c) $a_n = n^2 - 4/n + 2^n$
divide the numerator and denominator $(=1)$ by 2^n

$$a_n = \frac{n^2/2^n - 4/n2^n + 2^n/2^n}{1/2^n} = \frac{n^2/2^n - 4/(n \times 2^n) + 1}{1/2^n}$$

$n^p c^n$ is a null sequence, $p > 0$, $|c| < 1$
Compare this with $n^2/2^n = n^2 0.5^n$. It is a null sequence, with $c = 0.5$, $p = 2$
$4/(n \times 2^n) \leq 4/2^n = 4 \times 0.5^n$. Now c^n is a null sequence, $|c| < 1$, and a constant multiple of a null sequence is also a null sequence, by the multiple rule (P.S. $4/(n \times 2^n) \leq 4/2^n$, since $n = 1, 2, 3, \ldots$)
The term in the denominator $1/2^n$ is a null sequence (compare with c^n, $|c| < 1$, $1/2^n = 0.5^n$) $\frac{2\frac{1}{2}}{5}$
then putting all this into the expression for a_n

$$\lim_{n \to \infty} a_n = \frac{0 - 0 + 1}{0} = \infty$$ NO!!! We cannot divide by $\underline{\underline{0}}$ Use Reciprocal Rule

The sequence tends to infinity. See pp 30-31 of Unit 2

$\frac{12}{20}$

3) $\sum_{n=1}^{\infty} \frac{n^4}{3^n n!}$

Use the ratio test ✓ OK since each $a_n > 0$.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^4}{3^{n+1}(n+1)!} \Big/ \frac{n^4}{3^n n!} = \frac{(n+1)^4 \times 3^n n!}{n^4 \times 3^{n+1}(n+1)!} = \frac{(n+1)^4}{n^4} \times \frac{1}{3(n+1)}$$

as $n \to \infty$, $(n+1)^4/n^4 \to 1$

hence $\frac{a_{n+1}}{a_n} \to \frac{1}{3(n+1)} < 1$ ✗ $\frac{a_{n+1}}{a_n} \to 0$ which is < 1 so...

So the series is convergent. $4/5$ series is cgt.

b) $\sum \frac{(-1)^{n+1}}{3n - \sqrt{n}}$

$$a_n = \frac{(-1)^{n+1}}{3n - \sqrt{n}}$$