks

(a) (i) $C_1$ and $C_2$ are the circles punctured at the origin given by the equations

$$C_1 : x^2 + y^2 - 3y = 0$$
$$C_2 : x^2 + y^2 + \tfrac{3}{2}x - y = 0.$$

Find the equations of the images of $C_1$ and $C_2$ under inversion in the unit circle and draw a sketch showing both the circles and their images. [7]

(ii) The clockwise angle of intersection, $\alpha'$, from the image of $C_1$ to the image of $C_2$ in part (i) is the image of one of the angles $\alpha$ between $C_1$ and $C_2$. Mark $\alpha$ and $\alpha'$ on your sketch from part (i). Calculate the angle $\alpha'$ using the equations of the images, and hence determine the anticlockwise angle $\alpha$ from $C_1$ to $C_2$. [5]

(iii) Write down the equations of the images of the circles punctured at the origin, given by the equations

$$C_3 : x^2 + y^2 - 2ax - 2by = 0$$
$$C_4 : x^2 + y^2 - 2a'x - 2b'y = 0$$

under inversion in the unit circle. If $C_3$ and $C_4$ touch at the origin, show that

$$\frac{a}{b} = \frac{a'}{b'}.$$ [5]

(b) Find the Möbius transformation, $M$, that maps $z_1 = 2 - i$ to $0$, $z_2 = 1 + 3i$ to $1$ and $\infty$ to $\infty$. Describe the effect of the transformation $M$ on the line through $z_1$ and $z_2$. Use $M$ to determine whether the point $z_3 = -\frac{1}{2} + 9i$ is collinear with $z_1$ and $z_2$, justifying your answer. (You should explain your method and conclusion rather than just quote the result of a strategy.) [8]

**Question 3** (Unit 3) – 15 marks

Denote by $K$ the Euclidean circle given by the equation

$$x^2 + y^2 + x - y + \tfrac{7}{16} = 0.$$

(a) Express $K$ as a Euclidean circle in $\mathcal{D}$ in the form

$$K = \{z : |z - \alpha| = r\},$$

where $\alpha$ is a complex number and $r$ is a real number, and where $z = x + yi$. [2]

(b) Find the non-Euclidean centre and radius of $K$. [13]