Question 4 (Unit 2) – 30 marks

For each of the following sets, with the binary operation given, decide whether it forms a group, justifying your answer.

(a) $(\{1,3,5,9,11,13\}, \times_{14})$ [12]

(b) $(\mathbb{Z}, *)$ where $x * y = x - y + 1$ [4]

(c) $(\mathbb{Z}, *)$ where $x * y = x + y - 1$ [14]

TMA M203 02 (Introduction Block)

Cut-off date 14 March 1996

Question 1 (Unit 3) – 20 marks

(a) (i) Show that the curve given by the parametric equations

$$x = a + r\cos t, \qquad y = b + r\sin t \qquad (t \in \mathbb{R}),$$

is a circle with centre (a, b) and radius r. [2]

(ii) Show that the gradient of the tangent to the circle in part (i) at the point (x_1, y_1) is $\dfrac{-(x_1 - a)}{y_1 - b}$. [3]

(b) This part of the question concerns the circle C given by

$$C = \{(x, y) : x^2 + y^2 - 5x - 2y - 9 = 0\}.$$

(i) Find the centre and radius of the circle C. [2]

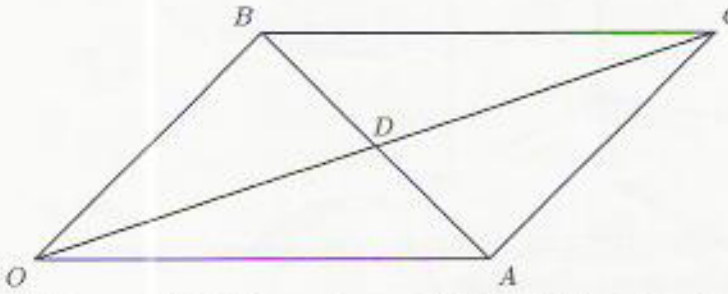
(ii) Verify that the points $P(-1, 3)$ and $Q(2, 5)$ lie on the circle C and find the equation of the line through P and Q. [3]

(iii) Find the equations of the tangents to the circle C at the points P and Q and the coordinates of their point of intersection, R. [7]

(iv) Verify that $RP = RQ$, i.e. that the tangents from R are of equal length. [3]

Question 2 (Unit 3) – 20 marks

(a) In this part of the question you are asked to prove by vector methods that the diagonals of a parallelogram bisect one another.



The parallelogram $OACB$ has vertices at O (the origin) and at points A, B and C with position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. The diagonals OC and AB intersect at D with position vector $\mathbf{d}$.

(i) Write down $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) Write down the position vector of a general point on the line OC. [1]

(iii) Write down the position vector of a general point on the line AB. [1]

(iv) Show that D is the midpoint of OC and of AB, i.e. that the diagonals bisect one another. [5]