So $|\varepsilon| > \frac{1}{N^3-4}$

$$N^3 - 4 > \frac{1}{|\varepsilon|}$$

$$N^3 > \frac{1}{|\varepsilon|} + 4 = \frac{1+4|\varepsilon|}{|\varepsilon|}$$

$$N > \sqrt[3]{\frac{1+4|\varepsilon|}{|\varepsilon|}}$$

$\underline{n}$ rather than $\underline{N}$

Then choose $N = \left[\left(4+\frac{1}{\varepsilon}\right)^{1/3}\right]$

So ~~for~~ any $|\varepsilon|$, $\sqrt[3]{\frac{1+4|\varepsilon|}{|\varepsilon|}}$ will return a number

integer part

and an integer for N greater than this will return a term $|a_N| < |\varepsilon|$. Hence a_n is a null sequence

3/5

b)i) $a_n = \frac{3n^3 - 4n + 4}{5n^3 + n^2 - 2n - 3}$ $n = 1, 2, 3 \ldots$

divide through by n^3 ~~(Quotient Rule)~~

NOT an application of Quotient Rule

$$a_n = \frac{3n^3/n^3 - 4n/n^3 + 4/n^3}{5n^3/n^3 + n^2/n^3 - 2n/n^3 - 3/n^3}$$

$$= \frac{3 - 4/n^2 + 4/n^3}{5 + 1/n - 2/n^2 - 3/n^3}$$

3½/5

as $n \to \infty$, all the terms containing a -ve power of $n \to 0$, since $1/n^p, p>0$ is a basic null sequence

$$\lim_{n\to\infty} a_n = \frac{3 - 4/n^2 + 4/n^3}{5 + 1/n - 2/n^2 - 3/n^3} = \frac{3 - 4\times 0 + 4\times 0}{5 + 0 - 0 - 0} = \frac{3}{5}$$

By COMBINATION RULES

ii) $a_n = \frac{3^n - 4(n!) + 2}{2(n!) + n^2 - 2^n}$ $n = 1, 2, 3 \ldots$

Divide through by $n!$ ~~(Quotient rule)~~

$$a_n = \frac{3^n/n! - 4(n!)/(n!) + 2/(n!)}{2(n!)/(n!) + n^2/(n!) - 2^n/(n!)} = \frac{3^n/n! - 4 + 2/(n!)}{2 + n^2/(n!) - 2^n/(n!)}$$

$c^n/n!$ and $n^p/n!$ $(p>0)$ are null sequences $\to 0$ as $n \to \infty$

$$\lim_{n\to\infty} a_n = \frac{0 - 4 + 0}{2 + 0 - 0} = -2$$

By COMBINATION RULES

4/5