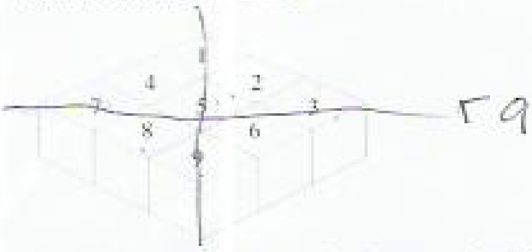
Question 3 (*Unit GE1*)

This question tests your ability to derive the cycle index and pattern inventory for a group action, and your knowledge of how to use these to determine the number of ways of colouring a solid object.

We start with a collection of unit cubes, each coloured either blue or yellow. Then we select nine such cubes and fasten them together to form a slab S of size $3 \times 3 \times 1$:



The lables 1 to 9 are purely there to identify the *positions* of the individual unit cubes: they are *not* marked on the cubes themselves. It may be helpful to think of the labels as being in the centres of the cubes rather than on the faces to remind yourself that the numbers label the cubes not the faces.

Let G denote the group of rotational symmetries of S, including those that turn the slab over.

(a) Using the given numbering, write down all eight of the elements of G as permutations of the cube positions 1 to 9. [5]

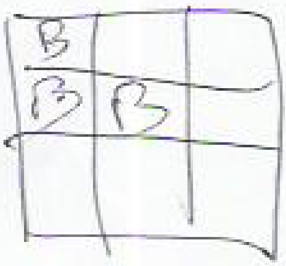
(b) Without using any results from *Unit GE1*, determine how many distinguishable slabs can be made using exactly two blue unit cubes with the remaining unit cubes yellow. Explain your answer. [4]

(c) Find the cycle index of the action of G on the cube positions. [4]

(d) Use your answer to part (c) to determine the number of distinguishable slabs that can be made when m colours are available for the constituent unit cubes. [2]

(e) (i) Write down the pattern inventory of all distinguishable slabs that can be made using the two colours blue (B) and yellow (Y) for the constituent unit cubes. (There is no need to expand your answer.) [2]

(ii) Use your answer to part (e)(i) to verify your answer to part (b). [8]



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