

If we take $g(x)=0$ and $h(x)=x^4$, the conditions of the squeeze rule:

① $g(x) \leq f(x) \leq h(x) \quad x \neq 0$

② $g(0) = f(0) = h(0)$

③ $g(x), h(x)$ are continuous at 0.

all hold

($f(0)=0$ from the definition of the function given at the start, and $g(0)=0, h(0)=0^4=0$ so ② holds. ③ holds since a constant function (ie $g(x)=0$) is continuous, and polynomials are basic continuous functions. ① has already been shown)

b) $2\sin x - \cos x = x$

This is a continuous function since polynomials and trigonometric functions are continuous.

Express as $*f(x) = 2\sin x - \cos x - x$

$f(0) = 2\sin 0 - \cos 0 - 0 = -1 < 0$

$f(\pi) = 2\sin \pi - \cos \pi - \pi = 0 - (-1) - \pi = 1 - \pi < 0$

$f(\pi/2) = 2\sin(\pi/2) - \cos(\pi/2) - \pi/2 = 2 - 0 - \pi/2 = 2 - \pi/2 > 0$

Hence the graph looks something like

*(So we want solutions $f(x)=0$).

