To find the image under inversion of
$C_1: x^2+y^2-3y=0$
let (x,y) be an arbitrary point of C_2, and let (x',y') be its image under inversion in the unit circle.

Then $(x,y)=\left(\frac{x'}{(x')^2+(y')^2}, \frac{y'}{(x')^2+(y')^2}\right)$

x and y are related by $x^2+y^2-3y=0$
so x' and y' are related by

$$\left(\frac{x'}{(x')^2+(y')^2}\right)^2+\left(\frac{y'}{(x')^2+(y')^2}\right)^2-\frac{3y'}{(x')^2+(y')^2}=0$$

$$\frac{(x')^2+(y')^2}{((x')^2+(y')^2)^2}-\frac{3y'}{(x')^2+(y')^2}=\frac{1}{(x')^2+(y')^2}-\frac{3y'}{(x')^2+(y')^2}=0$$

$$1-3y'=0$$

Dropping the dash and rearranging, the image of C_1 under inversion is $y=1/3$

$C_2: x^2+y^2+3/2\,x-y=0$

As before, let $(x,y)=\left(\frac{x'}{(x')^2+(y')^2}, \frac{y'}{(x')^2+(y')^2}\right)$

Then $$\left(\frac{x'}{(x')^2+(y')^2}\right)^2+\left(\frac{y'}{(x')^2+(y')^2}\right)^2+\frac{3/2\,x'}{(x')^2+(y')^2}-\frac{y'}{(x')^2+(y')^2}=0$$

$$\frac{(x')^2+(y')^2}{((x')^2+(y')^2)^2}+\frac{3x/2}{(x')^2+(y')^2}-\frac{y'}{(x')^2+(y')^2}=0$$

$$\frac{1}{(x')^2+(y')^2}+\frac{3x/2}{(x')^2+(y')^2}-\frac{y'}{(x')^2+(y')^2}=0$$

Hence $1+3x'/2-y'=0$

Dropping the dashes, the image under inversion of C_2 is the line $y=3x/2+1$.