

2b) 111110011000111111110000000010111101111110010

From the above sequence

$$n_{00}=12, n_{01}=6, n_{10}=7, n_{11}=24$$

(Check $r = n_{01} + n_{10} + 1 = 6 + 7 + 1 = 14$, from the above sequence there are 14 runs)

The matrix of transition counts is given by

$$N = \begin{bmatrix} 12 & 6 \\ 7 & 24 \end{bmatrix}$$

$$\text{and then } \hat{M} = \begin{bmatrix} 12/18 & 6/18 \\ 7/31 & 24/31 \end{bmatrix} = \begin{bmatrix} 0.6667 & 0.3333 \\ 0.2258 & 0.7742 \end{bmatrix}$$

The switching probabilities α and β are 0.3333 and 0.2258 respectively.

For the sequence to have been generated by a Bernoulli model α should be equal to $1-\beta$, and β should be equal to $1-\alpha$.

$$\beta = 0.2258 \neq 1-\alpha = 0.6667$$

$$\text{and } \alpha = 0.3333 \neq 1-\beta = 0.7742$$

It seems unlikely that the sequence could have been generated by a Bernoulli process, not least since β and $1-\alpha$, and α and $1-\beta$ are so different in magnitude.

ii)

`v = 1,1,1,1,1,1,0,0,1,1,0,0,0,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,1,0,0,1,1,`
`1,1,0,1,1,1,1,1,1,1,0,0,1,0`

`runstest(v)`

Exact test:

$$r = 14 \quad [n_0 = 19, n_1 = 31]$$

$$SP \text{ (total)} = 0.001982$$

Normal approximation:

$$r = 14 \quad [\mu = 24.56, \sigma = 3.294]$$

$$z = -3.206$$

$$SP \text{ (total)} = 0.001345$$

The significance probability is very small, at 0.001982, and the hypothesis that the sequence