for $\phi_3 : (\mathbb{Z}_7^*, \times_7) \to (\mathbb{Z}_6, +_6)$

$$\begin{aligned} 1 &\to 0 \\ 2 &\to 2 \\ 3 &\to 1 \\ 4 &\to 4 \\ 5 &\to 5 \\ 6 &\to 3 \end{aligned}$$

$\mathrm{Ker}(\phi_3) = \{1\}$

$\mathrm{Im}(\phi_3) = \{0, 1, 2, 3, 4, 5\} = \mathbb{Z}_6$

for $\phi_4 : (M, +) \to (\mathbb{R}, +)$

$A \to tr(A)$

$\mathrm{Ker}\left(tr\begin{pmatrix} a & c \\ b & d \end{pmatrix}\right) = \begin{pmatrix} a & c \\ b & -a \end{pmatrix}$, since $tr\begin{pmatrix} a & c \\ b & d \end{pmatrix} = a+d$, $a+d=0$

$\Rightarrow a = -d$.

Since $a, b, c, d \in \mathbb{R}$, $a+d \in \mathbb{R}$, so $\mathrm{Im}(\phi) = \mathbb{R}$

$G/N = G/\mathrm{Ker}(\phi) \cong \mathrm{Im}(\phi)$

for ϕ_1, $\mathrm{Ker}(\phi_1) = (k, -2k)$

The kernel of ϕ_1 is the line $y = -2x$ or $y + 2x = 0$

A coset of $(k, -2k)$ is $(1, 2) + (k, -2k) = (1+k, 2-2k)$

$y + 2x = 2 - 2k + 2 + 2k = 4 \Rightarrow y + 2x = 4$

Another coset is $(1, 1) + (k, -2k) = (1+k, 1-2k)$

$y + 2x = 1 - 2k + 2 + 2k = 3 \Rightarrow y + 2x = 3$.

The cosets of $(k, -2k)$ are a series of parallel lines. The homomorphism is onto, so

$R^2 / \mathrm{Ker}(\phi_1) \cong R^2$ $\cong \mathrm{Im}\,\phi_1 \cong \mathbb{R}$.