

5) a) $g \cdot (x, y) = (ax, y)$ for $g = \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$

i) $g \cdot (1, 0) = \{a, 0\} : a \in \mathbb{R}, a \neq 0\}$

$a \in \mathbb{R}$, and can take any value, so the orbit of $(1, 0)$ is the x -axis, excluding origin (since $a \neq 0$)

ii) $g \cdot (0, 1) = \{0, 1\}$

This is the only point produced by the action of G on $(0, 1)$, so the orbit of $(0, 1)$ is the point $(0, 1)$

iii) $g \cdot (1, 1) = \{a, 1\} : a \in \mathbb{R}, a \neq 0\}$

$a \in \mathbb{R}$, and can take any value, so the orbit of $(1, 1)$ is the line $y=1$, excluding $(0, 1)$

$a, x, y \in \mathbb{R}$, so $(ax, y) \in \mathbb{R}^2$. The orbits of the action is the union of all the orbits of each point acted on by G , and the union of those orbits is \mathbb{R}^2 .

OK, But what are these!!! — P.T.O. 5/8

b) i) The condition for a point to be stabilized is $g \cdot (1, 0) = (1, 0)$, but $g \cdot (1, 0) = (a, 0)$ from the definition of the action. For these two conditions to hold, $a=1$.

Hence the stabilizer of $(1, 0)$ is $\left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}$

ii) $g \cdot (0, 1) = (0, 1)$ and

this is the condition; it is also the result.

Hence the stabilizer of $(0, 1)$ is the whole of G

ie $\text{stab}(0, 1) = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \right\} = G$

iii) $g \cdot (1, 1) = (1, 1)$ (condition)

$g \cdot (1, 1) = (a, 1)$ (result)

The condition and result, taken together means that $a=1$. Hence

$\text{stab}(1, 1) = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}$

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