Put this into the equation for C_3

$$\left(\frac{x'}{(x')^2+(y')^2}\right)^2+\left(\frac{y'}{(x')^2+(y')^2}\right)^2-\frac{2ax'}{(x')^2+(y')^2}-\frac{2by'}{(x')^2+(y')^2}=0$$

$$\frac{(x')^2+(y')^2}{((x')^2+(y')^2)^2}-\frac{2ax'}{(x')^2+(y')^2}-\frac{2by'}{(x')^2+(y')^2}=0$$

$$1-2ax'-2by'=0$$

Dropping the dashes

$$1-2ax-2by=0$$

Similarly for C_4

$$1-2a'x-2b'y=0$$

The tangent to C_3 is given by

$$2x+2y\frac{dy}{dx}-2a-2b\frac{dy}{dx}=0.$$

$$(2y-2b)\frac{dy}{dx}=2a-2x$$

$$\frac{dy}{dx}=\frac{a-x}{y-b}$$

At the origin $\frac{dy}{dx}=\frac{-a}{b}$

The tangent to C_4 is given by

$$\frac{dy}{dx}=\frac{a'-x}{y-b'}$$

At the origin $\frac{dy}{dx}=\frac{-a'}{b'}$

The circles touch at the origin, so, equating gradients, $\frac{-a}{b}=\frac{-a'}{b'}$, or $\frac{a}{b}=\frac{a'}{b'}$

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b) $z_1=2-i\to 0$

$z_2=1+3i\to 1$

$z_3=\infty\to\infty$

Since $\infty\to\infty$, the Möbius transformation is of the form $M(z)=K(z-m)$