

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ a & 1 & 1 \end{vmatrix} = x \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - y \begin{vmatrix} 0 & 0 \\ a & 1 \end{vmatrix} + z \begin{vmatrix} 0 & 1 \\ a & 1 \end{vmatrix} = 0$$

$$= x - az$$

The equation of YW is given by

$$\begin{vmatrix} x & y & z \\ 0 & p & p \\ a & 1 & a \end{vmatrix} = x \begin{vmatrix} p & p \\ 1 & a \end{vmatrix} - y \begin{vmatrix} 0 & p \\ a & a \end{vmatrix} + z \begin{vmatrix} 0 & p \\ a & 1 \end{vmatrix} = 0$$

$$= x(ap - p) - y(-ap) + z(-ap) = 0$$

$$= x(a-1) + ay - az = 0$$

The coordinates of C are given by the solution to $x - az = 0$, $x(a-1) + ay - az = 0$

Sub $x = az$ into $x(a-1) + ay - az = 0$.

$$az(a-1) + ay - az = 0$$

$$z(a^2 - 2a) + ay = 0 \Rightarrow y = \frac{z(2-a)}{a}$$

Hence coordinates of C are $(a, \frac{2-a}{a}, 1)$ ✓

d) ABC are collinear if the matrix of their coordinates has determinant zero.

$$\begin{vmatrix} 1 & -1 & 0 \\ 2a-1 & 1 & a \\ a & 2-a & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & a \\ 2a & 1 \end{vmatrix} - 1 \begin{vmatrix} 2a-1 & a \\ a & 1 \end{vmatrix} + 0 \begin{vmatrix} 2a-1 & 1 \\ a & 2-a \end{vmatrix}$$

$$= 1(1 - a(2a)) + 1(2a-1 - a^2)$$

$$= (1 - 2a + a^2) + (-1 + 2a - a^2) = 0$$

Hence A, B, C are collinear. ✓

The equation of line through A, B, C is given by,

$$\begin{vmatrix} x & y & z \\ 1 & -1 & 0 \\ a & 2-a & 1 \end{vmatrix} = x \begin{vmatrix} -1 & 0 \\ 2-a & 1 \end{vmatrix} - y \begin{vmatrix} 1 & 0 \\ a & 1 \end{vmatrix} + z \begin{vmatrix} 1 & -1 \\ a & 2-a \end{vmatrix}$$