

$$\phi_3: (\mathbb{Z}_7^* \times \mathbb{Z}_7) \rightarrow (\mathbb{Z}_6, +_6)$$

1	$\rightarrow$	0
2	$\rightarrow$	2
3	$\rightarrow$	1
4	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	3

This is an isomorphism, from the Cayley tables constructed to show that it was a homomorphism. In fact, since it is an isomorphism, it is also a homomorphism, with the mapping of one element to another as shown.

$$\phi_4: (M, +) \rightarrow (\mathbb{R}, +)$$

$$A \mapsto \text{tr}(A)$$

$$\text{where } \text{tr} \begin{pmatrix} a & c \\ b & d \end{pmatrix} = a + d$$

$$\text{Ker}(\phi_4) = \left\{ \begin{pmatrix} a & c \\ b & -a \end{pmatrix} : a \in \mathbb{R} \right\}$$

This homomorphism is onto, so  
 $M / \text{Ker}(\phi_4) \cong \mathbb{R}$   
 ie the quotient group is isomorphic to  $\mathbb{R}$ .

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$$\frac{17\frac{1}{2}}{25}$$