

H's right cosets are given by

$$Ha = \{a, c, f\}$$

$$Hg = \{g, i, k\}$$

$$Hj = \{j, l, h\}$$

and H itself

$$K = \{e, c, i, l\}$$

H's left cosets are given by

$$aK = \{a, d, j, g\}$$

$$bK = \{b, f, k, h\}$$

and K

(NS)

and its right cosets are given by

$$Ka = \{a, d, h, k\}$$

$$Kb = \{b, f, g, j\}$$

and K

←

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c) The normal <sup>sub</sup>group is the one for which its distinct right and left cosets are the same. H is the normal subgroup.

The quotient group is constructed by taking the subgroup and its cosets as elements and filling in the entries. The identity is H.

$\circ$	H	aH	gH	jH
H	H	aH	gH	jH
aH	aH	H	jH	gH
gH	gH	jH	aH	H
jH	jH	gH	H	aH

The table is completed

using  $xH \circ yH = (x \circ y)H$ ,

reading the composition  $x \circ y$  from the Cayley table in the question, then composing with H to find the coset formed.

The group is cyclic generated by gH. It is therefore isomorphic to any cyclic group of order 4 eg  $(\mathbb{Z}_4, +_4)$

$$\begin{aligned} gH &= gH^1 \rightarrow 1 \\ aH &= gH^2 \rightarrow 2 \\ jH &= gH^3 \rightarrow 3 \\ H &= gH^4 \rightarrow 4 = 0 \end{aligned}$$

$$\text{or } \begin{aligned} gH &\rightarrow 1 \\ aH &\rightarrow 2 \\ jH &\rightarrow 3 \\ H &\rightarrow 0 \end{aligned}$$

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