Plan of Investigation

In this experiment I am going to require the following:

A calculator

A pencil

A pen

Variety of sources of information

Paper

Ruler

In this investigation I have been asked to find out how many squares would be needed to make up a certain pattern according to its sequence. The pattern is shown on the front page. In this investigation I hope to find a formula which could be used to find out the number of squares needed to build the pattern at any sequencial position. Firstly I will break the problem down into simple steps to begin with and go into more detail to explain my solutions. I will illustrate fully any methods I should use and explain how I applied them to this certain problem. I will firstly carry out this experiment on a 2D pattern and then extend my investigation to 3D.

The Number of Squares in Each Sequence

I have achieved the following information by drawing out the pattern and extending upon it.

Seq. no. 1 2 3 4 5 6 7 8

No. Of cubes 1 5 13 25 41 61 85 113

I am going to use this next method to see if I can work out some sort of pattern:

Sequence Calculations Answer

1 =1 1

2 2(1)+3 5

3 2(1+3)+5 13

4 2(1+3+5)+7 25

5 2(1+3+5+7)+9 41

6 2(1+3+5+7+9)+11 61

7 2(1+3+5+7+9+11)+13 85

8 2(1+3+5+7+9+11+13)+15 113

9 2(1+3+5+7+9+11+13+15) +17 145

What I am doing above is shown with the aid of a diagram below;

If we take sequence 3:

2(1+3)+5=13

2(1 squares)

2(3 squares)

1(5 squares)

The Patterns I Have Noticied in Carrying Out the Previous Method

I have now carried out ny first investigation into the pattern and have seen a number of different patterns.

Firstly I can see that the number of squares in each pattern is an odd number.

Secondly I can see that the number of squares in the pattern can be found out by taking the odd numbers from 1 onwards and adding them up (according to the sequence). We then take the summation (å) of these odd numbers and multiply them by two. After doing this we add on the next consecutive odd number to the doubled total.

I have also noticied something through the drawings I have made of the patterns. If we look at the symetrical sides of the pattern

and add up the number of squares we achieve a square number.

Attempting to Obtain a Formula Through the Use of the Difference Method

I will now apply Jean Holderness' difference method to try and find a formula.

Pos.in seq. 1 2 3 4 5 6

No.of squar. (c) 1 5 13 25 41 61

1st differ. (a+b) 0 4 8 12 16 20

2nd differ. (2a) 4 4 4 4 4

We can now use the equation

an² + bn + c

'n' indicating the position in the sequence.

If a = 2 then c = 1 and a + b = 0 If

2 is equal to b- then b = -2

I will now work out the equation using the information I have obtained

through using the difference method:

1) 2(n -1) (n - 1) + 2n - 1

2) 2(n² - 2n + 1) + 2n - 1

3) 2n² - 4n + 2 + 2n - 1

4) 2n² - 2n + 1

Therefore my final equation is:

2n² - 2n + 1

Proving My Equation and Using it to Find the Number

of Squares in Higher Sequences

I will now prove my equation by applying it to a number of sequences

and higher sequences I have not yet explored.

Sequence 3:

1. 2(3²) - 6 + 1

2. 2(9) - 6 + 1

3. 18 -5

4. = 13

The formula when applied to sequence 3 appears to be successful.

Sequence 5:

1. 2(5²) - 10 + 1

2. 2(25) - 10 + 1

3. 50 - 10 + 1

4. 50 - 9

5. = 41

Successful

Sequence 6:

1. 2(6²) - 12 + 1

2. 2(36) - 12 +1

3. 72 - 12 + 1

4. 72 - 11

5. = 61

Successful

Sequence 8:

1. 2(8²) - 16 + 1

2. 2(64) - 16 + 1

3. 128 - 16 + 1

4. 128 - 15 5. = 113

Successful

The formula I found seems to be successful as I have shown on the previous page. I will now use the formula to find the number of squares in a higher sequence.

So now I wil use the formula 2n² - 2n + 1 to try and find

the number of squares contained in sequence 20.

Sequence 20:

2 (20²) - 40 + 1

2(400) - 40 + 1

800 - 40 + 1

800 - 49

= 761

Instead of illustrating the pattern I am going to use the method

I used at the start of this piece of coursework. The method in which

Iused to look for any patterns in the sequences. I will use this

to prove the number of squares given by the equation is correct.

As shown below:

2(1+3+5+7+9+11+13+15+17+19+21+23+25+27+29+31+33+35+37) + 39 = 761

I feel this proves the equation fully.

Using the Difference Method to Find an Equation to Establish

the Number of Squares in a 3D Version of the Pattern

Pos.in seq. 0 1 2 3 4 5

No.of squar. -1 1 7 25 63 129

1st differ. 2 6 18 38 66

2nd differ. 4 12 20 28 36

3rd differ. 8 8 8 8

So therefore we get the equation;

anƒ + bn² + cn + d

We already know the values of 'n' (position in sequence) in the

equation so now we have to find out the values of a, b, c, and d.

If n = 0 then d = -1 and if n = 1 then d = 1

I can now get rid of d from the equation to make it easier to find

the rest of the values. I will will take n = 2 to do this in the following way:

1st calculation

_ 8a + 4b + 2c + d

a + b + c +d

7a + 3b + c

D will always be added to each side of the equation.

2nd

8a + 4b + 2c = 8 = 4a + 2b + c = 4

2

So then n = 2 8a + 4b + 2c = 8 = 4a + 2b + c = 4

n = 3 27a + 9b + 3c = 26

n = 4 64a + 16b +4c = 64 = 16a + 4b + c = 16

4

To get rid of 'c' I will use this calculation;

_16a + 4b + c = 16

4a + 2b + c = 4

12a + 2b = 12

We can simplify this equation to:

6a + b = 6

My next calculation is below:

N =3 <eth> _27a + 9b + 3c = 26

12a + 6b + 3c = 12 15a + 3b =14

(15a + 3b = 14) ÷ 3

= 5a + b = 4Y

If I use the equation above 6a + b = 6. I can take my latest

equation and subtract it from it to find 'a'.

So, _6a + b = 6

5a + b = 4Y

a = 15

Now that we nave obtained 'a' we can now substitute its value into

the equation to find the other values.

We can now find 'b' by substituting in 15 into the equation as follows:

5(_) + b = 4Y

5(_) = 6Y

b = 4Y - 6Y

b= -2

We have now found values a & b.

I will now attempt to find values c and d by substituting in the two values I now possess.

So we will now sub. in the values to the equation

If we take sequence 2 for our value of n with our present values we will get:

_(8) - 2(4) + c = 8

this can then be simplified to,

_(2) - 2(2) + c = 4

by dividing the equation by two.

We will continue with the calculation using the simplified version of the equation;

To find 'c' we will use the following calculations:

1) _(4) - 2(2) + c = 4

2) 55 - 4 + c = 4

3) 55 + c = 8

4) c = 8 - 55

5) c = 2Y

Therefore my four values are:

a = 15

b = -2

c = 2Y

d = -1

My equation for the working out of the number of cubes in a 3D version of the pattern is:

_(nƒ) - 2n² + 2Y(n) - 1

Testing out the New Equation

I will take sequence 10 to try and test this new equation.

Sequence 5 :

N = 5

_(5ƒ) - 2(5²) + 2Y(5) - 1 =

166Y - 50 + 135 - 1 =

129

The formula on this sequence seems to be successful.

I will now apply it to another sequence to be 100% correct:

N = 4

_(4ƒ) - 2(4²) + 2Y(4) - 1 =

855 - 32 + 10Y - 1 =

63

The formula again proves to be successful.

Using the formula to find the number of squares in a higher sequuence not yet explored in this investigation.

Sequence 10:

N = 10

_(10ƒ) - 2(10²) + 2Y(10) - 1 =

13335 - 200 + 26Y - 1 =

1159

Sequence 15:

N = 15

_(15ƒ) - 2(15²) + 2Y(15) - 1 =

4500 - 450 + 40 - 1 =

4089

This equation has correctly given me the number of squares in each

sequence which again proves it can be applied to any of the 3D sequences to give the correct answer.

My Conclusions

I have made a number of conclusions from the investigation I have carried out.

Firstly I have deciferred that the equation used in the 2D pattern

was a quadratic. This can be proven through the fact that the 2nd

difference was a constant, a necessary element of any quadratic

and also the fact that the first value has to be squared. This can

also be proved by illustrating the equation on the graph, creating a curve.

I have also established that the top triangular half of the 2D pattern

always turns out to be a square number.

If we now look at the 3D pattern, the equation I achieved for it has turned out to a cubic equation. This can be proven through the constant, again a necessary characteristic of any cubic equation and also the fact that its 1st value must be cubed and its second squared. If we drew a graph we would get a ccurved graph in which the line falls steeply, levels off and then falls again.
The Differentiation Method developed by Jean Holderness played a very important role in this investigation. It helped us to gain knowledge of any pattern and anything that would help in the invetigation, giving us our constant, but most importantly it gave us the equation on which to base our solutions.

It was:

an² + bn + c

This proved very helpful.
To find our equation we then substituted in different values which we could find in our differentiation table.
I have concluded that both the equations proved to be very successful.
Therefore the equations are:

For the 2D pattern the equation is;

2n² - 2n + 1

For the 3D pattern the equation is;

_(nƒ) - 2n² + 2Y(n) - 1