

iii) tangent given by $S_1 = 0$ at $(x_1, y_1) = (-1/2, 1/2)$

$$S_1 = Axx + B(x_1y + xy_1)/2 + Cyy + F(x_1 + x)/2 + G(y_1 + y)/2 + H$$

$$= 3(-1/2x) + (-2)(-1/2y + x/2)/2 + 3(1/2y) + 2(-1/2 + x)/2$$

$$+ (-6)(1/2 + y)/2 + 2$$

$$= -3x/2 + y/2 - x/2 + 3y/2 - 1/2 + x/2 - 3/2 - 3y/2 + 2$$

$$= -3x/2 + y/2 + 3/2 = 3x - y - 3 = 0$$

equation of tangent is $3x - y - 3 = 0$ $\times \frac{1}{3}$

b) i) Tangent given by $S_1 = 0$
where $S_1 = Axx + \frac{1}{2}B(x_1y + xy_1) + \frac{1}{2}Cyy + \frac{1}{2}F(x_1 + x) + \frac{1}{2}G(y_1 + y) + Hzz$

$$(x_1, y_1, z_1) = (-1, 1, 2)$$

ii) If conic is given by

$$3x^2 - 2xy + 3y^2 + 2xz - 6yz + 2z^2 = 0$$

$$A=3, B=-2, C=3, F=2, G=-6, H=2$$

$$S_1 = -3x + (-2)\frac{1}{2}(-y + x) + 3(y) + \frac{1}{2} \times 2(-z + 2x) - \frac{1}{2} \times 6(z + 2y) + 2 \times 2z$$

$$= -3x - x + y + 3y - z + 2x - 3z - 6y + 4z$$

$$= -2x - 2y = 0$$

$$x + y = 0$$

$$ii) x + y = 0$$

hence tangent passes through $(1, -1, 0)$.

The tangents are given by $S_1^2 = S \times S_{11}$

$$S_1 = 3x + \frac{1}{2} \times 2(-x + y) + 3x - y + \frac{1}{2} \times 2(z + 0) + \frac{1}{2} \times 6(-z + 0) + 0$$

$$= 3x + x - y - 3y + z + 3z = 4x - 4y + 4z$$

$$S_{11} = 3(1)^2 - 2 \times 1 \times -1 + 3(-1)^2 + 2 \times 1 \times 0 - 6 \times -1 \times 0 + 2 \times 0^2$$

$$= 3 + 2 + 3 = 8$$

$$\text{Then } S_1^2 = S \times S_{11}$$