(ii) the transition matrix from the basis

$$X = \{1, 2+x, 2x+x^2\}$$

to the standard basis for P_3; *[2]*

(iii) the matrix of f with respect to the basis X in the domain and the standard basis in the codomain. *[4]*

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(Analysis Block A)

Question 1 (Unit 1) – 20 marks

(a) Solve the inequality

$$2|x-1| < |x+3|.$$ *[6]*

(b) Use Bernoulli's inequality, first with $x = -4/(5n)$, and then with $x = 4/n$, to prove that

$$1 + \frac{4}{5n-4} \le 5^{1/n} \le 1 + \frac{4}{n}, \quad \text{for } n \ge 1.$$ *[7]*

(c) Determine the greatest lower bound of the set

$$E = \left\{ 4 + \frac{2}{n^2} : n = 1, 2, \ldots \right\}.$$ *[7]*

Question 2 (Unit 2) – 20 marks

(a) Use the definition of null sequence to prove that the sequence

$$a_n = \frac{(-1)^n}{n^3 - 4}, \quad n = 1, 2, \ldots,$$

is null. *[5]*

In the rest of this question you may use the basic null sequences listed in the final frame of the audio-tape section of *Unit 2*, but you should state clearly which results or rules you use.

(b) Determine the limits of the following sequences:

(i) $a_n = \dfrac{3n^3 - 4n + 4}{5n^3 + n^2 - 2n - 3}, \quad n = 1, 2, \ldots,$

(ii) $a_n = \dfrac{3^n - 4(n!) + 2}{2(n!) + n^2 - 2^n}, \quad n = 1, 2, \ldots.$ *[10]*

(c) Prove that the sequence

$$a_n = n^2 - 4/n + 2^n, \quad n = 1, 2, \ldots,$$

tends to infinity. *[5]*