TMA M203 05 **Cut-off date** 11 July 1996

(Group Theory Block)

Question 1 (Unit 1) – 25 marks

(a) Find all elements in S_6 that conjugate (153642) to itself. [5]

(b) For each element in your solution to part (a), give its order and its parity. [5]

(c) Prove directly (that is, verify that the subgroup axioms hold) that the set of elements that you found in part (a) is a subgroup of S_6 and identify this subgroup up to isomorphism. (In other words, identify a group of symmetries of some figure to which your subgroup is isomorphic.) [6]

(d) Find all the elements in S_6 that conjugate (153642) to (123456). [5]

(e) Choose one element, p, that conjugates (153642) to (123456) and show that the elements $p \circ x$, where x conjugates (153642) to itself (from part (a)), are precisely the elements found in part (d). [4]

Question 2 (Unit 2) – 20 marks

The Cayley table for a group G is shown below.

	e	a	b	c	d	f	g	h	i	j	k	l
e	e	a	b	c	d	f	g	h	i	j	k	l
a	a	b	c	d	f	e	h	i	j	k	l	g
b	b	c	d	f	e	a	i	j	k	l	g	h
c	c	d	f	e	a	b	j	k	l	g	h	i
d	d	f	e	a	b	c	k	l	g	h	i	j
f	f	e	a	b	c	d	l	g	h	i	j	k
g	g	l	k	j	i	h	e	b	a	e	f	d
h	h	g	l	k	j	i	d	c	b	a	e	f
i	i	h	g	l	k	j	f	d	c	b	a	e
j	j	i	h	g	l	k	e	f	d	c	b	a
k	k	j	i	h	g	l	a	e	f	d	c	b
l	l	k	j	i	h	g	b	a	e	f	d	c

(a) Subsets H_1 to H_6 are specified below:

$$H_1 = \{e, a, b, c, d\};$$
$$H_2 = \{e, c\};$$
$$H_3 = \{i, j, k, l\};$$
$$H_4 = \{e, a, b, c, d, f\};$$
$$H_5 = \{e, c, g, j\};$$
$$H_6 = \{e, c, i\}.$$

For each of the subsets given, state whether or not it is a subgroup of G. For each subset that is a group, write out its Cayley table. For each subset that is not a subgroup, give one reason why it is not. [9]

(b) The subsets $H = \{e, b, d\}$ and $K = \{e, c, i, l\}$ are subgroups of G. For each of H and K list all its distinct left cosets and all its distinct right cosets. [6]

(c) One of the subgroups H and K is normal. For this subgroup construct the quotient group and identify the quotient group up to isomorphism. [5]

Question 3 (Unit 3) – 20 marks

The following figure is a square based pyramid in which all edges have the same length. The positions of the faces of the pyramid have been numbered so that we can represent the group, G, of all symmetries of the pyramid as permutations of the set $\{1, 2, 3, 4, 5\}$.