For a homomophism

$\phi_2(A) \circ \phi_2(B) = \phi_2(A \circ B)$

$\det(A) + \det(B) = \det(A + B)$

This does not hold. Consider for example the diagonal matrices $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$$\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \det\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 1 + 4 = 5$$

$$\det\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right) = \det\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = 9$$

$5 \neq 9$, so the function is not a homomophism.

iii) $\phi_3 : (\mathbb{Z}_7^*, \times_7) \to (\mathbb{Z}_6, +_6)$

$1 \to 0$
$2 \to 2$
$3 \to 1$
$4 \to 4$
$5 \to 5$
$6 \to 3$

Draw up the Cayley tables for $(\mathbb{Z}_7^*, \times_7)$ and $(\mathbb{Z}_6, +_6)$

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$(\mathbb{Z}_7^*, \times_7)$ is cyclic, generated by 3 and 5.

$(\mathbb{Z}_6, +_6)$ is cyclic, generated by 1 and 5

for $(\mathbb{Z}_7, \times_7)$

$3^1 = 3$
$3^2 = 2$
$3^3 = 6$

for $(\mathbb{Z}_6, +_6)$

$1^1 = 1$
$1^2 = 2$
$1^3 = 3$