

$$\begin{aligned} 3^4 &= 4 \\ 3^5 &= 5 \\ 3^6 &= 1 \end{aligned}$$

$$\begin{aligned} 1^4 &= 4 \\ 1^5 &= 5 \\ 1^6 &= 0 \end{aligned}$$

Matching up the corresponding powers under composition

$$\begin{aligned} \phi_3: 1 &\rightarrow 0 \\ 2 &\rightarrow 2 \\ 3 &\rightarrow 1 \\ 4 &\rightarrow 4 \\ 5 &\rightarrow 5 \\ 6 &\rightarrow 3 \end{aligned}$$

Hence  $\phi_3$  is a homomorphism, since it is the mapping of the corresponding powers under composition of generators of two cyclic groups. Write out the Cayley tables, using the mapping above.

$x_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

$+_6$	0	2	1	4	5	3
0	0	2	1	4	5	3
2	2	4	3	0	1	5
1	1	3	2	5	0	4
4	4	0	5	2	3	1
5	5	1	0	3	4	2
3	3	5	4	1	2	0

With this mapping, the Cayley tables have identical patterns. We can say, therefore, more than that the function is a homomorphism; it is an isomorphism, (and alternatively, that since it is an isomorphism, it is also a homomorphism).

$$\text{iv) } \phi_4: (M, +) \rightarrow (\mathbb{R}, +)$$

$$A \rightarrow \text{tr}(A)$$

If the function is a homomorphism, then  $\phi(M_1) \circ \phi(M_2) = \phi(M_1 \circ M_2)$  ✓