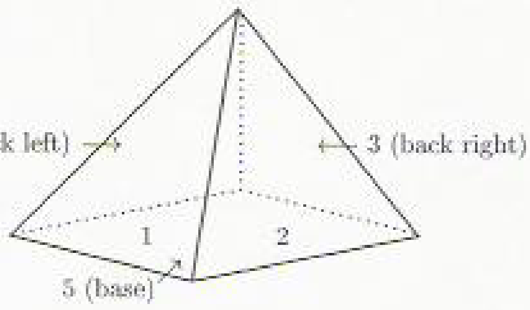

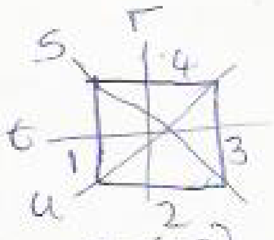

$r=(13)$
$s=(14)(23)$
$t=(24)$
$u=(12)(34)$

(a) Describe the symmetries of the pyramid represented by the permutations (13)(24) and (14)(23). [2]

(b) List all symmetries of the pyramid as permutations of $\{1,2,3,4,5\}$. [4]

(c) List all the conjugacy classes of G. [4]

(d) Find a subgroup of G which has order 2 and is not normal. You should explain why your choice of subgroup is not normal. [2]

(e) Find two different subgroups of G of order 4 and explain whether or not each of these subgroups is normal. [4]

(f) Give an example of a group from the course units which is isomorphic to G, justifying your answer briefly. [4]

Question 4 (Unit 4) – 20 marks

In this question

$$M = \left\{ \begin{pmatrix} a & c \\ b & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \right\},$$

that is, all 2×2 matrices. The set M forms a group under the operation of matrix addition. You are NOT asked to verify this fact.

(a) For each of the following functions, decide whether or not it is a homomorphism, justifying your answer.

(i) $\phi_1 : (\mathbb{R}^2, +) \longrightarrow (\mathbb{R}^2, +)$
$(x, y) \longmapsto (2x + y, 4x + 2y)$

(ii) $\phi_2 : (M, +) \longrightarrow (\mathbb{R}, +)$
$\mathbf{A} \longmapsto \det(\mathbf{A})$

(iii) $\phi_3 : (\mathbb{Z}_7^*, \times_7) \longrightarrow (\mathbb{Z}_6, +_6)$
$1 \longmapsto 0$
$2 \longmapsto 2$
$3 \longmapsto 1$
$4 \longmapsto 4$
$5 \longmapsto 5$
$6 \longmapsto 3$

(iv) $\phi_4 : (M, +) \longrightarrow (\mathbb{R}, +)$
$\mathbf{A} \longmapsto \operatorname{tr}(\mathbf{A})$ [12]

Note: $\det \begin{pmatrix} a & c \\ b & d \end{pmatrix} = ad - bc$, $\operatorname{tr} \begin{pmatrix} a & c \\ b & d \end{pmatrix} = a + d$.

(b) For each homomorphism ϕ in part (a), determine $\operatorname{Ker}(\phi)$ and $\operatorname{Im}(\phi)$ and identify the quotient group $G/\operatorname{Ker}(\phi)$ up to isomorphism, where G is the group that is the domain of the homomorphism. [8]