ie $\phi_4(M_1)+\phi_4(M_2)=\phi(M_1+M_2)$

Put $M_1=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $M_2=\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

$tr(M_1)=a+d$, $tr(M_2)=e+h$; $tr(M_1)+tr(M_2)=a+d+e+h$.

$tr(M_1+M_2)=tr\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}+\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right)=tr\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}=a+e+d+h$

The function is a homomorphism ✓ $\frac{1\frac{1}{2}}{2}$

b) The kernel of a function is the set of all x such that $\phi(x)=e$, the identity of the codomain

for $\phi_1:(\mathbb{R}^2,+)\to(\mathbb{R}^2,+)$

$(x,y)\to(2x+y,\ 4x+2y)$

the kernel of ϕ_1 is the set of points satisfying

$(2x+y, 4x+2y)=(0,0)$

$2x+y=0 \Rightarrow y=-2x$

$4x+2y=0$

ie the set of points $(k,-2k)$, $k\in\mathbb{R}$ ✓ $Ker(\phi_1)=(k,-2k)$

$x\in\mathbb{R}$, $y\in\mathbb{R}$, so any linear combination of x and $y\in\mathbb{R}$. $2x+y\in\mathbb{R}$ and $4x+2y\in\mathbb{R}$, so the image of the function is $\mathbb{R}^2$.

$Im\,\phi_1=\{(k,2k): k\in\mathbb{R}\}$ $(\neq\mathbb{R}^2)$