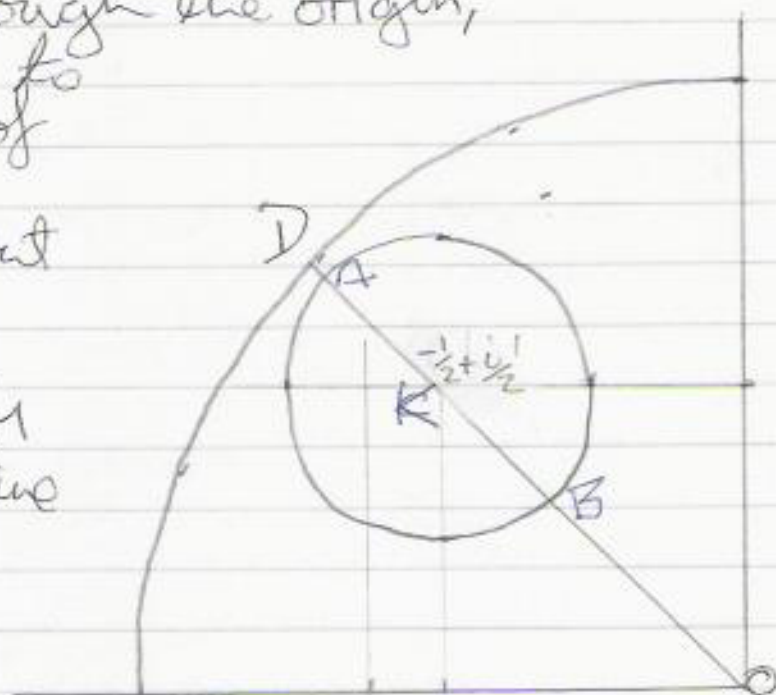


b) The line through the origin, the centre of K to the boundary of the disc is symmetric about the line $y = -x$.

We can find a transformation M that maps the line $y = -x$ or equivalently the line OD to the x axis.



It is just a rotation through $\frac{3\pi}{4}$ anti-clockwise.

$$\begin{aligned} t(z) &= (\cos 45^\circ + i \sin 45^\circ) \left(\frac{z}{2} - i \frac{z}{2} \right) \\ &= \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) \left(\frac{z}{2} - i \frac{z}{2} \right) \\ &= \frac{z}{2\sqrt{2}} - i \frac{z}{2\sqrt{2}} + i \frac{z}{2\sqrt{2}} + \frac{z}{2\sqrt{2}} \\ &= \frac{z}{\sqrt{2}} \end{aligned}$$

After transformation the line OD is:

$$\begin{array}{ccccccc} D' & A' & K' & B' & O' & A' \\ -1 - \frac{(4+i\sqrt{2})}{4\sqrt{2}} & -\frac{1}{2} & -\frac{(4-i\sqrt{2})}{4\sqrt{2}} & 0 & 0 \end{array}$$

Euclidean length = tanh (non Euclidean length)

non Euclidean centre given by

$$\frac{1}{2} \left(\tanh^{-1} \left(\frac{4+i\sqrt{2}}{4\sqrt{2}} \right) + \tanh^{-1} \left(\frac{4-i\sqrt{2}}{4\sqrt{2}} \right) \right) = -1.202 \quad \checkmark$$

non Euclidean radius given by

$$\frac{1}{2} \left(\tanh^{-1} \left(\frac{4+i\sqrt{2}}{4\sqrt{2}} \right) - \tanh^{-1} \left(\frac{4-i\sqrt{2}}{4\sqrt{2}} \right) \right) = 0.7083$$

Now we have to rotate by 315° anticlockwise to find the non Euclidean centre C

$$\begin{aligned} C &= (\cos 315 + i \sin 315) (-1.202) = -0.8599 + 0.5899i \\ &= -0.85 + i0.85 = -0.85 + 0.85i \end{aligned}$$

(The radius is a magnitude of length,