**Question 3** (Unit 3) – 20 marks

Determine which of the following series are convergent. You may use the basic series listed in the final frame of the audio-tape section of *Unit 3*, but you should state clearly which results or rules you use.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4}{3^n n!}$ [5]

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n - \sqrt{n}}$ [6]

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1 - n^3}{1 + n^2 + 2n^3}$ [4]

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ [5]

**Question 4** (Unit 4) – 25 marks

(a) Show that each of the following functions is continuous. You may use the basic continuous functions listed in the final frame of the audio-tape section of *Unit 4*, but you should state clearly which results or rules you use.

(i) $f(x) = \begin{cases} x^2 + 2x, & x < 0, \\ \cos x - 1, & x \geq 0; \end{cases}$ [9]

(ii) $f(x) = \begin{cases} x^4 \sin^2\left(\dfrac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$ [9]

(b) Show that there are at least two values of $x \in [0, \pi]$ for which

$$2 \sin x - \cos x = x.$$

State clearly any results that you use. [7]

**Question 5** (Unit 5) – 15 marks

(a) Determine whether the following limits exist.

(i) $\displaystyle\lim_{x \to 0} 3 \sin\left(\frac{1}{|x|}\right)$ [5]

(ii) $\displaystyle\lim_{x \to 0} |x| \cos\left(\frac{3}{x}\right)$ [5]

(b) In this part of the question you may assume that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

Prove that the following limit exists, and determine its value. State clearly which rules and results you use.

$$\lim_{x \to 0} \frac{\tan x}{x}$$ [5]

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