<u>Mas3039 Mathematics: History and Culture</u>

Topic 1: The History of Numbers

Essay (2): Describe in detail the Egyptian number system and how it was used to perform calculations. Give examples from the existing mathematical papyri. What role did mathematics play in ancient Egyptian society?

The Egyptians have left us with some of the oldest writing in the world on a form of paper made from papyrus reeds that grew along the River Nile in Egypt. The reeds were pressed and left to dry in the sun creating long sheets of valuable writing material called papyri. The papyri would be rolled up and easily carried or stored as scrolls. The Egyptians used this very durable papyrus to display their writings. Most of the Egyptian papyrus has not withstood the rigour of time. However the very dry Egyptian climate has preserved some papyri. The most important being that of the Moscow and Rhind papyruses leaving us with very important information of their mathematics. Other sources of Egyptian mathematics come from various carvings on buildings.

Egyptian mathematics dates back to about 2000 BC, although there is earlier evidence of numbers in the form of tags from around 3000 BC.[1] The Egyptian number system was initially based upon a series of pictures/symbols known as hieroglyphics, whereby each number was represented by a hieroglyph. It was based on powers of ten from one to one million. The number one was illustrated by a short length of rope. The number ten was shown as a horseshoe of a longer piece of rope. One hundred, a coil of rope. Rope was used as a hieroglyphic in different shapes and lengths to signify the key role of the “rope stretchers” of ancient Egypt.[2] The hieroglyphic for one thousand was a lotus flower, a finger represented ten thousand, one hundred thousand was pictured as a tadpole/frog, and anything as large one million or more was illustrated by a man with raised arms. Hieroglyphics were written from left to right in ascending order of magnitude and each symbol was written as many times as needed. So using the Egyptian number system a large number like 125368 was written in hieroglyphics as, 8 lengths of rope, 6 horseshoes of rope, 3 coils of rope, 5 lotus flowers, 2 fingers and 1 tadpole/frog. This is illustrated below;

125368 = 8 + 6*(10) + 3*(10^2) + 5*(10^3) + 2*(10^4) + 1*(10^5)

[1] Nigel Byott, lecture on the history of numbers handout
[2] G.G. Joseph, 'The Crest of the Peacock: Non-European Roots of Mathematics', Penguin (1991)

This method of writing is clearly very time consuming and involves a lot of symbols. So a new number system was developed, the hieratic.

The hieratic is an adaptation of the hieroglyphics. They are far more economical because a number of the same hieroglyphs can be replaced with fewer or just one symbol. The only drawback being that is was far more taxing on the memory as there were now far more symbols to remember. Its early use was due to the fact that it was much more suitable for writing in pen and ink. For example 999 would have been illustrated with 27 hieroglyphics compared to just 6 in hieratic representations or 3 in present day numerals.[2]

The Egyptians did not think of numbers as set values. They thought of the number eighteen as the area and perimeter of a rectangle with length 6 and width 3. An example from their architecture is a vestibule in the temple of Hathor at Dendera, which contains 6 by 3 columns in 2 squares of 9. 42 signified the number of lost books of learning, of gods who assess the dead, and of the sins one might commit.[3]

Addition was very simple; it consisted of literally adding the symbols as demonstrated below.

263 + 179

263 =

179 =

263 + 179 =

This can be tidied up and simplified further.

263 + 179 =

Subtraction is much of the same, simply cancelling out the terms subtracted. In the example below the horseshoe of rope must be replaced by ten lengths of rope in order to complete the calculation.

312 – 29

312 =

=

29 =

312 – 29 =

[3] I. Grattan-Guinness, 'The Fontana History of the Mathematical Sciences', Fontana (1997)

The Egyptian method of multiplication only required the knowledge of addition and the two times table. This method was needed to overcome the deficiencies of their number system and is shown in the Rhind papyrus. To multiply 43 by 23 the following method was used.

1	23
2	46
4	92
8	184
16	368
32	736
	989

They simply doubled each column successively until a combination of numbers in the left column can be added to make the multiplier which in this case is 43. The corresponding values in the right column are added to obtain the answer to the multiplication.

Division was very similar; it was again based upon the process of doubling and halving. According to the Rhind papyrus a division such as x/y is described as, "reckon y as to obtain x".[2] So for the division 143/11 Egyptians would be thinking, "starting with 11 how many times should I add it to itself to get 143" instead of, "dividing 143 by 11". Below is an example, 143 divided by 11, of how it worked.

/	1	11
	2	22
/	4	44
/	8	88
	13	143

These methods were devised because basic arithmetic was needed by the Egyptians for every day life. Mathematics was used in measurement, building the pyramids, ritual practices and also to develop a good solar calendar that consisted of 12 months and 5 extra feast days.[4] The government also

[4] C.B. Boyer, 'A History of Mathematics', Wiley (1968)

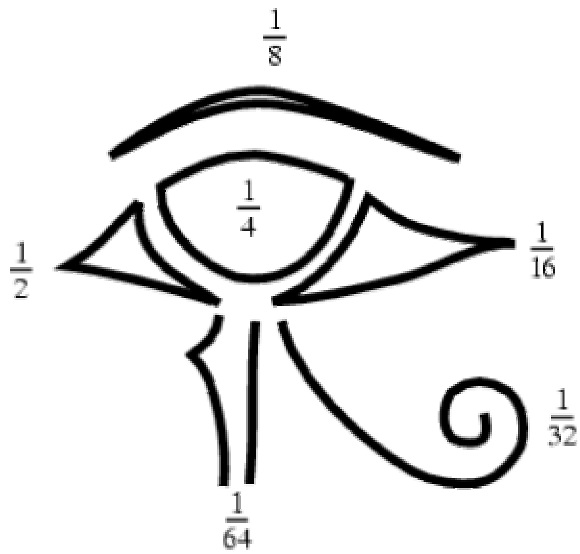
benefited as basic arithmetic aided them in tax collection. But it was in trade that the idea of fractions was required.

Fractions were used because of the need for calculations in real life situations. Food and land needed to be shared out equally and the ingredients for bread and beer measured out accurately. The Egyptians probably didn't think of fractions as rational numbers instead relating them to gradients and ratios. They only used unit fractions, meaning fractions with the numerator 1, the only exception being that of 2/3. This was considered as a special fraction and given its own symbol. Unit fractions were written in two ways as pictured below.

Only the Egyptians used the system of unit fractions. It meant that fractions with the numerator greater than 1 were broken down and expressed as the sum of unit fractions. For example 2/7, which we today see as irreducible, would be written as 1/4 + 1/28. For some unknown reason it was not possible for the Egyptians to express a fraction such as 2/7 as 1/7 + 1/7 or (1/7) * 2. We call a formula representing a sum of distinct unit fractions an Egyptian fraction.[5] Fractions also occur due to division. To help the Egyptians to divide and multiply fractions a scribe created a table of the decompositions of 2/n into unit fractions for all odd values of n from 3 to 101. This is on view at the start of the Rhind papyrus and contains no errors. The table is very useful because of the notation of the Egyptian fraction is very cumbersome and difficult to compute with. Arithmetic problems could be solved by looking up the answer in the tables. This table can be compared to that of tables we use today such as a log table and statistics tables. A number of views have been put forward as to how the Egyptians decomposed fractions into unit fractions. They did prefer to keep to even fractions if possible and use no more than 4 fractions with small denominators. Unit fractions were clearly very important to The Egyptians as a hieroglyphic was created called the Horus eye. Each of the sacred unit fractions which the ancient Egyptians attributed to the six parts of the eye of the god Horus, all with powers of two in their denominators, were used to represent the fractions of Hekat, the unit measure of capacity for grains.[6] This can be seen below;

[5] www.ics.uci.edu/~eppstein/numth/egypt/

[6] http://mathworld.wolfram.com/EyeofHorusFraction.html



The Rhind papyrus is the richest single mathematical text of early Egyptian maths and is named after its English purchaser, however it is also known as the Ahmes papyrus after the scribe who composed it. The other important text was the Moscow papyrus that was composed by an unknown and less able scribe. It is similar to the Rhind papyrus in that it covers the same topics but it is poorly set out. The Rhind papyrus is of size 18 foot by 1 foot and the Moscow papyrus is 18 foot by 3 foot.[7] Together the 2 papyruses contain 112 real life problems and solutions, 87 in the Rhind papyrus and 25 in the Moscow papyrus.[2] They both contain problems that would be set in the training of scribes. The scribes would then go on to work in taxation and surveying with the information they had gained from completing the problems set in the papyruses. Most problems concerned the distribution of bread and beer as this is what early Egyptian maths was primarily used for, everyday life situations.

The first six problems in the Rhind papyrus are examples of this. It asks how to divide n loaves of bread between 10 men where n =1 for Problem 1, n = 2 for Problem 2, n = 6 for Problem 3, n = 7 for Problem 4, n = 8 for Problem 5, and n = 9 for Problem 6.[8] So for problem 6, sharing 9 loaves between 10 men, we would simply give 9 men 9/10's of a loaf each and the other man 9 pieces of size 1/10 of a loaf. The Egyptians worked otherwise deeming this an unfair way of sharing. They preferred to look up the decomposition of 9/10 to find 9/10 = 2/3 + 1/5 + 1/30 so that 7 men would get 3 pieces each with sizes 2/3 + 1/5 + 1/30 of a loaf and the other 3 men receiving 4 pieces, 1/3 + 1/3 + 1/5 + 1/30 of a loaf.[2] 81 of the 87 problems in the Rhind papyrus involve operating with fractions. This tells us

[7] V.J. Katz, 'A History of Mathematics', Addison-Wesley (1998)
[8] Mactutor website: http://www-groups.dcs.st-and.ac.uk/~history/

just how important a role mathematics played in Egyptian society because most of these problems involved measuring, sharing and counting out quantities so basic arithmetic and fractions were needed. Another example is problem 26; a quantity added to a quarter of that quantity becomes 15. What is that quantity?[8]

A table in Gillings's book shows a record of payments in loaves of bread and beer to various temple personnel at Illahun around 2000bc. 70 loaves, 35 jugs of beer 1 and 115 and a 1/2 jugs of beer 2 was shared between 21 people. The unit of distribution being 1/42 of the following, 1 + 2/3 of bread, 2/3 + 1/6 of beer 1 and 2 + 2/3 + 1/10 of beer 2. From this table we can get an idea of the status of personnel at the temple. The Temple Director was at the top receiving 10 portions of each and the scribes lower down. The table is very interesting and revealing as it contains a small arithmetic error which is carried through. If the scribe added up the portions he would have spotted his error. Despite this single error the table proves just how well Egyptians could handle fractions despite the deficiencies in their number system. The division of beer and bread into very small amounts gives an indication of how accurate the Egyptians were at measuring and weighing.[9]

Another real life situation is Problem 64 which involves geometric series', "divide 10 hekats of barley among 10 men so that each gets $^{1}/_{8}$ of a hekat more than the one before."[8] Other problems involved geometry, namely Problem 50, "a round field has diameter 9 khet. What is its area?"[8]

A counter game that was found preserved in the tomb of Tutankhamen suggests the Egyptians had begun to think about probability but there is no evidence of workings to support this.[3] They also had a little understanding of trigonometry through astronomy as proved by the construction of pyramids.

Some believe Egypt is where the earliest mathematics took place. The Egyptians achieved many things by creating their own methods of arithmetic which were based around addition because of the limitations of their number systems. They posed many questions and puzzles which were used for educational purposes but were only concerned with arithmetic that helped them in every day life. They had no concept of zero, used no placeholder or decimal point and lacked in abstract thought, generalizations and theory.[10]

Overall the Egyptians played a very important part in the history of mathematics and there may be more to their mathematics than evidence suggests as we know that Pythagoras and others

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[9] R.J.Gillings, 'Problems 1 to 6 of the Rhind Mathematical Papyrus', The Mathematics Teacher (1962)
[10] Robbins, Gay and Charles Shue, 'The Rhind Mathematical Papyrus', New York: Dover, (1987)

visited Egypt to study. If there was only the everyday problems we see in the Rhind and Moscow papyri the trip would have had little value. But as most papyrus has not withstood the test of time we have no evidence of more complex mathematics. It is possible that Egyptian mathematics was rewritten by the Greeks who then continued the good work of the Egyptians.

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