Question 2 (*Chapter 12 Random processes*)

(a) The following sequence of 0s and 1s was generated by computer using an underlying Markov chain model (possibly, but not necessarily, a Bernoulli model) to drive the simulation.

```
11111100110001111111111000000001001111011111110010
```

(i) Use the data to estimate the transition probability matrix $\mathbf{M}$, and write down your estimates for the switching probabilities α and β. On the basis of your estimated transition probabilities, comment informally on whether you think the simulated realization could have been generated by a Bernoulli model. [4]

(ii) Test the hypothesis that the data were generated as a realization of a Bernoulli process, quoting the *SP* for the test and your conclusions from it. [5]

(iii) Use your computer together with your estimates from part (a)(i) to generate a sequence of 1000 observations. (Try querying the routine `chain.()`.) How many runs were there? Compare this informally with the expected number and say whether your findings from part (a)(ii) are confirmed by this additional realization. [6]

(b) A realization of events in a Poisson process, starting at time 0, can be generated by successively adding interevent waiting times drawn from the appropriate exponential distribution. The following program, with syntax `pp(tt,rr)`, returns a vector of times of occurrence of events in a Poisson process which is observed for a total time `tt` and in which events occur at average rate `rr`. (If, by chance, no events occur, then the function `pp()` returns a vector of length 1, containing the number 0.)

```
func pp(){
  local(tt,rr,et,vv)
  tt= $1$; rr= $2$; et= mrand(1/rr)
  if(et <= tt) vv= (et,) else vv= (0,)
  while(et<=tt){
    et= et+mrand(1/rr)
    if(et <= tt) vv= vec(vv,et)
  }
  return vv
}
```

Copy the program into your worksheet. (You will need `pp()` for parts (i) and (iii).)

(i) Then use it to generate 4000 days' (about ten years') earthquake times, assuming that observation starts at time 0 and that earthquakes occur as a Poisson process in time with an expected 437 days between occurrences. Obtain five realizations and in each case say how many earthquakes there were. From what distribution are these five counts drawn? [6]

The data set `spikes` in the SSC data subdirectory gives consecutive motor cortex neuron interspike time intervals (in milliseconds) for a monkey at rest, collected as part of a study of brain activity. Observation ceased after the last spike was recorded.

(ii) Use the data to estimate the spike rate, taking particular care over the units of measurement. [3]

(iii) Assuming that a Poisson model is adequate for the incidence of spikes, write down the SSC command that would return the simulated times of occurrence of spikes in the same monkey, observed for one second, tomorrow. List the times of occurrence for one realization, and say how many spikes there were. [6]