a_n is an alternating sequence of the form $a_n = (-1)^{n+1} b_n$, where $b_n = \frac{1}{3n - \sqrt{n}}$. b_n is a +ve decreasing null sequence, since $b_n = \frac{1}{3n-\sqrt{n}} = \frac{1/n}{3 - 1/\sqrt{n}}$

Numerator is a null sequence of the form $1/n^p$, $p>0$
$\frac{1}{\sqrt{n}}$ is a null-sequence (of the form $1/n^p$, $p>0$.)
hence $\lim_{n\to\infty} a_n = \frac{0}{3-0} = 0$. Prove b_n is decreasing
[Show $3(n+1) - \sqrt{n+1} > 3n - \sqrt{n}$]

By the alternating test, if $a_n = (-1)^{n+1} b_n$, then $\sum_{n=1}^{\infty} a_n$ is convergent (for b_n, a null sequence).

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c) $\sum_{n=1}^{\infty} \frac{1-n^3}{1+n^2+2n^3}$

Divide each term by n^3 in numerator and denominator

$a_n = \frac{1-n^3}{1+n^2+2n^3} = \frac{1/n^3 - 1}{1/n^3 + 1/n^2 + 2}$

$\lim_{n\to\infty} a_n = \frac{1/\infty - 1}{1/\infty + 1/\infty + 2} = \frac{0-1}{0+0+2} = \frac{-1}{2}$

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hence $\lim_{n\to\infty} a_n \to -1/2 \neq 0$, so a_n is not a null sequence and so is not convergent (The non null test).

d) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

$|\cos n| \leq 1$ so,

$\left|\frac{\cos n}{n^2}\right| \leq \left|\frac{1}{n^2}\right|$ (Comparison test. Since $(\cos n)/n^2$ has +ve ~~-ve~~ terms, we can simplify the answer by testing for absolute convergence).

$\sum 1/n^2$ converges (It is a basic convergent series of the form $1/n^p$, $p>0$). Hence by the compar-