Calculus III, 2011: Coursework 2

Will Sutherland.

The deadline is 4PM on Thursday Oct 13th. Hand in your solution for **Question 4 only** *to the green box on the basement floor of the Maths building.*

1. Given a plane defined by $2x - 2y + z = 12$, find:

(a) The unit vector normal to the plane ;

(b) The point in the plane closest to the origin.

(c) The perpendicular distance from the point $(6, -6, 3)$ to the plane.

2. Find the gradient vector (∇f) for $f(x, y, z) = 2x^3y + 3(x^2 - y^2) + z$.

3. Consider a sphere of radius 9 centred at the origin.

(a) Express the surface of the sphere in the form $f(x, y, z) = 0$.

(b) Find a plane tangent to the sphere at the point $\mathbf{r}_1 = (7, 4, -4)$.

(c) Find the position vector of points on a line through $\mathbf{r}_1$ and normal to the sphere.

(d) Find the position vector of points on the line through $\mathbf{r}_1$ and $\mathbf{r}_2 = (4, 7, 4)$.

4. (*) Hand-in question:

(a) Calculate the volume of a parallelepiped (the 3D version of a parallelogram), where 3 edges are given by the vectors $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. [5]

(b) Find equations for the (i) tangent plane and (ii) normal line, to the surface $3x^2 - y^2 + 2z^2 = 1$, at the point $P_0 = (1, 2, -1)$. [5]

5. Find the directional derivative of the scalar field $V = 4y^2z + xyz^2$ in the direction of the vector $(2, -1, -1)$ at the point $P = (1, -2, -1)$. [Hint: write down the unit vector $\mathbf{n}$ which has the same direction as the given vector, and evaluate $\mathbf{n} \cdot \nabla V$ at the point P.]

6. Prove that $\nabla(r^n) = nr^{n-2}\mathbf{r}$, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is the usual position vector and $r = |\mathbf{r}| = (x^2 + y^2 + z^2)^{1/2}$.