School of Engineering and Mathematical Sciences

Part 2: Numerical Methods Supplementary Coursework

Coursework

Coursework should be e-mailed to s.yan@city.ac.uk and should consist of a spreadsheet file accompanied by a word document explaining the equations used in the spreadsheet. The coursework is in three sections. All the sections require knowledge you have been taught in the lectures. For the first two sections, all the calculation can be done using Excel spreadsheet files which are similar to those used in the tutorial sessions. For the last section, you will need to use finite difference method to solve a boundary value problem as discussed in the Lecture 4.

The coursework need to be finished **individually**. I suggest that anyone who is uncertain about how to undertake the coursework sees me rather than consulting their colleagues. If students submit identical spreadsheets/word documents it will be assumed that they have copied and they will receive a mark of zero.

Deadline of Submission

The coursework need to be submitted on or before **Monday, 23th, March, 2009**. Any late submission will not be accepted.

Marks Required to Pass

The final mark consists of those (50 marks) you obtained from 4 tutorial sessions (Original Mark) and those (50 marks) from this coursework. To pass the course, you will need to achieve enough mark from this coursework as listed below,

ORIGINAL MARK	MARK REQUIRED FROM THIS COURSEWORK
Greater than or equal to 30	50%
Greater than 20 and less than 30	65%
Less than or equal to 20	80%

Section 1: Numerical Integration (15 marks)

For the open channel flow, the equation governing the depth of the water, h, with distance along the channel, x, can be written as

$$\frac{dh}{dx} = S_0\left(\frac{1-(h_n/h)^{10/3}}{1-(h_c/h)^3}\right)$$

A sketch of this problem is shown in Fig.1, in which h_n = 1.5m, h_c = 0.86m, S_0 = 0.0015

Fig. 1 sketch of the open channel flow

If the height of water (h) at a given point is 2.0m, find the distance x along the channel to the point where h is 2.5m? The answer needs to keep 4 decimal places. Finish the following three tasks.

1) Write the mathematical formula (integral formulation) which you used to integrate to achieve the results.

2) Using Simpson's 1/3 rule estimate the required distance x.

3) Using repeated trapezium rule with a proper n give an answer accurate to 3 decimal places. Explain why your selected n is big enough to achieve an accuracy of 3 decimal places.

Section 2: Initial Value Problem (20 marks)

The following differential equation can be used to estimate the radial displacement, u, at a radius, r , from the centre of a tunnel,

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0$$

The tunnel is 3.5m in diameter and the soil, which has a limiting strength of 150kPa, is just failing around the circumference of the tunnel so that at a radius r=1.75m the radial stress is 190kPa and the circumferential stress is 490kPa. If the soil has a Young's Modulus of 20MPa using Hookes laws at r=1.75m, du/dr = - 0.011 and u=0.02m. Calculate the radial displacement 7m from the centre of the tunnel using Euler's method with a step size that gives an answer accurate to 3 decimal places. **You will need to convert equation 1 into two first order differential equations following the approach given in the lecture notes for week 4 of the course.** For example, you can introduce a new variable $s = du/dr$ (first differential equation). Substituting this equation to the above one, you can get the second differential equation.

Plot a graph of computed movement against distance from the centreline of the tunnel. Note that the displacement will decreases with distance from the tunnel.

In your calculation, you need to compare your result with another more accurate approximation (using smaller step size) to make sure the first three decimals are the same.

Section 3: Boundary Value Problem (15 marks)

The temperature distribution along a heated rod (as sketched in Fig. 2) satisfies the following governing differential equation,

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0$$

where h' is a coefficient and T_a is the ambient temperature. The length of the rod is L. The origin of the coordinate system is located at the centre of the left side of the rod and the x-axis points toward the right side of the rod. On the boundaries, the temperatures satisfy

$T = T_0$ at $x=0$ and $T = T_L$ at $x=L$

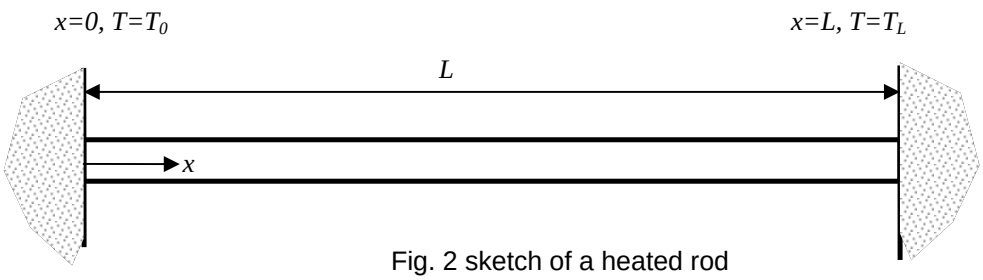

Fig. 2 sketch of a heated rod

To find the temperature distribution along the rod, we are required to use the finite difference method. To do so, we need to separate the whole rod into n elements with equally spaced (n+1) nodes.

For the boundary nodes, 1 and (n+1), the boundary conditions are imposed. Therefore, we can have two equations.

For the other nodes, the governing differential equation is applied, in which the derivative terms are replaced by the corresponding numerical difference schemes. Suppose the central finite difference scheme is chosen, we can rewrite the governing equation to be,

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} + h'(T_a - T_i) = 0$$

for an internal node *i*. In the above equation, Δx is the step size. Thus, we can have (n-1) equations for all n-1 internal nodes. By simultaneously solving the (n+1) equations, consisting of 2 equations for boundary nodes and (n-1) equations for internal nodes, we can get the temperature at all nodes.

If h'=0.01m^{-1}, T_a=20°, L=10m, T_0=40°, T_L=100°, use the finite difference method with the central finite difference scheme as described above to find the temperature distribution along the rod. All results require 3 decimals.

1) Use $\Delta x = 2$ m. List the equations for all nodes, i.e. 2 equations for boundary nodes (nodes 1 and 6) and 4 equations for internal nodes (nodes 2-5). Solve the equations to find the temperatures at these 6 nodes. Put your data into an Excel table as followed.

Node No.	x	T_i
1	0.000	
2	2.000	
3	4.000	
4	6.000	
5	8.000	
6	10.000	

2) Plot a curve using Ti against x.

3) Choose $\Delta x = 1$ m and repeat the procedures in Step 1) to find the temperature at different nodes.

4) Plot another curve using the results you obtained in Step 3), together with the curve you plotted in Step (2). Describe the difference and explain the reason which causes the difference.