(a) For each of the following sets $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ of vectors in $\mathbb{R}^3$, write down the (plane) lattice type of $L(\mathbf{a}, \mathbf{b})$; find the offset of $\mathbf{c}$ relative to $\{\mathbf{a}, \mathbf{b}\}$; and hence determine the Bravais lattice type of $L(\mathbf{a}, \mathbf{b}, \mathbf{c})$.

(i) $\mathbf{a} = (2, 3, 0)$, $\mathbf{b} = (3, 2, 0)$, $\mathbf{c} = (5, 5, 5)$

(ii) $\mathbf{a} = (3, 4, 0)$, $\mathbf{b} = (4, -3, 0)$, $\mathbf{c} = \left(\frac{7}{2}, \frac{1}{2}, \frac{5}{2}\right)$

(b) Let $\mathbf{a} = (0, 4, 5)$, $\mathbf{b} = (3, 0, 5)$, $\mathbf{c} = (3, 4, 0)$.

(i) Find vectors $\mathbf{d}$, $\mathbf{e}$ and $\mathbf{f}$ such that
$\mathbf{a} = \frac{1}{2}(\mathbf{e} + \mathbf{f})$, $\quad \mathbf{b} = \frac{1}{2}(\mathbf{d} + \mathbf{f})$ $\quad$ and $\quad \mathbf{c} = \frac{1}{2}(\mathbf{d} + \mathbf{e})$.

(ii) Hence determine the Bravais lattice type of $L(\mathbf{a}, \mathbf{b}, \mathbf{c})$.

(c) Each face of a solid cube is divided into two triangles, in such a way that the diagonal dividing lines form the edges of a regular tetrahedron:

Each of the twelve triangles so formed may be coloured either black or white. Find the number of equivalence classes, under rotations of the configuration so formed, of such colourings of the triangles.

(a) Let $\mathbf{a} = (3, 3, 0)$, $\mathbf{b} = (3, -3, 0)$, $\mathbf{c} = (5, 4, 2)$. Find:

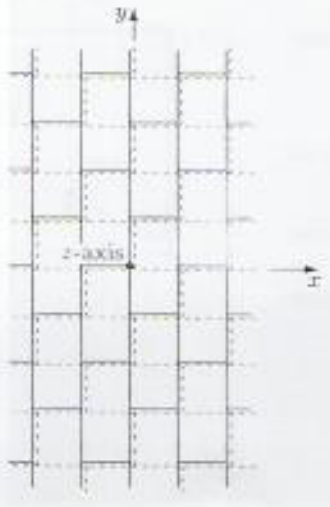
(i) the offset of $\mathbf{b}$ relative to $\mathbf{a}$;

(ii) the offset of $\mathbf{c}$ relative to $\{\mathbf{a}, \mathbf{b}\}$.

(b) A crystal pattern C is constructed from rectangular slabs, each of length 2 units, width 1 unit and depth 1 unit, as follows.

A layer is laid on the xy-plane so that, looking down from above, the arrangement is as shown below.

A second layer is laid on the first; the arrangement of this second layer is obtained from the first by an anticlockwise rotation through $\pi/2$ about the z-axis and then a translation by 1 unit in the x-direction.



Successive layers are constructed in the same way: each layer is rotated anticlockwise through $\pi/2$ about the z-axis relative to the layer immediately underneath.

(i) Specify a basis $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ for a space lattice L associated with C. Explain why it is a basis.

(ii) Find the Bravais type of L.

(iii) Specify a symmetry of L that is not a symmetry of C.