By the intermediate value theorem, if $f(x)$ is a continuous function on $[a, b]$ and k is a number $f(a) < k < f(b)$ then there is a number c in I such that $f(c) = k$

In the question, put $a_1 = 0$, $b_1 = \pi/2$, $f(a_1) = -1$, $f(b_1) = 2 - \pi/2$

so $f(a_1) < k < f(b_1)$

implies $f(0) < k < f(\pi/2)$

$-1 < k < 2 - \pi/2$

Put $k = 0$, then there must be a value c_1 such that $f(c_1) = 0$.

On the interval $[\pi/2, \pi]$, $a_2 = \pi/2$, $b_2 = \pi$, $f(a_2) = 2 - \frac{\pi}{2}$

$f(b_2) = 1 - \pi$

$(2 - \pi/2 > 0$, $1 - \pi < 0)$

so the intermediate value theorem states that $f(a_2) > k > f(b_2)$

$2 - \pi/2 > k > 1 - \pi$

7/7

Put $k = 0$, then there is a number c_2 such that $f(c_2) = 0$

16/25

5) $\lim_{x \to 0} 3 \sin\left(\frac{1}{|x|}\right)$

Choose two null sequences x_1 and x_2

$x_{n_1} = \frac{1}{(2n + 1/2)\pi}$ $x_{n_2} = \frac{1}{(2n + 3/2)\pi}$

for x_{n_1}

$3 \sin\left(\frac{1}{|x|}\right) = 3 \sin(2n + 1/2)\pi = 3 \times 1 = 3$

for x_{n_2}

$3 \sin\left(\frac{1}{|x|}\right) = 3 \sin(2n + 3/2)\pi = 3 \times -1 = -3$

the limits for the two sequences are different hence $\lim_{x \to 0} 3 \sin\left(\frac{1}{|x|}\right)$ does not exist.

5/5