

The subgroup axioms are satisfied, therefore the set forms a subgroup.

But the subgroup is not a normal subgroup, since it is not a union of conjugacy classes. The conjugacy class containing (13) is $\{(13), (24)\}$; a normal subgroup must contain the whole conjugacy class of every member of the subgroup.

e) $\{e, (1234), (13)(24), (1432)\}$. Form the Cayley table.

\circ	e	(1234)	(1432)	$(13)(24)$
e	e	(1234)	(1432)	$(13)(24)$
(1234)	(1234)	$(13)(24)$	e	(1432)
(1432)	(1432)	e	$(13)(24)$	(1234)
$(13)(24)$	$(13)(24)$	(1432)	(1234)	e

SG1 The set is closed under composition

SG2 The identity is a member of the set

SG3 Each element has an inverse. The identity appears once in each column and once in each row.

This subgroup is normal, since it is a union of conjugacy groups.

$$SG = \{e^3\} \cup \{(1234), (1432)\} \cup \{(13)(24)\}$$

\circ	e	$(13)(24)$	$(14)(23)$	$(12)(34)$
e	e	$(13)(24)$	$(14)(23)$	$(12)(34)$
$(13)(24)$	$(13)(24)$	e	$(12)(34)$	$(14)(23)$
$(14)(23)$	$(14)(23)$	$(12)(34)$	e	$(13)(24)$
$(12)(34)$	$(12)(34)$	$(14)(23)$	$(13)(24)$	e

SG1 The set is closed under composition

SG2 The identity is a member of the set

SG3 Each element has an inverse - the identity