M203 TMA 04

1) $2|x-1| < |x+3|$

Square both sides

$$2^2|x-1|^2 < |x+3|^2$$

$$4(x^2-2x+1) < (x^2+6x+9)$$

$$4x^2-8x+4 < x^2+6x+9$$

$$3x^2-14x-5<0$$

$$(3x+1)(x-5)<0 \quad (\text{LHS}=0 \Rightarrow x=-\tfrac{1}{3}, \text{ or } x=5)$$

To solve this equation, draw up a sign line

x		$-\frac{1}{3}$		5	
$3x+1$	$-$		$+$		$+$
$x-5$	$-$		$-$		$+$
$(3x+1)(x-5)$	$+$		$-$		$+$

From the diagram $-\frac{1}{3}<x<5$. If $x=-\frac{1}{3}$ or 5, then $(3x+1)(x-5)=0$, so the inequality does not hold.

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2) $1+\dfrac{4}{5n-4} \leq 5^{1/n} \leq 1+\dfrac{4}{n}$

Bernoulli's inequality: $(1+x)^n \geqslant 1+nx$

Put $x=-4/5n$

$$\left(1-\frac{4}{5n}\right)^n \geqslant 1-\frac{4}{5n}\times n = 1-\frac{4}{5} = \frac{1}{5}$$

$$1-\frac{4}{5n} \geqslant \left(\tfrac{1}{5}\right)^{1/n}$$

$$\frac{5n-4}{5n} \geqslant \left(\tfrac{1}{5}\right)^{1/n}$$

$$\frac{5n}{5n-4} \leqslant 1/\left(\tfrac{1}{5}\right)^{1/n} = 5^{1/n} \qquad (1)$$

$x=4/n$

$$\left(1+\frac{4}{n}\right)^n \geqslant 1+n\times\frac{4}{n} = 1+4=5$$

$$1+\frac{4}{n} \geqslant 5^{1/n} \qquad (2)$$