*University of Edinburgh*

# Computer Methods in Mechanical Engineering 3 Computing Project

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# 1. INTRODUCTION

In the construction of any type of frame structure it is necessary to investigate the effects of any loads that may be acting on the structure and how it reacts against these loads. This is very important to investigate as it determines whether or not the structure will fail or deform in a critical way. MASTAN2 is a program within MATLAB, which allows the user to create a model of the frame to be analysed and apply the loads that will be acting on it to determine important information such as the bending moments, axial forces, shear forces and deflected shape to name a few.

The tasks for this project have been split into two questions; the aims of the first question are as follows:

- Create a plane frame in MASTAN2 with the specifications detailed in the question under the full factored load (fig. 1) and perform a first order analysis
- Determine the size of the joint stiffness matrix K and its partitions $K_{FF}$, $K_{FR}$, $K_{RF}$, $K_{RR}$
- Write the full member stiffness matrix for member 4 and rotate it to structure directions
- Draw complete bending moment diagrams for all the members and indicating the maximum bending moment values and the locations of the points of contraflexure
- Locate the maximum bending moments along the span of members 3 and 6
- Draw an exaggerated deflection shape of the beam under the load, locating the points of contraflexure
- Locate the points of maximum deflection along members 3 and 6
- Draw the axial and shear force diagrams for all the members, indicating the maximum values
- Apply combinations of the full factored load and the dead load to members 3 and 6 to find the worst permutations that lead to the greatest bending moments and shear forces at midspan and supports, indicating their values
- Draw a bending moment envelope for members 3 and 6 based on the analysis

For the second question, the aim was as follows:

- Investigate the effects of different types of arch (sin curve, parabola and circular arc) under numerous point loads (fig. 2) and evaluate which is most suitable when H=L/2, H=L/4 and H=L/8 (where H = Height and L = Length)

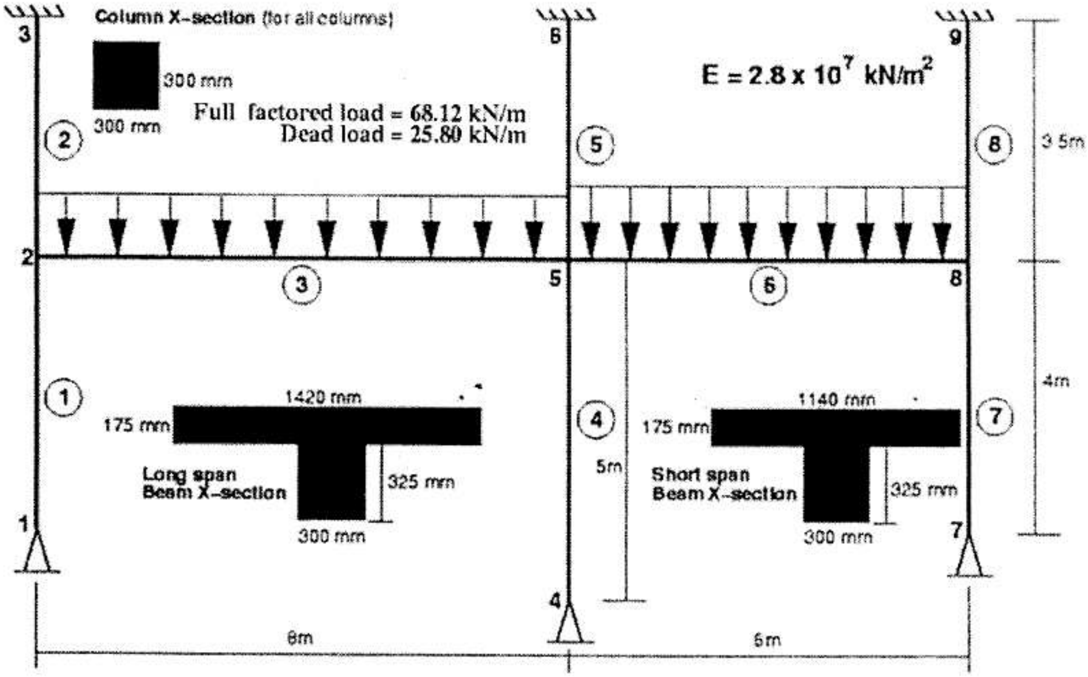


Figure 1 – Frame with beam cross sections and material properties

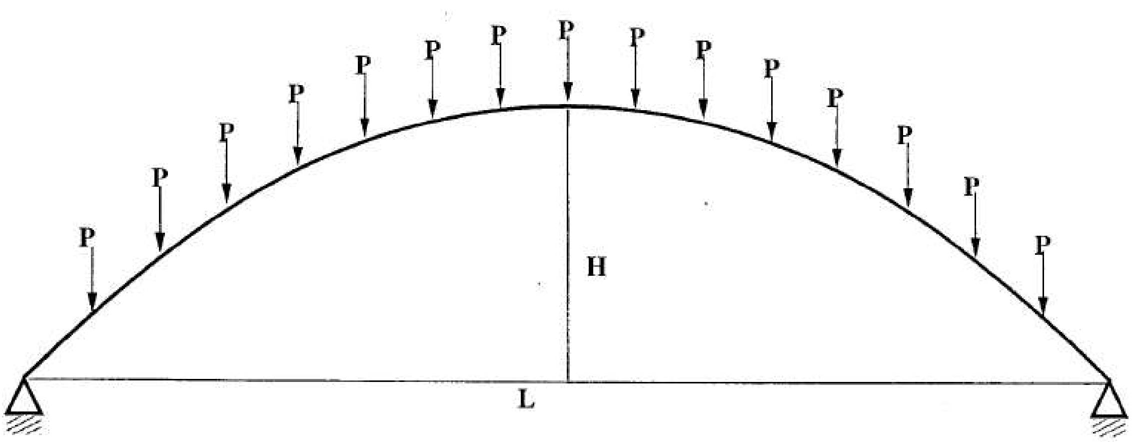


Figure 2 – Arc indicating height and length and point loads

# 2. SOLUTIONS TO PROBLEM 1

The first task was to find the size of the joint stiffness matrix, K, using the direct stiffness method approach. As the structure has nine joints and each joint has three Degrees of Freedom (DOF) (when analysed as a planar frame) the complete joint stiffness matrix will therefore be K = [27 x 27]. Looking at the frame and noting the type of supports used it can be seen that there are 12 free DOF and therefore its partitions are as follows: $K_{FF} = [12 \times 12]$ (12 stiffness terms in DOF direction due to 12 free displacements), $K_{RF} = [15 \times 12]$ (15 stiffness terms in restrained direction due to 12 free displacements), $K_{FR} = [12 \times 15]$ (12 stiffness terms in DOF direction due to 15 restrained displacements) and $K_{RR} = [15 \times 15]$ (15 stiffness terms in restrained direction due to 15 restrained displacements). The full member stiffness matrix and supporting calculations for member 4 can be found in appendix 1.

$$E = 2.8 \times 10^{7}\, kN/m^2 = 28\, kN/mm^2$$

Full Factored Load = 68.12 kN/m = 0.06812 kN/mm, Dead Load = 25.80 kN/m = 0.0258 kN/mm

<u>Column X-Section:</u>

$$A = bd = 300 \times 300 = 90000\ mm^2 \qquad I = \frac{1}{12}bd^3 = 6.75 \times 10^8\, mm^4$$

<u>Member 3 X-Section:</u>

$$A = 34500\ mm^2 \qquad I = 586897333\ mm^4 \quad \left(I = \sum \frac{1}{12}bd^3 + A\bar{y}\right)$$

<u>Member 6 X-Section:</u>

$$A = 29700\ mm^2 \qquad I = 540662025\ mm^4$$

Once these values had all been placed into MASTAN2, the full factored load was placed onto members three and six and the first order analysis was performed on the structure. A bending moment diagram was the produced, using the facilities on the program, which can be seen in figure 3. Points of contraflexure are indicated by circles, there is one point on each of the vertical members and two points on each of the horizontal members. The locations of the points of contraflexure can be seen in table 1. Table 2 shows the maximum positive bending moments within the spans of members 3 and 6.

| Element | Point | Location |
|---|---|---|
| 1 | 1 | 0m from bottom |
| 2 | 2 | 1.12m from top |
| 3 | 3 | 0.56m from left |
| | 4 | 6.4m from left |
| 4 | 5 | 0m from bottom |
| 5 | 6 | 1.26m from top |
| 6 | 7 | 1.8m from left |
| | 8 | 5.7m from left |
| 7 | 9 | 0m from bottom |
| 8 | 10 | 1.26m from top |

Table 1 – Locations of points of contraflexure in frame in Figure 1

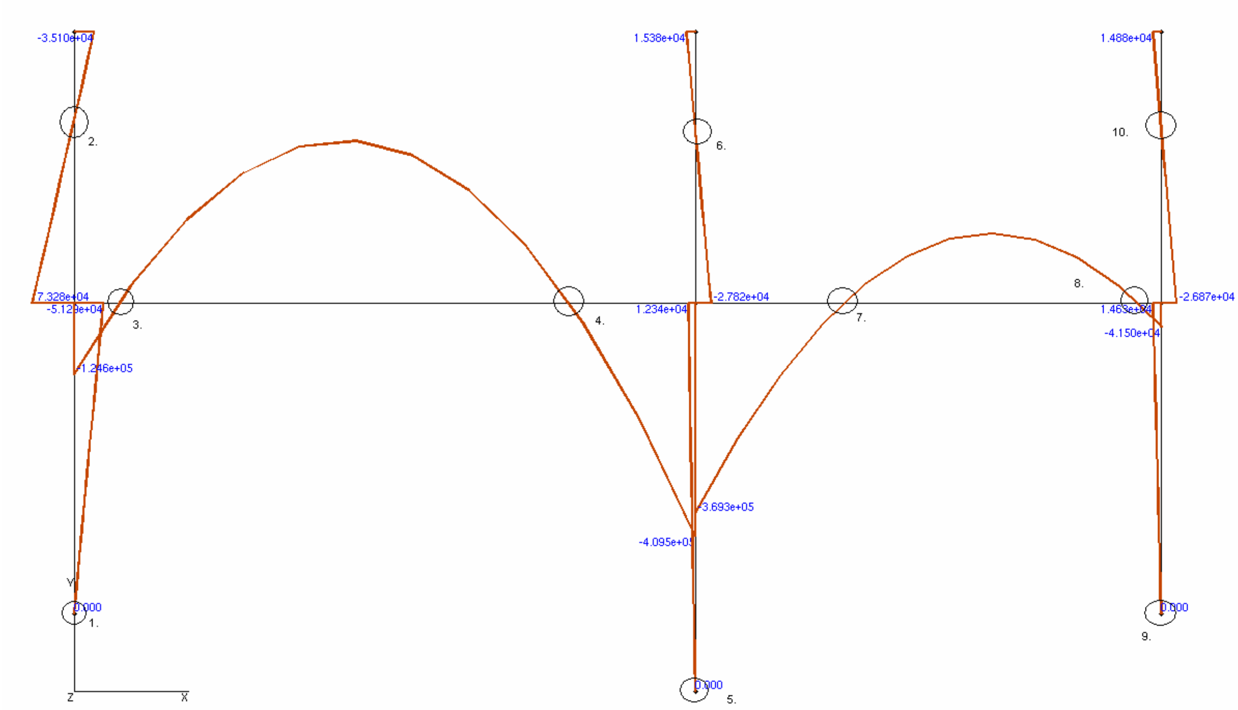


Figure 3 – Frame under full factored load on members 3 and 6, showing bending moments and locations of contraflexure

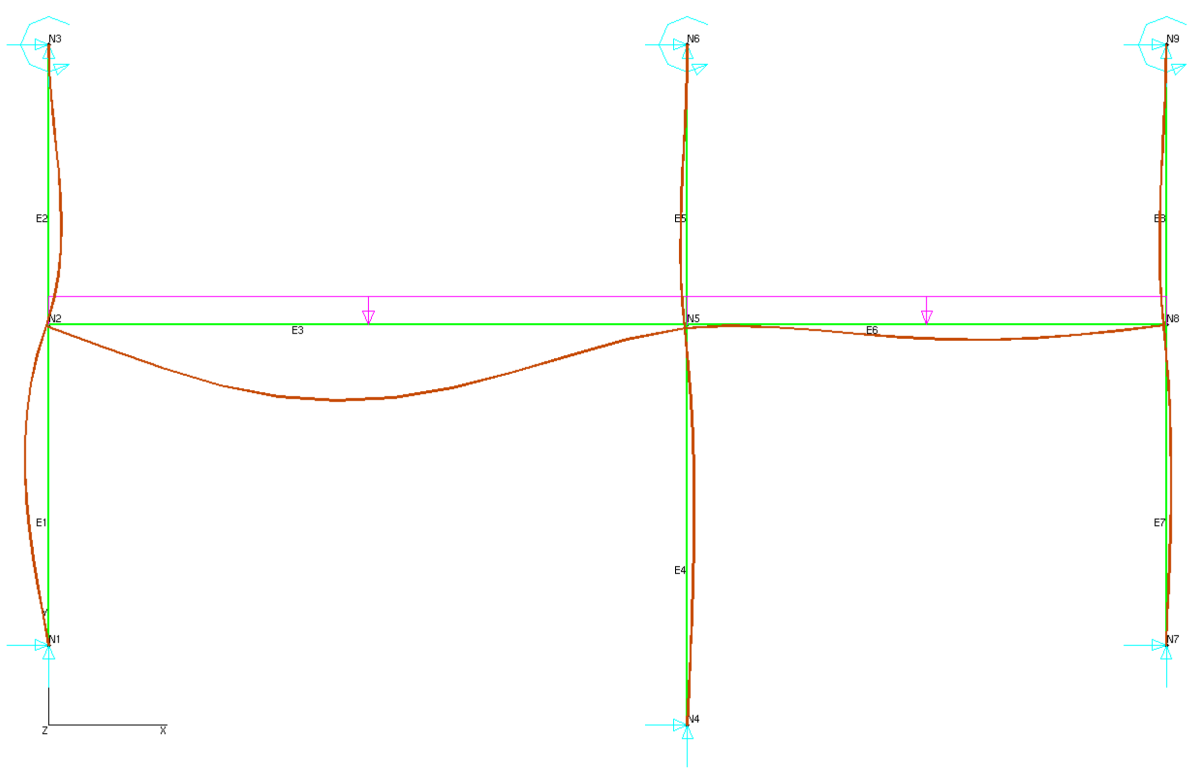


Figure 4 – Frame under full factored load on members 3 and 6, showing exaggerated deflection

| Element | Value (kNmm) | Location |
|---|---|---|
| 3 | 2.872*10^5 | 3.44m from left |
| 6 | 1.23*10^5 | 3.84m from left |

Table 2 – Maximum positive bending moments along span and location

Figure 4 shows the exaggerated deflected shape of the frame under the full factored load. It can be clearly seen from the comparison of figures 3 and 4 that the location of the maximum deflection is at the point of maximum bending moment along the horizontal spans. This is what would be expected as the larger the bending moment then the greater the effect on the member and thus a larger deflection. The values and locations of the maximum deflections in members 3 and 6 can be found in table 3.

| Element | Value (mm) | Location |
|---|---|---|
| 3 | 9.5 | 3.7m from left |
| 6 | 1.92 | 3.6m from left |

Table 3 – Maximum deflection along members 3 and 6 and location

Figures 5 and 6 show the complete shear force and axial force diagrams for the frame under the full factored load respectively with the maximum values indicated on the diagrams.

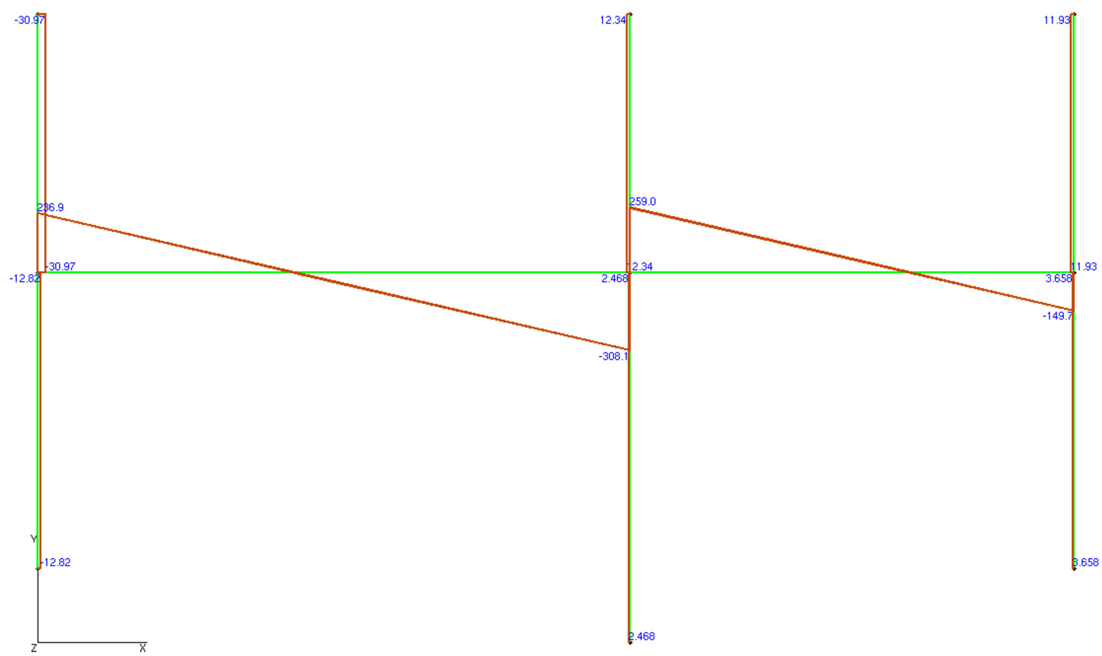


Figure 5 – Frame showing shear force diagram and values of maximum shear force

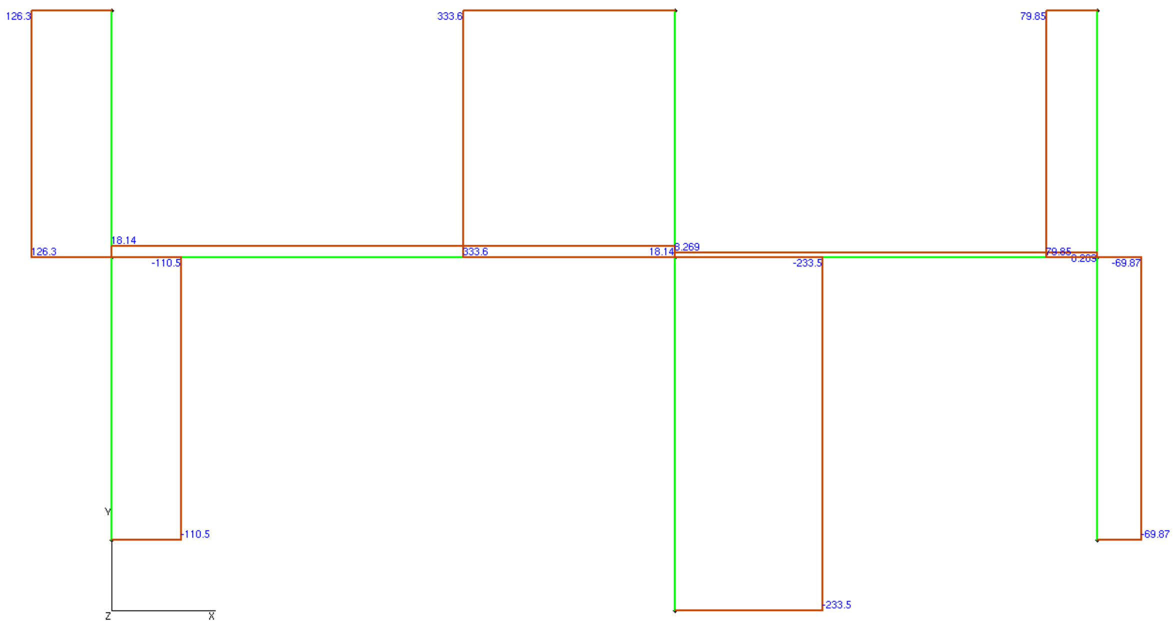


Figure 6 – Frame showing axial force diagram and values of maximum axial force

There are a number of different loading combinations that can be applied to the frame that will affect the bending moments and shear forces of the different members. The following figures (7 – 12) show the bending moments and shear forces of all the different loading permutations.

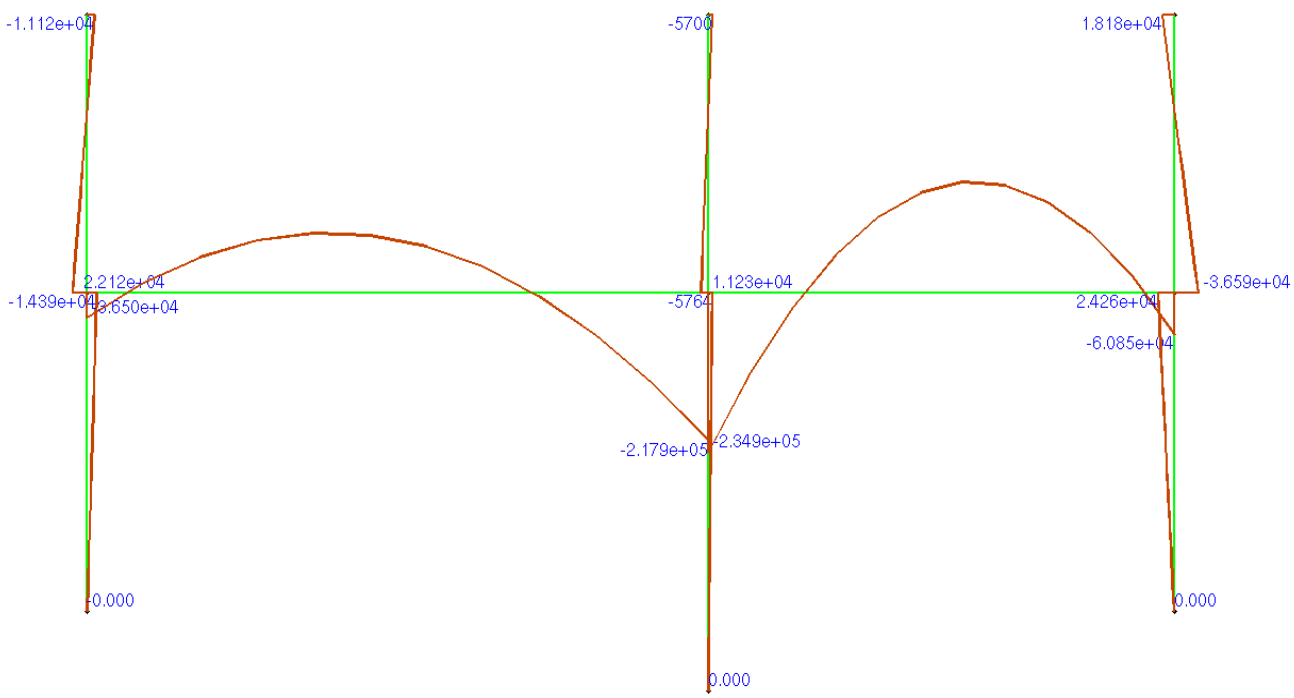


Figure 7 – Bending moment diagram under dead load on member 3 and full load on member 6

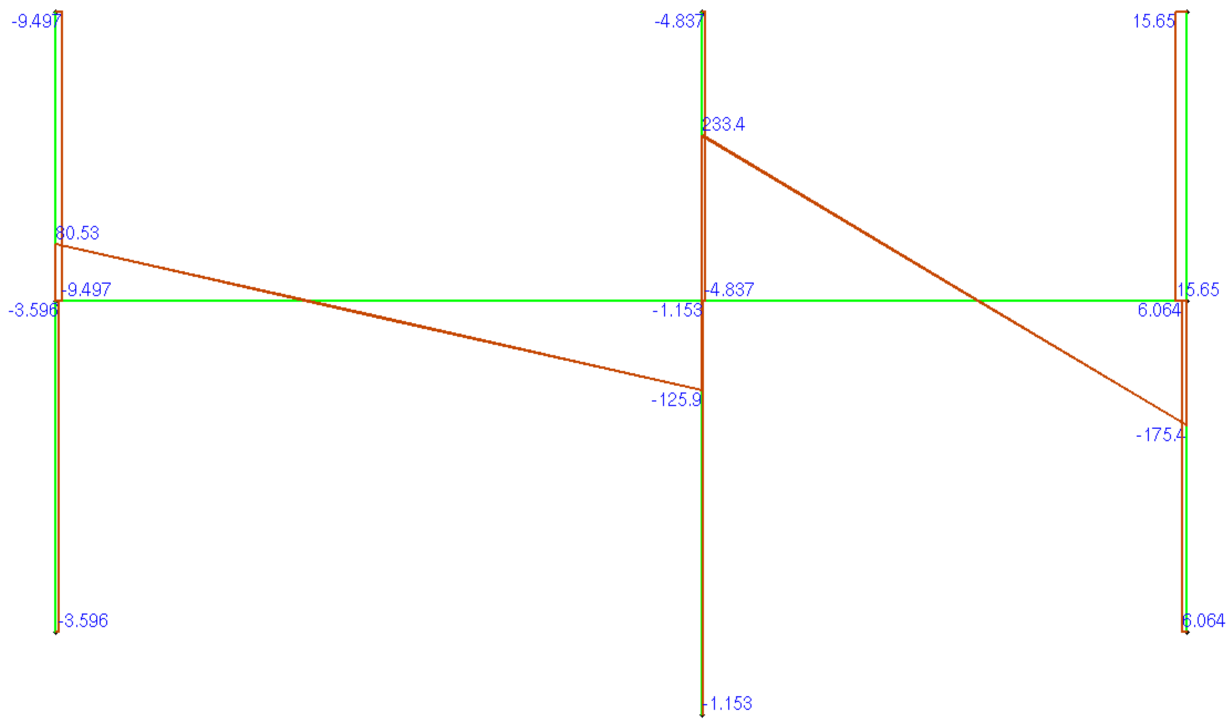


Figure 8 – Shear force diagram under dead load on member 3 and full load on member 6

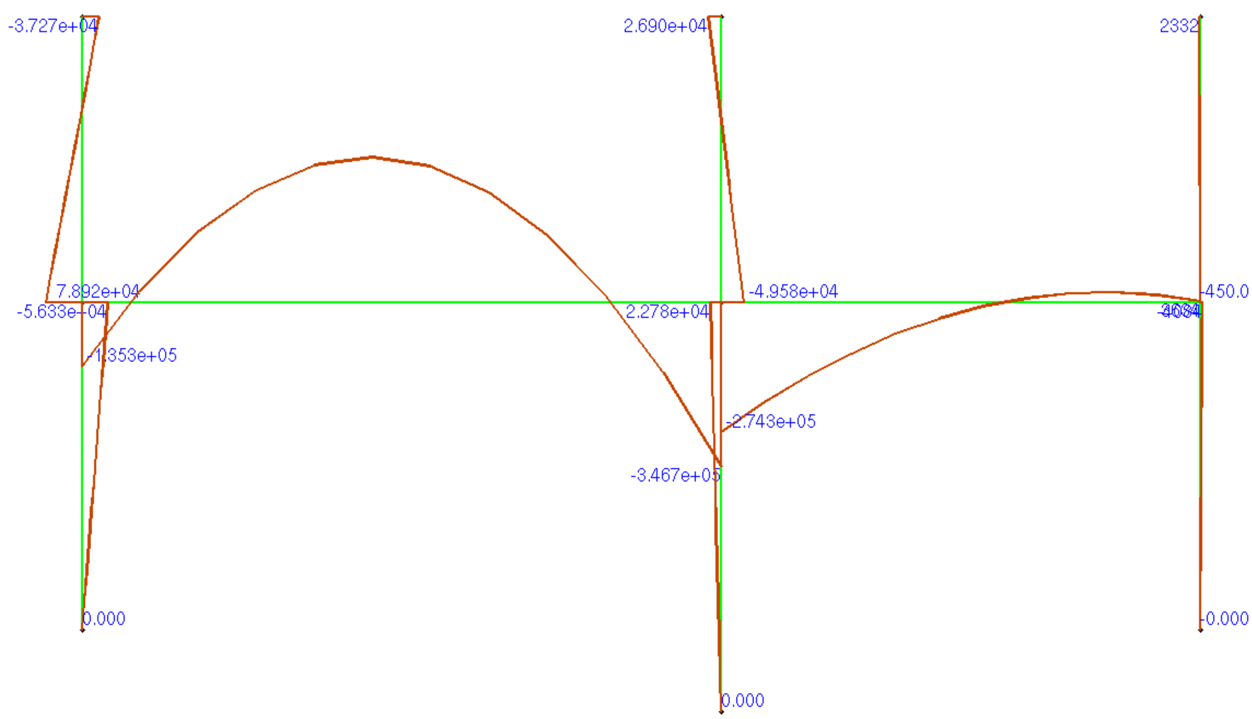


Figure 9 – Bending moment diagram under full load on member 3 and dead load on member 6

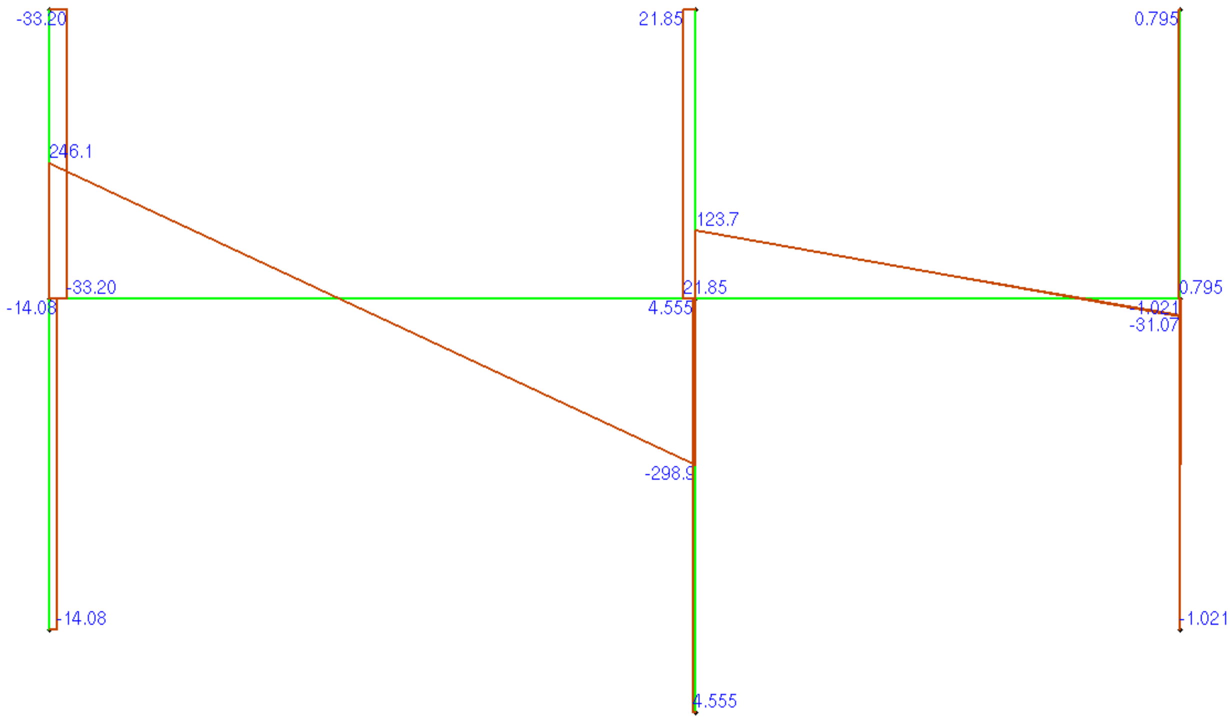


Figure 10 – Shear force diagram under full load on member 3 and dead load on member 6

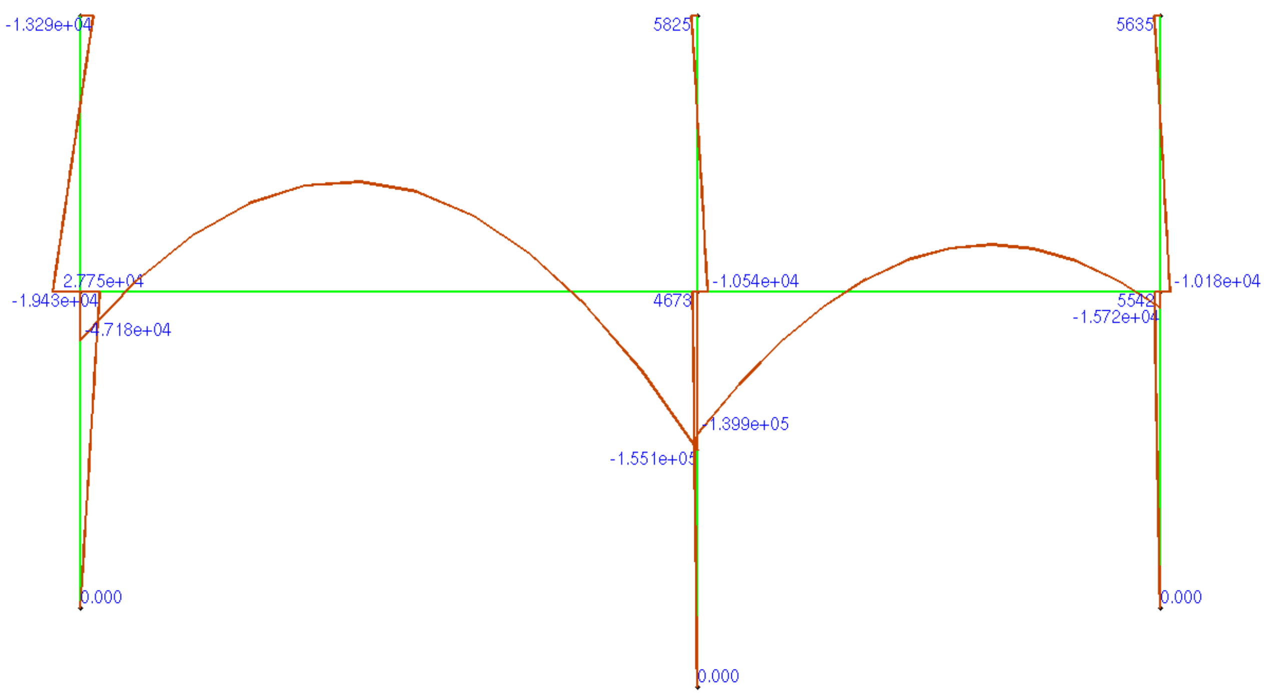


Figure 11 – Bending moment diagram under dead load on member 3 and dead load on member 6

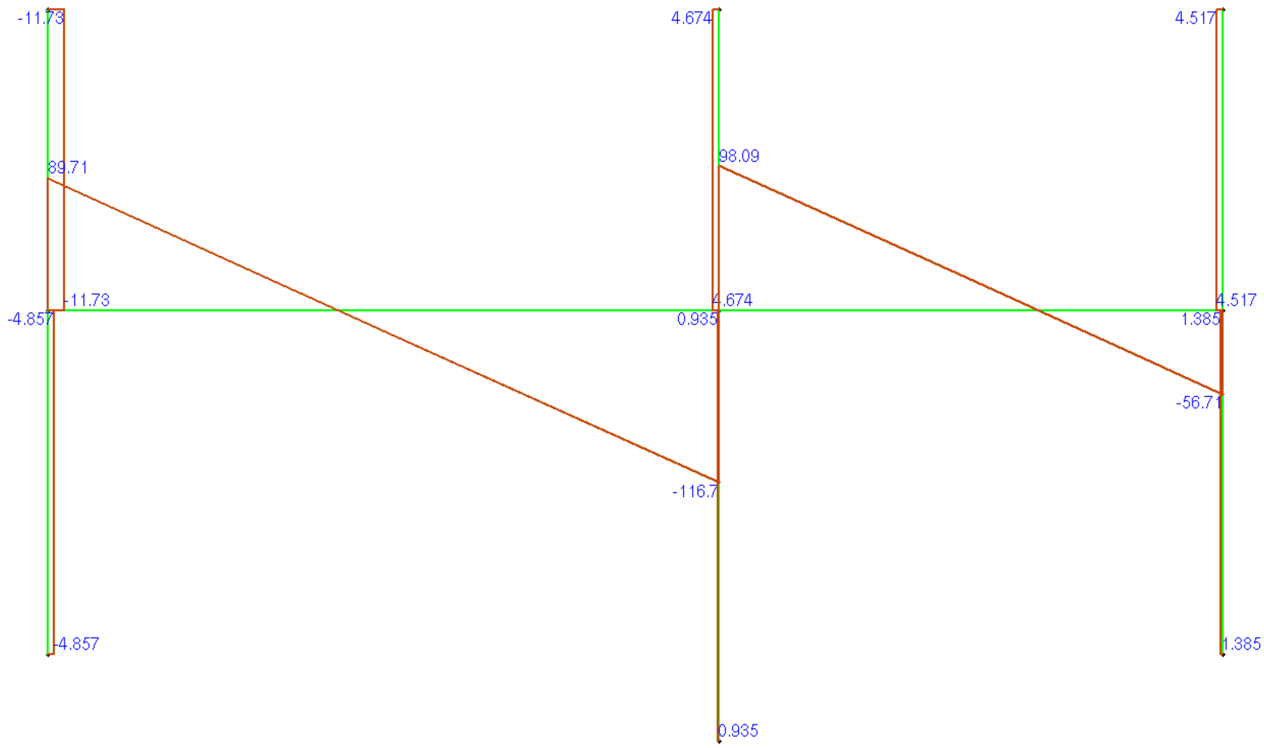


Figure 12 – Shear force diagram under dead load on member 3 and dead load on member 6

| Member | Node | FF Moment (kNmm) | FD Moment (kNmm) | DF Moment (kNmm) | DD Moment (kNmm) |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 |
| | 2 | -510000 | -560000 | -140000 | -190000 |
| 2 | 2 | 730000 | 790000 | 220000 | 280000 |
| | 3 | -350000 | -370000 | -110000 | -130000 |
| 3 | 2 | -1200000 | -1400000 | -370000 | -470000 |
| | 5 | -4100000 | -3500000 | -2200000 | -1500000 |
| 4 | 4 | 0 | 0 | 0 | 0 |
| | 5 | 120000 | 230000 | -5764 | 4673 |
| 5 | 5 | -280000 | -500000 | 110000 | -10000 |
| | 6 | 150000 | 270000 | -5700 | 5825 |
| 6 | 5 | -3700000 | -2700000 | -2300000 | -1400000 |
| | 8 | -410000 | 3604 | -600000 | -160000 |
| 7 | 7 | 0 | 0 | 0 | 0 |
| | 8 | 150000 | -4084 | 240000 | 5542 |
| 8 | 8 | -270000 | -450 | 370000 | 10000 |
| | 9 | 150000 | 2332 | 180000 | 5635 |

Table 4 – Bending moments at each node due to different load combinations (FF = Full load on member 3 and Full load on member 6, FD = Full load on member 3 and Dead load on member 6 etc.)

| Load type | Member | Mid Span Maximum Bending Moment | Location |
|---|---|---|---|
| FD | 3 | 3.96*10^5 kNmm | 3.44m from left |
| | 6 | 1.7*10^5 kNmm | 3.84m from left |
| DF | 3 | 8.9*10^5 kNmm | 3.12m from left |
| | 6 | 1.648*10^5 kNmm | 3.42m from left |
| DD | 3 | 1.071*10^5 kNmm | 3.12m from left |
| | 6 | 4.659*10^5 kNmm | 3.78m from left |

Table 5 – Maximum bending moments at mid span for members 3 and 6 under different loading combinations

With the information showing in tables 4 and 5 it is possible to draw a bending moment envelope for members 3 and 6 as can be seen in figure 13 below.

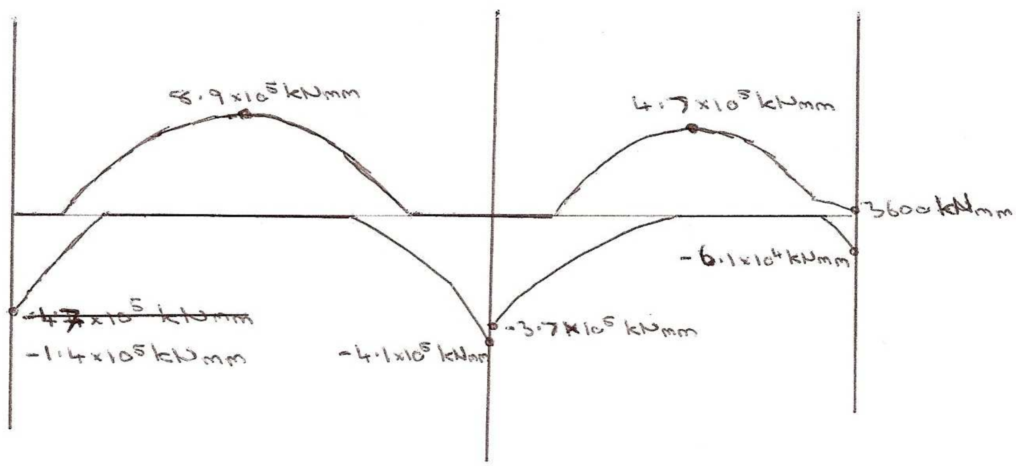


Figure 13 – Bending moment envelope for different loading permutations

## 3. SOLUTIONS TO PROBLEM 2

For this task we were required to analyse a bridge structure. The structure (figure 2) is used to support a bridge deck that is suspended from cables at the locations of the loads (15 in total), which are equally spaced apart. It was decided that the bridge be made from a rectangular cross section of concrete measuring 0.25m by 0.25m and as such has the following properties:
$E = 2.8*10^7$ kN/m$^2$, $I = bd^3/12 = 3.255*10^{-4}$ m$^4$ and $A = 0.0625$ m$^2$ .

The arch was to be constructed from three different shapes (a sin curve, a parabola and a circular arc) and for each case three different dimensional conditions were applied (H = L/2, H = L/4, H = L/8). Each arch was to be compared against one another for each dimensional condition to find the most efficient one. The following equations were created to find the y-coordinates of the arches that were to be created in MASTAN2:

| | |
|---|---|
| a) H = L/2 | Sin Curve: $y = 8\sin(11.25x)$<br>Parabola Curve: $y = -\frac{x^2}{8} + 2x$<br>Circular Arc: $y = \sqrt{64 - (x-8)^2}$ |

| | |
|---|---|
| b) H = L/4 | Sin Curve: $y = 4\sin(11.25x)$<br>Parabola Curve: $y = -\frac{x^2}{4} + x$<br>Circular Arc: $y = \sqrt{100 - (x-8)^2} - 6$ |

| | |
|---|---|
| c) H = L/8 | Sin Curve: $y = 2\sin(11.25x)$<br>Parabola Curve: $y = -\frac{x^2}{2} + (x/2)$<br>Circular Arc: $y = \sqrt{289 - (x-8)^2} - 15$ |

Using the above equations a table of the coordinates to be used was created using excel as can be seen in table 6.

| X – Coordinate | Sin(L/2) | Sin(L/4) | Sin(L/8) | Parabola (L/2) | Parabola (L/4) | Parabola (L/8) | Circle (L/2) | Circle (L/4) | Circle (L/8) |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 | 1.561 | 0.780 | 0.390 | 1.875 | 0.938 | 0.469 | 3.873 | 1.141 | 0.492 |
| 2 | 3.061 | 1.531 | 0.765 | 3.500 | 1.750 | 0.875 | 5.292 | 2.000 | 0.906 |
| 3 | 4.445 | 2.222 | 1.111 | 4.875 | 2.438 | 1.219 | 6.245 | 2.660 | 1.248 |
| 4 | 5.657 | 2.828 | 1.414 | 6.000 | 3.000 | 1.500 | 6.928 | 3.165 | 1.523 |
| 5 | 6.652 | 3.326 | 1.663 | 6.875 | 3.438 | 1.719 | 7.416 | 3.539 | 1.733 |
| 6 | 7.391 | 3.696 | 1.848 | 7.500 | 3.750 | 1.875 | 7.746 | 3.798 | 1.882 |
| 7 | 7.846 | 3.923 | 1.962 | 7.875 | 3.938 | 1.969 | 7.937 | 3.950 | 1.971 |
| 8 | 8.000 | 4.000 | 2.000 | 8.000 | 4.000 | 2.000 | 8.000 | 4.000 | 2.000 |
| 9 | 7.846 | 3.923 | 1.962 | 7.875 | 3.938 | 1.969 | 7.937 | 3.950 | 1.971 |
| 10 | 7.391 | 3.696 | 1.848 | 7.500 | 3.750 | 1.875 | 7.746 | 3.798 | 1.882 |
| 11 | 6.652 | 3.326 | 1.663 | 6.875 | 3.438 | 1.719 | 7.416 | 3.539 | 1.733 |
| 12 | 5.657 | 2.828 | 1.414 | 6.000 | 3.000 | 1.500 | 6.928 | 3.165 | 1.523 |
| 13 | 4.445 | 2.222 | 1.111 | 4.875 | 2.438 | 1.219 | 6.245 | 2.660 | 1.248 |
| 14 | 3.061 | 1.531 | 0.765 | 3.500 | 1.750 | 0.875 | 5.292 | 2.000 | 0.906 |
| 15 | 1.561 | 0.780 | 0.390 | 1.875 | 0.938 | 0.469 | 3.873 | 1.141 | 0.492 |
| 16 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Table 6 – y-coordinates for all curve styles and dimensional conditions

Using the material properties, geometry and the information in table 6 the curves were created in MASTAN2 and first order analysis was performed on each one, assuming the arch to be a plane frame with straight segments between loads.

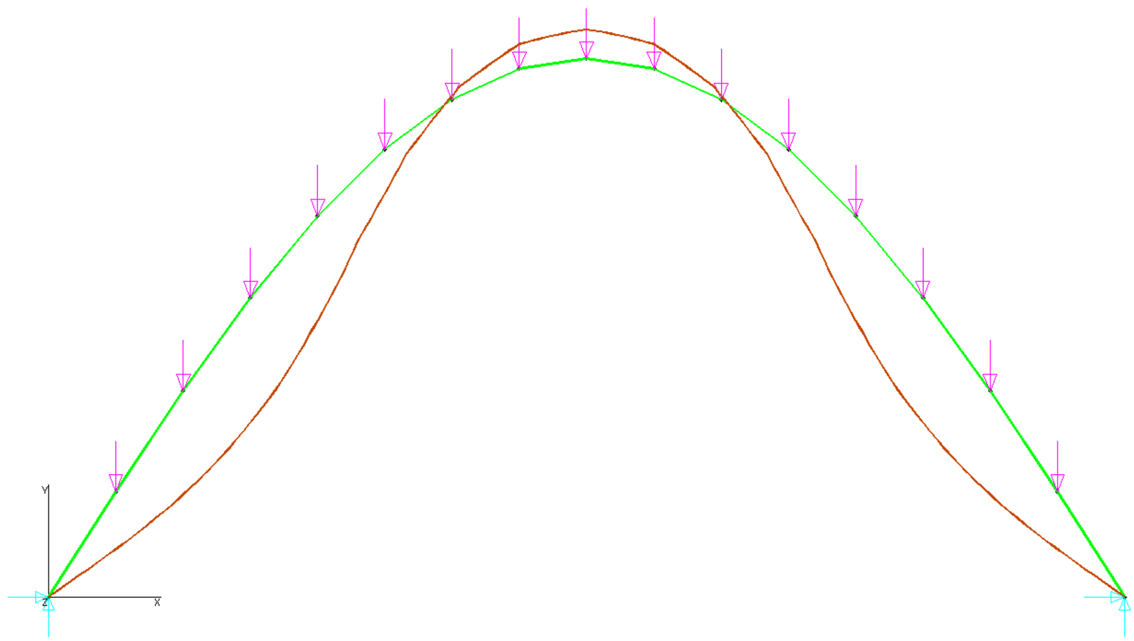


Figure 14 – H = L/2 Sin curve deflected shape

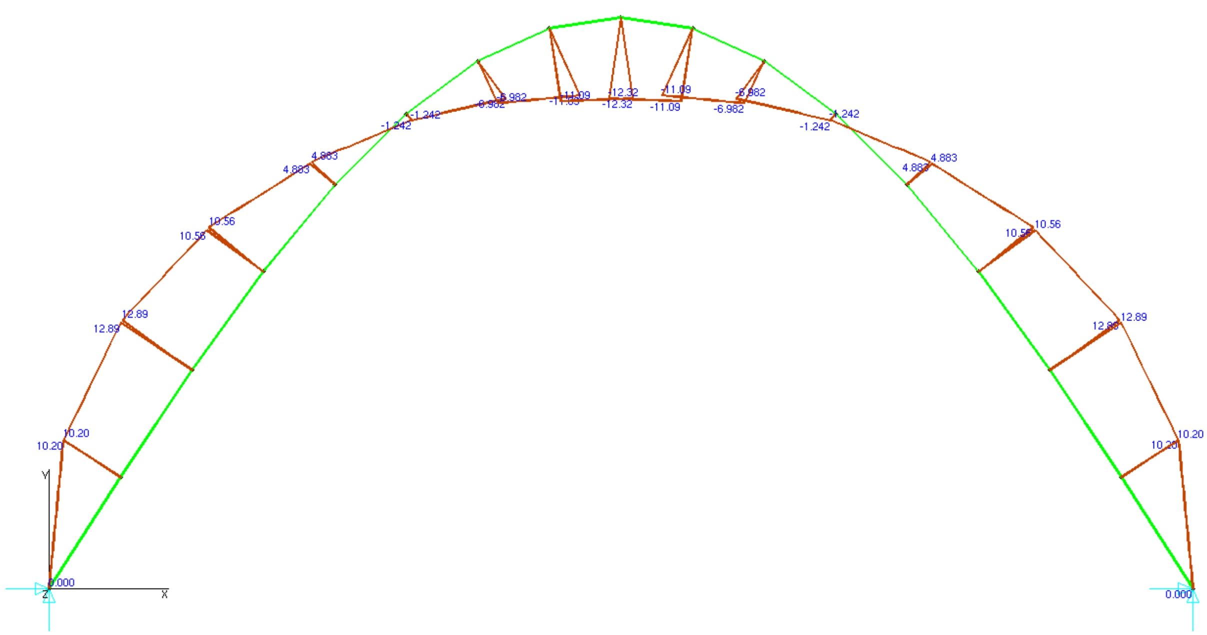


Figure 15 – H = L/2 Sin curve bending moment

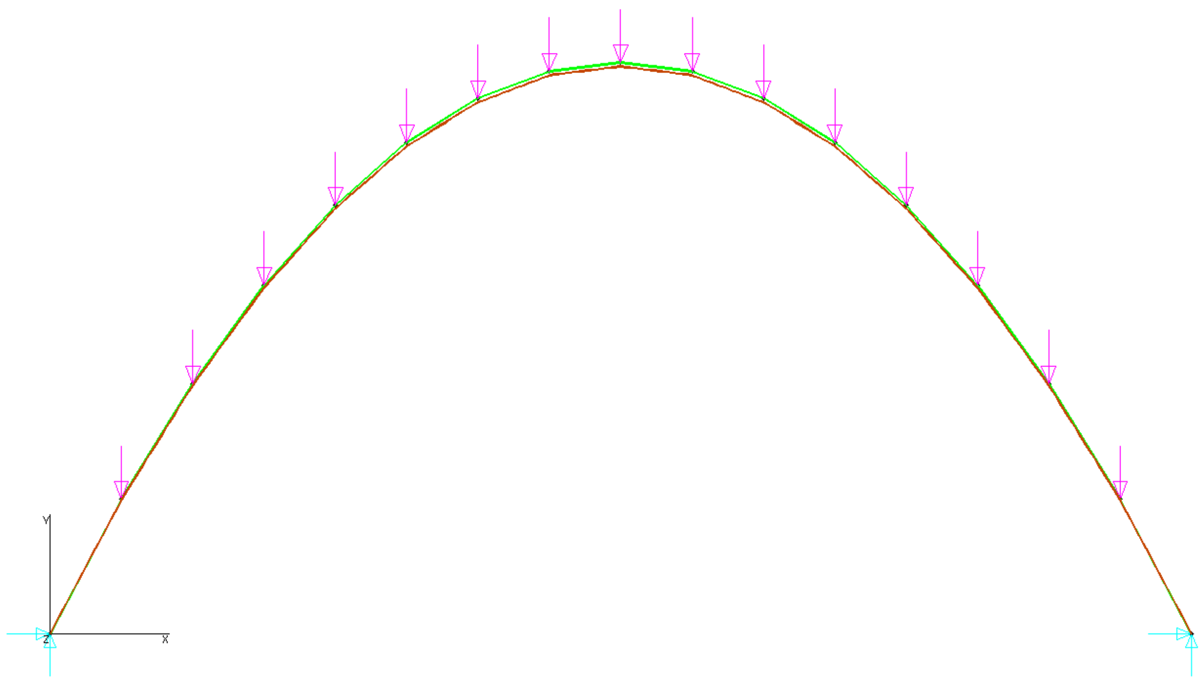

Figure 16 – H = L/2 Parabola Curve deflected shape

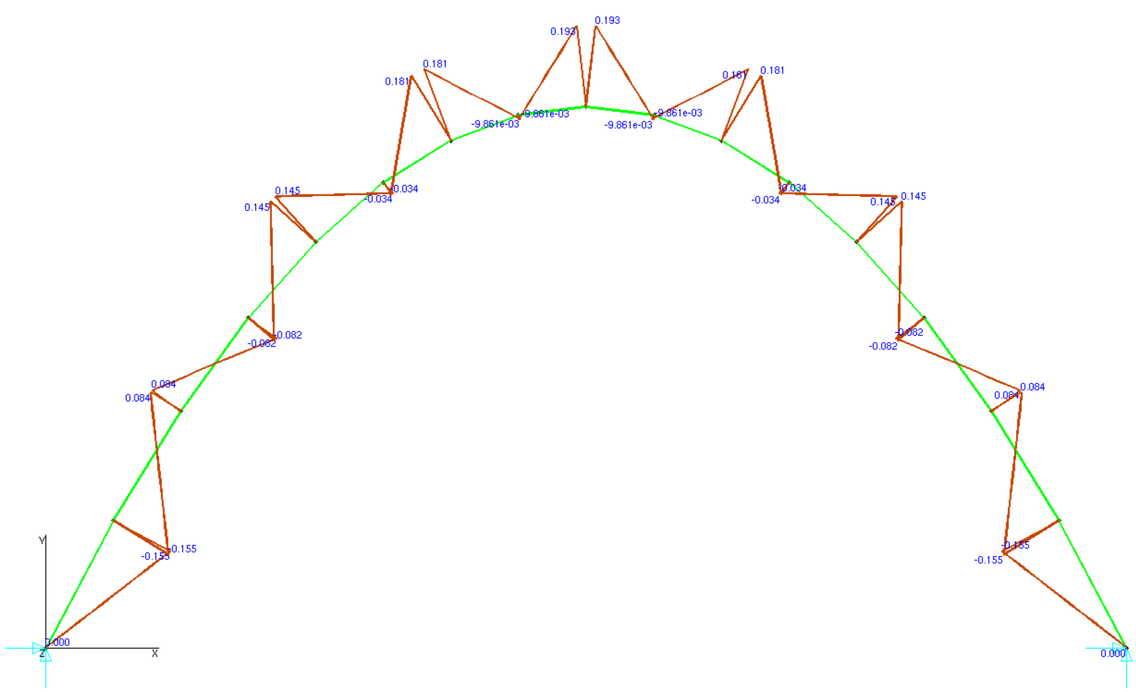

Figure 17 – H = L/2 Parabola curve bending moment

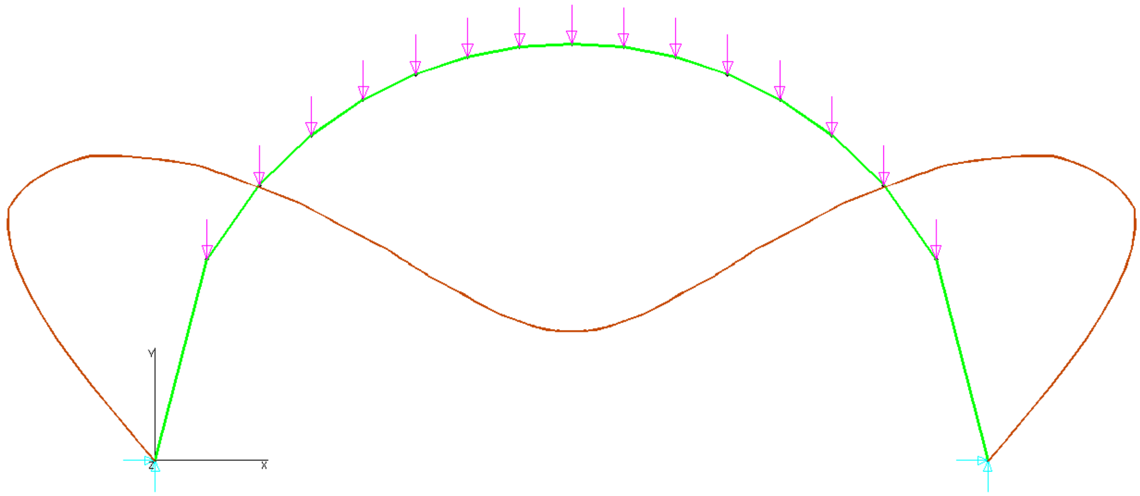


Figure 18 – H = L/2 Circular arc deflected shape

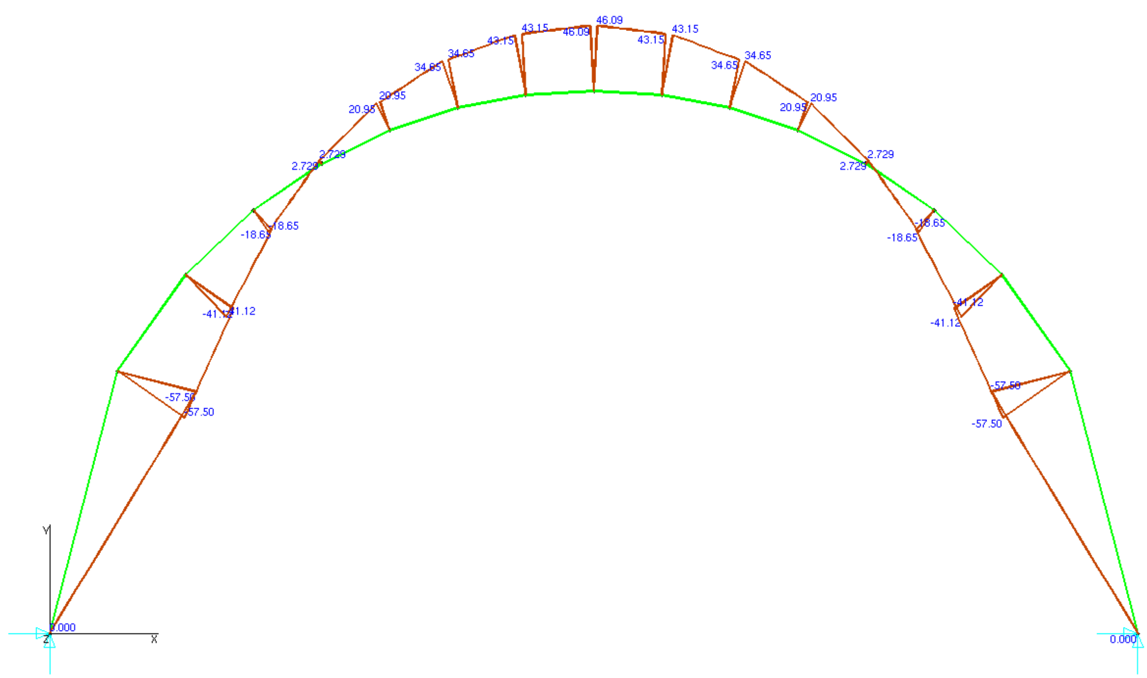


Figure 19 – H = L/2 Circular arc bending moment

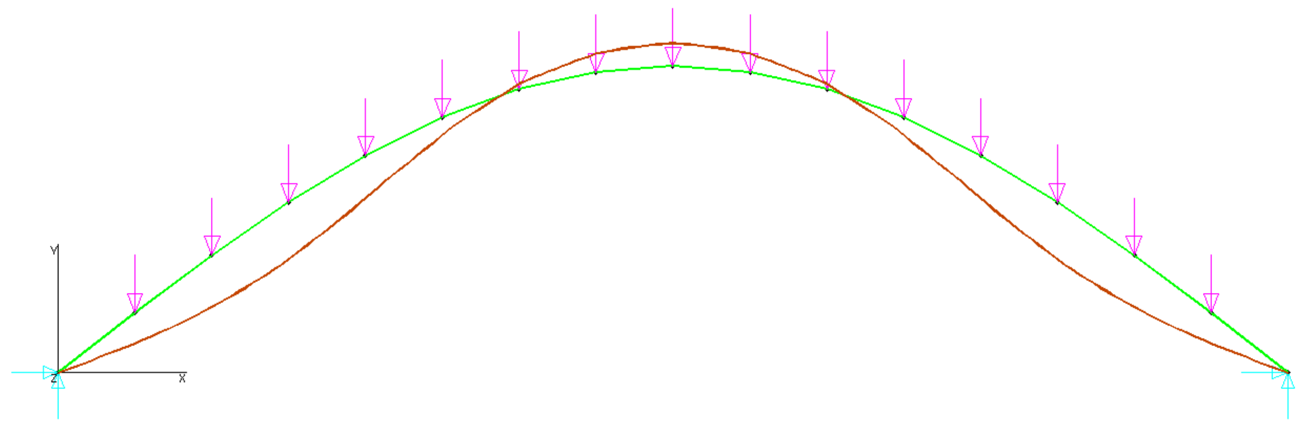


Figure 20 – H = L/4 Sin curve deflected shape

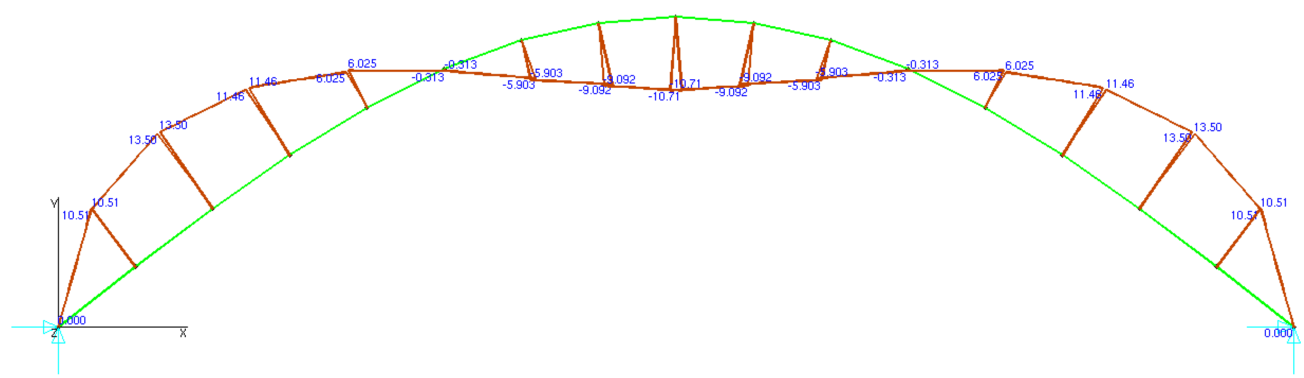


Figure 21 – H = L/4 Sin curve bending moment

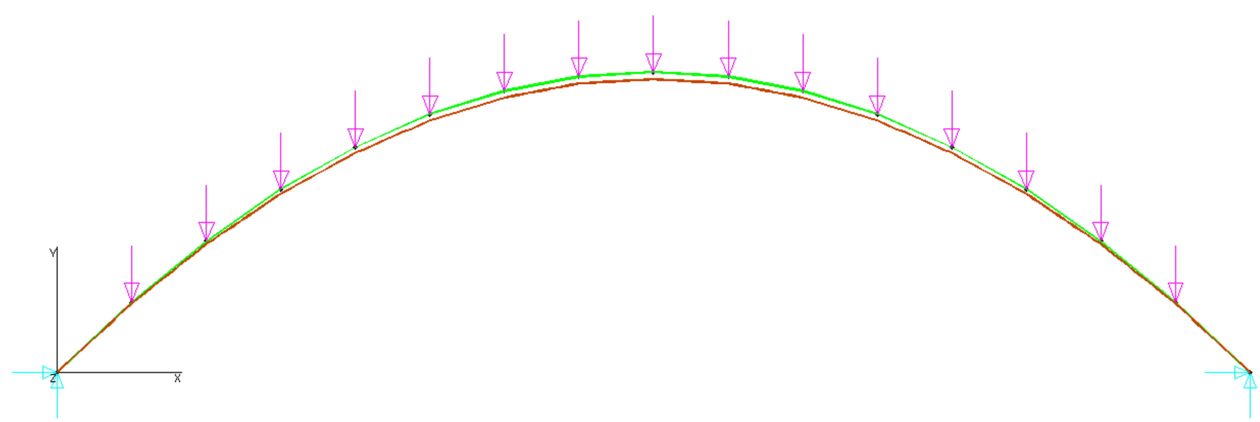


Figure 22 – H = L/4 Parabola curve deflected shape

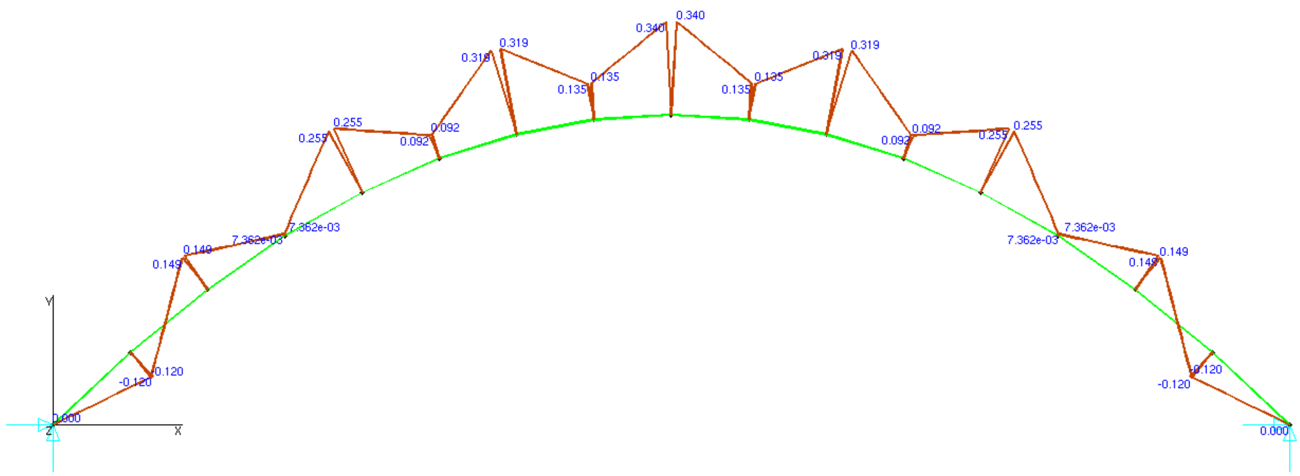


Figure 23 – H = L/4 Parabola curve bending moment

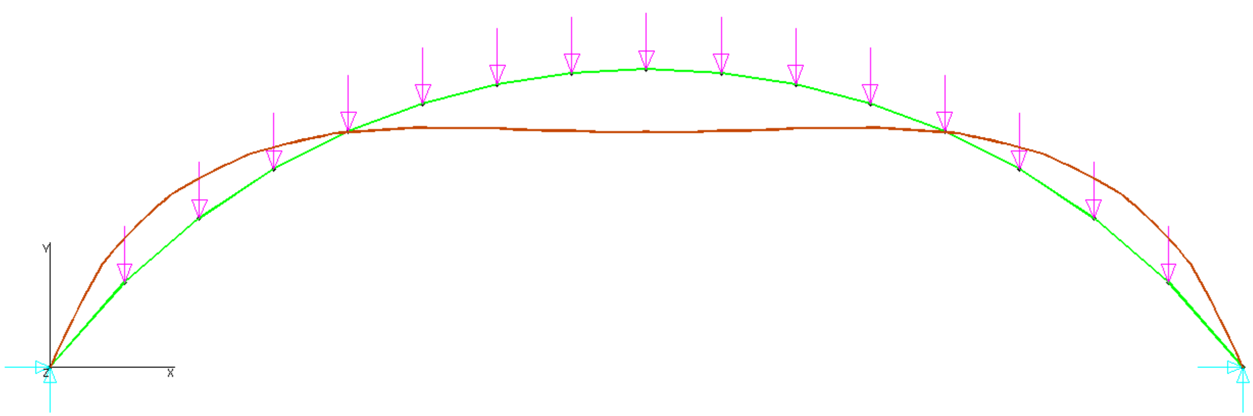


Figure 24 – H = L/4 Circular arc deflected shape

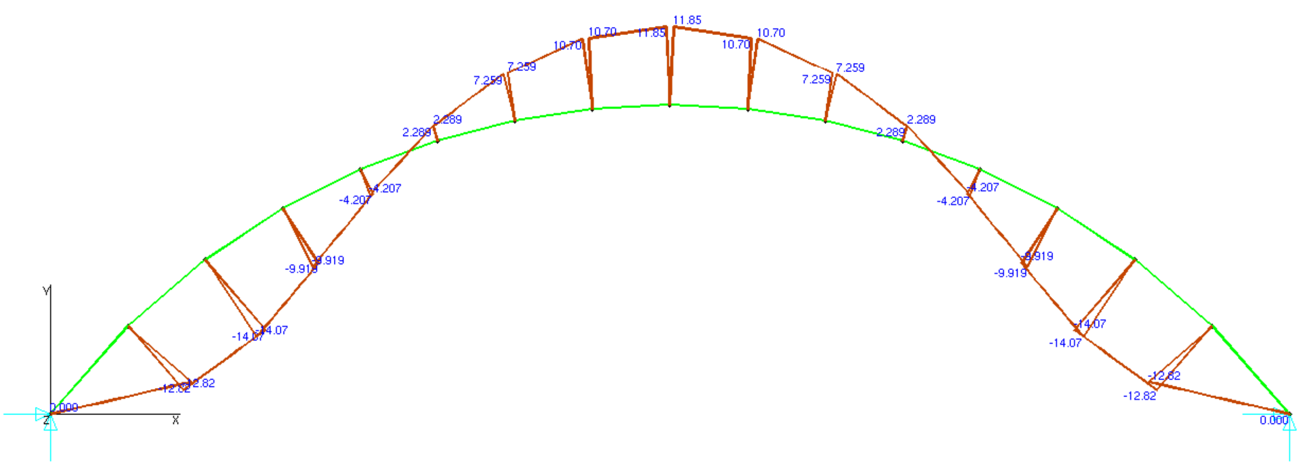


Figure 25 – H = L/4 Circular arc bending moment

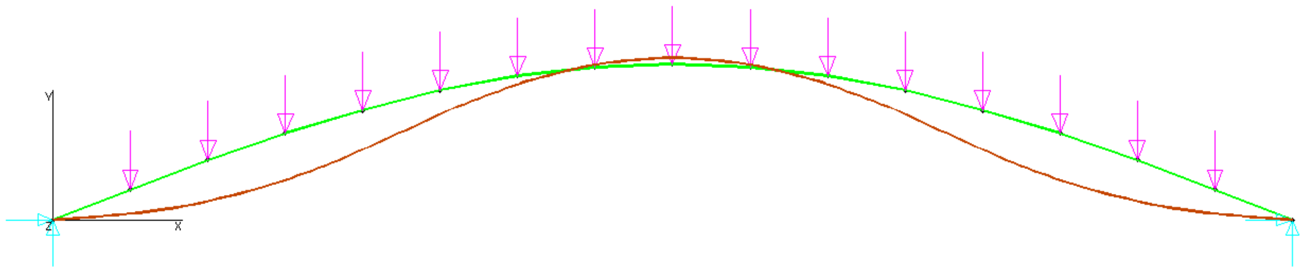


Figure 26 – H = L/8 Sin curve deflected shape

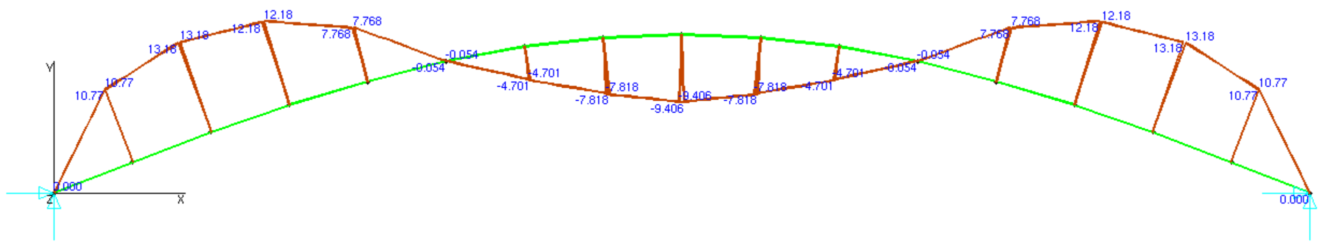


Figure 27 – H = L/8 Sin curve bending moment

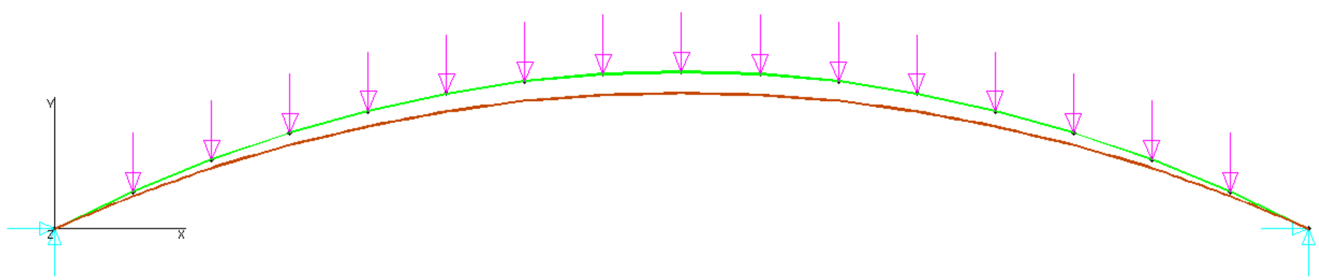


Figure 28 – H = L/8 Parabola curve deflected shape

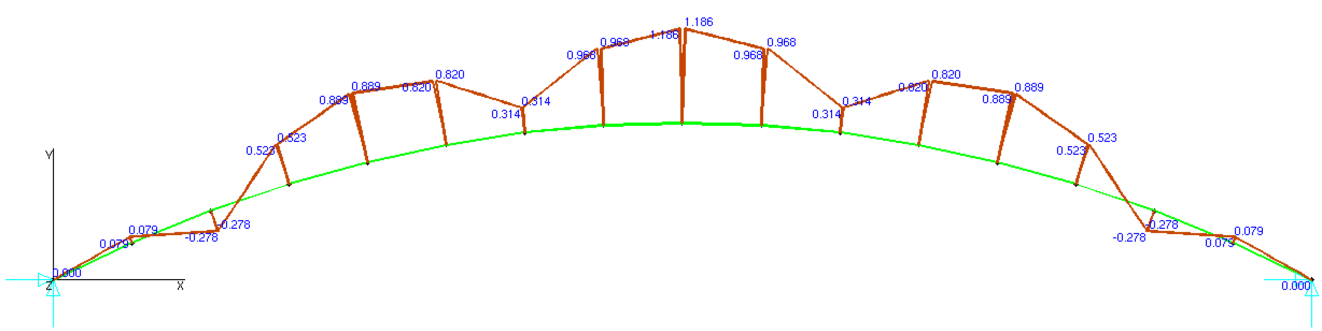


Figure 29 – H = L/8 Parabola curve bending moment

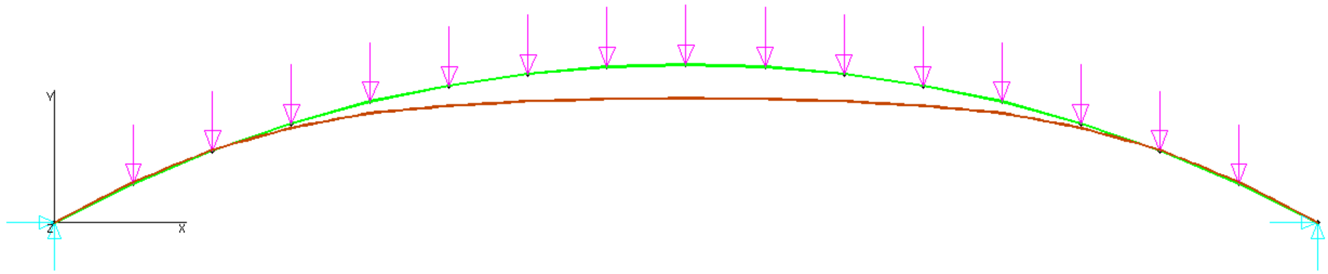

Figure 30 – H = L/8 Circular arc deflected shape

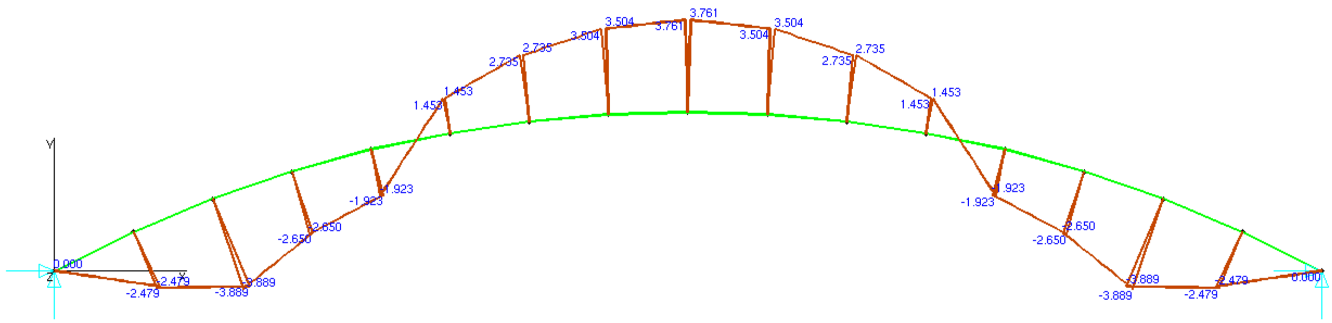


Figure 31 – H = L/8 Circular arc bending moment

| | Max bending moment (kNm) | Max axial stress (kN/m) | Min axial stress (kN/m) | Eccentricity |
|---|---|---|---|---|
| **H=L/2** | | | | |
| **Circle** | -57.5 | -81.18 | -34.48 | -46.7 |
| **Parabola** | 0.193 | -84.99 | -40.29 | -44.7 |
| **Sin** | 12.89 | -85.56 | -41.82 | -43.74 |
| | | | | |
| **H=L/4** | | | | |
| **Circle** | -14.07 | -107.2 | -77.19 | -30.01 |
| **Parabola** | 0.34 | -109.6 | -80.07 | -29.53 |
| **Sin** | 13.44 | -111.3 | -82.86 | -28.44 |
| | | | | |
| **H=L/8** | | | | |
| **Circle** | -3.889 | -175 | -158.2 | -16.8 |
| **Parabola** | 1.427 | -176.4 | -159.7 | -16.7 |
| **Sin** | 14.76 | -180.8 | -164.9 | -15.9 |

Table 7 – Maximum bending moments and axial stresses, minimum axial stresses and eccentricity between min and max axial stresses

From the analysis of the information in the bending moment diagrams and the maximum bending moments in table 7 the most suitable and efficient type of curve can be chosen for each dimensional condition. It is clear that for each case (H = L/2, H = L/4, H = L/8) that the most efficient curve is in fact the parabolic type as it has. In the case of H = L/2 the parabolic curve has a maximum bending moment lower than 0.5% of that of the circular arc. In the case of H = L/8 the parabolic curve is still the most efficient but the difference has been reduced compared to the circular arc with the parabola being 63.4% smaller compared to 95.5% smaller in the H = L/2 case.

# 4. CONCLUSIONS

- The full stiffness matrix, K, for the frame in figure 1 is K = [27x27], its partitions are $K_{FF} = [12 \times 12]$, $K_{RF} = [15 \times 12]$, $K_{FR} = [12 \times 15]$ and $K_{RR} = [15 \times 15]$
- The maximum bending moments in the spans of members 3 and 6 are $2.872 \times 10^5$ kNmm (3.44m from left) and $1.23 \times 10^5$ kNmm (3.84m from left) respectively
- The maximum deflections in the spans of members 3 and 6 are 9.5mm (3.7m from left) and 1.92mm (3.6m from left)
- The maximum deflections are roughly located at the points of maximum bending moment
- Using different combinations of the dead load and full factored load, values for the maximum bending moments and shear forces could be seen and a bending moment envelope could be created for members 3 and 6
- The difference between different types of curve (sin, parabola, circular) can greatly affect the size of the bending moments in an arch shaped support
- In the case of H = L/2, H = L/4 and H = L/8, the parabolic curve proved to be the most efficient type of curve as its bending moments were greatly lower than those of the sin curve or circular arc shaped arches