By examination, the set is closed.
The identity element is 1

element	1	3	5	9	11	13
inverse	1	5	3	11	9	13

Each element has an inverse
$a\circ(b\circ c) = (a\circ b)\circ c$ (test for associativity)
Mulitiplication is associative
Therefore the set is a group under $\times_{14}$.

b) $(\mathbb{Z}, *)$ where $x*y = x-y+1$
for $x,y \in \mathbb{Z}$, $x-y+1 \in \mathbb{Z}$ $\therefore$ the set is closed
identity $x*e = x = e*x$

$x - e + 1 = x = e - x + 1$

$x - e + 1 = e - x + 1$

$x - e = e - x = -(x-e)$

hence $x - e = 0$. But the identity in a set is unique hence the group is not a set, since x has an infinite number of values.

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c) $(\mathbb{Z}, *)$ $x*y = x+y-1$.
closure - for all $x,y \in \mathbb{Z}$, $x+y-1 \in \mathbb{Z}$
identity $x*e = x = e*x$

$x + e - 1 = x = e + x - 1$

LHS implies $e=1$, RHS implies $e=1$

$\therefore e = 1$

inverse; for all $x \in \mathbb{Z}$, there exists x^{-1} so that $x * x^{-1} = e$

$x * x^{-1} = x + x^{-1} - 1 = 1$

$x + x^{-1} = 2$

$x^{-1} = 2 - x$ for all x

Should show LHS = RHS

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associativity: $x*(y*z) = (x*y)*z$

$x*(y+z-1) = (x+y-1)*z$

$x+y+z-1-1 = x+y+z-1-1$ $= x+y+z-2$

Hence the set is a group under $*$