

①

a) linear $\Rightarrow \rho = a + bz$

$z = h \Rightarrow \rho = \rho_w$

$z = 0 \Rightarrow \rho = \rho_s$

$a = \rho_s$

and $\rho_w = \rho_s + bh$

so $b = \frac{\rho_s - \rho_w}{h}$

so $\rho = \rho_s - (\rho_s - \rho_w) \frac{z}{h}$

b) $dP = (\rho \, dm)g$ where dm is the mass

of water in a layer $1m^2$ of thick-
ness $dh \Rightarrow dm = \rho \, dh$
 $\int dP = \rho g \, dh = \left(\rho_s - (\rho_s - \rho_w) \frac{z}{h} \right) g \, dz$

$\therefore P - P_a = \rho_s g z - (\rho_s - \rho_w) \frac{g}{2} \frac{z^2}{h}$

$P = P_a + \rho_s g(z-h) - \frac{g}{2} (\rho_s - \rho_w) \frac{z^2 - h^2}{h}$

c) $dF = P \, dA \Rightarrow F = \int P \, dz$ per unit length

$F = P_a h + \rho_s g \left(-\frac{h^2}{2} \right) - \frac{g}{2} (\rho_s - \rho_w) \left(-\frac{2h^3}{3} \right)$