Equating ① and ②

$$\frac{5n}{5n-4} \leq 5^{1/n} \leq 1+\frac{4}{n}$$

$$\left(5n-4 \overline{)\,5n\,} = 1 \text{ R } 4 ;\quad 5n-4,\ 4 \Rightarrow \frac{5n}{5n-4} = 1 + \frac{4}{5n-4} \right)$$

So the equation may be restated:

$$1+\frac{4}{5n-4} \leq 5^{1/n} \leq 1+\frac{4}{n}$$

7/7

Please see definition of g.l.b. You must take $m' > 4$ and find $x \in E$ with $x < m'$

c) $E = \left\{ 4+\frac{2}{n^2} : n = 1, 2, 3 \ldots\ldots \right\}$

The Greatest lower bound of the set E is the largest number m for which all $E(n) \in E$ are greater than m.

$4+\frac{2}{n^2} > 4$ since $\frac{2}{n^2} > 0$ for all n. But $\frac{2}{n^2} \to 0$ 4/7 as $n \to \infty$, therefore $4+\frac{2}{n^2} \to 4+0 = 4$ Also $4+\frac{2}{n^2} > 4$ for all n, hence 4 is the greatest lower bound.

17/28 SEE EX 4, p32 of Unit 1

2) a) A null sequence is a sequence in which the terms tend to zero for increasing n. For each arbitrarily small +ve number ε, there is an integer N such that $|a_n| < ε$ for all $n > N$

For the sequence $a_n = \frac{(-1)^n}{n^3-4}$

$$|\varepsilon| > |a_n| \Leftarrow \Rightarrow |\varepsilon| > \left| \frac{(-1)^{n}}{n^3-4} \right|$$

For $N \geq 2$ the denominator of a_n on the RHS > 0, so if we put the numerator = 1, then

$$\left| \frac{(-1)^N}{N^3-4} \right| = \frac{1}{N^3-4} \qquad N \geq 2 \quad \Big\{ \ |a_n| = \frac{1}{n^3-4} \text{ for } n \geq 2.$$