Normal Modes of Vibration / Shear Frame

VIBRATION, NOISE & VEHCILE DYNAMICS

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INTRODUCTION

When certain systems have been disturbed in any manner, they have the ability to vibrate freely. The initial disturbance can be regarded as being a way of transferring energy to the system from an external source. Kinetic Energy is gained throughout some components of mechanical systems in motion. The forces on the components are a combination of internal and external sources. Examples of external forces are the effects of gravity, and the presence of a fluid environment (air or sea). Internal forces include the effects of elastic forces. A prime example would be in the deformed spring. Some solids have the ability to store energy in an elastic deformation state.

The experiment is conducted as to determine the normal modes of vibration of the frame. The frame is a three story shear frame which is attached with a magnetic accelerometer to shake the frame. By using the stroboscope nodes and anti nodes was recognized

When vibrating in a normal mode, the mode shape taken up by the system will be different to that occurring when vibrating in one of its other normal modes, also all real systems posses and infinite number of degrees of freedom and posses an infinite number of normal modes of vibration or natural frequencies.

- **Aim**

The aim of the experiment is to observe the Normal Modes of Vibration of a multi degree of freedom system by testing and evaluate these modes using a Matrix Method of Analysis.

- **Objectives**

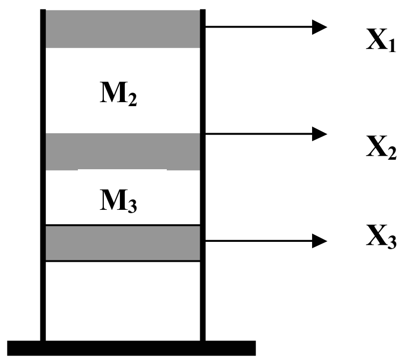
To determine the normal modes of vibration of the frame by experiment and by solving the matrix Eigen value equation.

General Background

Theory: When vibrating in a normal mode, the mode shape taken up by the system will be different to that occurring when vibrating in one of its other normal mode, also all real systems comprise of components that posses continuous, or distributed elastic and mass properties. Therefore, all real systems posses and infinite number of degrees of freedom and posses an infinite number of normal modes of vibration or natural frequencies.

Natural frequencies can basically be found using three different methods like

1. Impulse
2. Forced
3. Flexibility



The equations of motion of the system, for normal mode motion, take the form of an eigen – value equation;

$$[\mathbf{K}]^{-1}[\mathbf{M}]\{\mathbf{X}\} = (\mathbf{1/}\,\omega^{2})\{\mathbf{X}\}$$

$$\therefore [\mathbf{D}]\{\mathbf{X}\} = (\mathbf{1/}\,\omega^{2})\{\mathbf{X}\}$$

Where

- [**M**] = mass matrix
- [**D**] = Dynamic matrix.
- **[K**]$^{-1}$ = [**H**] = flexibility matrix, the elements of this matrix $\mathbf{a_{ij}}$ are specified by static deflection at position **I** for a **<u>unit</u>** force at **j**.

PROCEDURE

1) An accelerometer with a magnetic base was attached on the top end of seven story shear frame. Then, an impact excitation was applied. The natural frequencies were noted down. By using a shaker, each natural frequency was found and the nodes and anti-nodes were confirmed by using a stroboscope.

2) A 10 N force was applied to each floor in turn, and the experimental "flexibility matrix" [**H**] , of the frame was evaluated. The units of this matrix must be in m/N.

3) The dial gauges were moved away from the frame. A forced vibration test on the frame was performed and by using a stroboscope, its three normal mode frequencies and shapes were determined.

4) Mass of each floor was given as = 0.204 kg

5) Total mass of combined side walls = 0.207 kg, the mass matrix [**M**] was determined. An allowance for the side walls in [**M**] was included and was made sure diagonal.

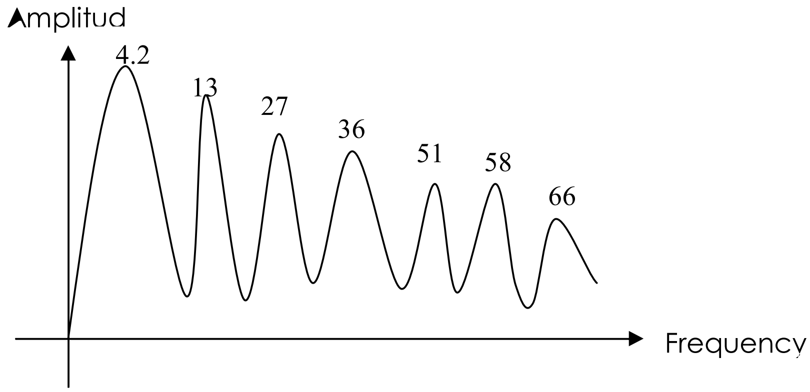
6) The computer program "frame" was used to solve the matrix eigen value problem and the theoretical normal modes of the frame was evaluated.

7) The sub group was made to perform the procedure (1) for a 3 storey shear frame assigned for the experiment.

Numerical Analysis

The seven storey shear frame has seven degrees of Freedom and hence seven natural frequencies. A accelerometer was placed on an anti-node of the frame to obtain maximum vibration and the structure was tapped with a steel rod to give the required information.

An approximation to the graph obtained on the Spectrum analyzer when the seven storey frame was tapped.



ANALYSIS ON THE THREE STOREY STRUCTURE

A Similar procedure was conducted on the three storey shear frame which has three degrees of freedom and three natural frequencies. A load of 10N was used.

	Top	**Middle**	**Bottom**
Top			
1	3.30	2.10	1.00
2	3.20	2.10	0.97
3	3.10	2.20	1.00
Avg	**3.20**	**2.13**	**0.99**
Middle			
1	2.10	2.17	1.10
2	2.15	2.16	1.05
3	2.12	2.27	1.07
Avg	**2.12**	**2.20**	**1.07**
Bottom			
1	1.23	1.10	1.12
2	1.20	1.10	1.10
3	1.13	1.15	1.11
Avg	**1.19**	**1.12**	**1.11**

The mass of M_1, M_2, and M_3 was found as below:

Mass	**Value**
M_1	0.204 + 0.207/6 = 0.2385
M_2	0.204 + 0.207/3 = 0.273
M_3	0.204 + 0.207/3 = 0.273

And therefore the mass matrix, $[M] = \begin{pmatrix} 0.2385 & 0 & 0 \\ 0 & 0.273 & 0 \\ 0 & 0 & 0.273 \end{pmatrix}$

Experimental values

Using the computer software the following results were given to us

```
                          Frame



                     FRAME    -    RESULTS
                     ======================
Input taken from data file named  :  V4B.dat
Mass Matrix :
       0.238500         0.00000         0.00000
        0.00000        0.273000         0.00000
        0.00000         0.00000        0.273000
Flexibility Matrix :
    0.000328400     0.000230800     0.000110900
    0.000230800     0.000217000     0.000103000
    0.000110900     0.000103000     8.10000e-05
Dynamic Matrix :
    7.83234e-05     6.30084e-05     3.02757e-05
    5.50458e-05     5.92410e-05     2.81190e-05
    2.64497e-05     2.81190e-05     2.21130e-05
Normal Mode Frequencies (Hz) are :
   13.371    46.1038    64.5523
Normal Mode Shapes are :
        1.00000         1.00000         1.00000
       0.807961       -0.592189        -2.24954
       0.411217       -0.960953         2.29541
```

Theoretical calculations

- **Lowest Mode**

Using the given formula, the following calculations were done:

$$[K]^{-1}\,[M]\,\{X\} = \frac{1}{\omega^2}\,\{X\}$$

Average flexibility matrix:

$$[H] = \begin{pmatrix} 0.000328 & 0.000235 & 0.000109 \\ 0.000226 & 0.000217 & 0.000109 \\ 0.000105 & 0.000103 & 0.000081 \end{pmatrix}$$

Symmetric matrix:

$$[H] = [K^{-1}] = \begin{pmatrix} 0.0003284 & 0.0002308 & 0.0001109 \\ 0.0002308 & 0.000217 & 0.0001030 \\ 0.0001109 & 0.000103 & 0.000081 \end{pmatrix}$$

Dynamic matrix:

$$[M] \times [H] = [D] = \begin{pmatrix} 0.000078324 & 0.0000630084 & 0.0000302757 \\ 0.0000550458 & 0.000059241 & 0.000028119 \\ 0.0000264497 & 0.0000281190 & 0.0000221130 \end{pmatrix}$$

$[\,D\,]\,\{\,X\,\} = (\,1/\,\omega^2\,)\,\{\,X\,\}$

$$\begin{pmatrix} 0.000078324 & 0.0000630084 & 0.0000302757 \\ 0.0000550458 & 0.000059241 & 0.000028119 \\ 0.0000264497 & 0.0000281190 & 0.0000221130 \end{pmatrix}\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \frac{1}{\omega^2}\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$

The equation above represents the Eigen-value of $[Q]\{y\} = [\gamma]\{y\}$ where the Eigen-value γ represents $1/\omega^2$ and the Eigen-vector $\{y\}$ represents the mode shape $\{X\}$.

Iteration was done by assuming a trial vector for the mode shape $\{X\}$ using:

$\{\overline{X}\} = \{1.0 \quad 1.5 \quad 2.0\}$

1st Iteration

$$\begin{pmatrix} 0.000078324 & 0.0000630084 & 0.0000302757 \\ 0.0000550458 & 0.000059241 & 0.000028119 \\ 0.0000264497 & 0.0000281190 & 0.0000221130 \end{pmatrix}\begin{Bmatrix} 1.0 \\ 1.5 \\ 2.0 \end{Bmatrix} = \begin{Bmatrix} .00023338 \\ .000200145 \\ .000112854 \end{Bmatrix}$$

i.e.

$$.00023338\begin{Bmatrix} 1 \\ .8576 \\ .4835 \end{Bmatrix}$$

The Iteration was repeated eight times to obtain the required result (see appendix 1)

8th Iteration

$$\begin{pmatrix} 0.000078324 & 0.0000630084 & 0.0000302757 \\ 0.0000550458 & 0.000059241 & 0.000028119 \\ 0.0000264497 & 0.0000281190 & 0.0000221130 \end{pmatrix} \begin{Bmatrix} 1 \\ 0.8079574202 \\ 0.4112161556 \end{Bmatrix} = \begin{Bmatrix} 0.0001416820 \\ 0.00011447 \\ 0.0005826 \end{Bmatrix}$$

i.e.

$$0.0001416820 \begin{Bmatrix} 1 \\ .8080 \\ .4112 \end{Bmatrix}$$

Thus the lowest value of ω can be calculated from:

$1/\omega^2 = 0.0001416820$,

$\omega = \sqrt{1/0.0001416820} = 84.0122718379$ rad/s

Frequency (f) = $(84.0122718379/2\pi) = 13.3709683434$Hz

The shape of the lowest normal mode frequency: $\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 1.0000000000 \\ 0.8079574401 \\ 0.4112159204 \end{Bmatrix}$

- **Highest mode**

The top node was calculated using the given equation

[K] [M]$^{-1}$ {X} = ω^2 {X}

$$[M]^{-1} = \begin{Bmatrix} 1/0.2385 & 0 & 0 \\ 0 & 1/0.273 & 0 \\ 0 & 0 & 1/0.273 \end{Bmatrix} = \begin{Bmatrix} 4.192872117 & 0 & 0 \\ 0 & 3.663003663 & 0 \\ 0 & 0 & 3.663003663 \end{Bmatrix}$$

The inverse of $[K]^{-1}$ was calculated using Matlab

```
b =

   1.0e-003 *

    0.3284    0.2308    0.1109
    0.2308    0.2170    0.1030
    0.1109    0.1030    0.0810

>> c=inv(b)

c =

   1.0e+004 *

    1.2068   -1.2594   -0.0507
   -1.2594    2.4769   -1.4252
   -0.0507   -1.4252    3.1164
```

i.e.

$$[K] = \begin{Bmatrix} 1.2068 & -1.2594 & -0.0507 \\ -1.2594 & 2.4769 & -1.4252 \\ -0.0507 & -1.4252 & 3.1164 \end{Bmatrix}$$

$$[M]^{-1}[K]\ \{X\} = \omega^2\ \{X\}$$

$$\begin{Bmatrix} 4.192872117 & 0 & 0 \\ 0 & 3.663003663 & 0 \\ 0 & 0 & 3.663003663 \end{Bmatrix} \begin{Bmatrix} 1.2068 & -1.2594 & -0.0507 \\ -1.2594 & 2.4769 & -1.4252 \\ -0.0507 & -1.4252 & 3.1164 \end{Bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$

$$= \omega^2 \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$

[M][K] = [A]

$$[A] = \begin{Bmatrix} 5.0600 & -5.2805 & -0.2126 \\ -4.6132 & 9.0729 & -5.2205 \\ -0.1857 & -5.2205 & 11.4154 \end{Bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \omega^2 \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$

Assuming a trial vector for the highest mode shape, {X} = {1.0 -2.0 2.0}

$$\begin{Bmatrix} 5.0600 & -5.2805 & -0.2126 \\ -4.6132 & 9.0729 & -5.2205 \\ -0.1857 & -5.2205 & 11.4154 \end{Bmatrix} \begin{Bmatrix} 1.0 \\ -2.0 \\ 2.0 \end{Bmatrix} = \begin{Bmatrix} 15.1959 \\ -33.2000 \\ 33.0860 \end{Bmatrix}$$

i.e.

$$15.1959 \begin{Bmatrix} 1 \\ -2.1848 \\ 2.1773 \end{Bmatrix}$$

The desired values was found after a total of 18 iterations

$$\begin{Bmatrix} 5.0600 & -5.2805 & -0.2126 \\ -4.6132 & 9.0729 & -5.2205 \\ -0.1857 & -5.2205 & 11.4154 \end{Bmatrix} \begin{Bmatrix} 16.4506 \\ -37.0057 \\ 37.7605 \end{Bmatrix} = \begin{Bmatrix} 16.4506 \\ -37.0058 \\ 37.7605 \end{Bmatrix}$$

And equals to; $16.4506 \begin{Bmatrix} 1 \\ -2.2495 \\ 2.2954 \end{Bmatrix}$

Thus the highest value of ω is:

$\omega^2 = 16.4506 \times 10^4$

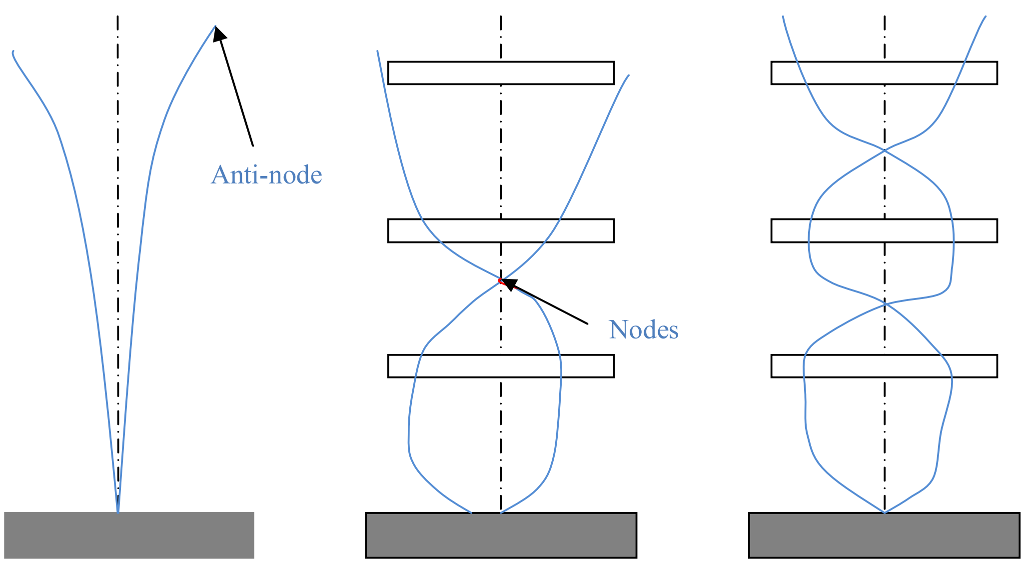
$\omega = \sqrt{164506} = 405.5935$ rad/s

Frequency (f) = 405.5935 / 2π = 64.5522 Hz

The shape of the mode for the highest normal mode frequency:

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -2.2495 \\ 2.2954 \end{Bmatrix}$$

Results



Forced response mode shapes

Forced 10 38.00 65.70

Impulse	9.75	39.2	66.7
Calculation	13.37	46.103	64.55

Discussion

Based on the above the calculations the following points were noted:

- The flexibility matrix was found to be symmetrical.
- The mode frequencies were compared with the analytical and the percentage error margin was really small.

  Top mode

  $$\% \, error = \frac{64.5523 - 64.5522}{64.5523} \times 100 = \ 0.0001549 \ \%$$

  Low mode

  $$\% \, error = \frac{13.371 - 13.37096}{13.371} \times 100 = \ 0.00028 \ \%$$

- The mode shapes were compared with the numerical data and was found to be similar.
- The side wall allowance is considerably important as the calculation were done as if it was based on real structures, hence achieving more accurate results.
- From the experiment and calculation done above, there are three degrees of freedom since it has three modes of vibration and three natural frequencies
- The frame can be associated with a real structural building like a multi-storey building based on practical applications.

Conclusion

This experiment is similar to the ones used by civil engineers or the construction industry. It was designed to represent a structure like a multi-storey building which would react under extreme conditions of weather. The model was forced to oscillate to obtain the Natural Frequencies of the three storey frame by 'frame' software analysis and by analytical calculations.

The aims & objectives of the experiment were achieved by solving the matrix Eigen value equation and determining the normal modes of vibration of the frame. By testing and evaluating the modes using a Matrix Method of Analysis, the Normal Modes of Vibration of a multi degree of freedom system was observed

References

- Badi, M. N. M. (2008) Mechanical Vibrations 1: Vibration Analysis.
- Vibrations Analysis by Rao V. Dukkipati, Alpha science 2004

Appendix-1

[H] or [K]^-1	3.284000	2.308000	1.109000	**[M]**	0.238500	0.000000	0.000000					
	2.308000	2.170000	1.030000		0.000000	0.273000	0.000000					
	1.109000	1.030000	0.810000		0.000000	0.000000	0.273000					
[D]	0.783234	0.630084	0.302757		1.000000							
	0.550458	0.592410	0.281190		1.500000							
	0.264497	0.281190	0.221130		2.000000							
1st	2.3339			1.0000	**5th**	1.4168			1.0000			
	2.0015		2.3339	0.8576		1.1448		1.4168	0.8080			
	1.1285			0.4835		0.5826			0.4112			
	1.4700			1.0000	**6th**	1.4168			1.0000			
2nd	1.1945		1.4700	0.8126		1.1447		1.4168	0.8080			
	0.6126			0.4167		0.5826			0.4112			
3rd	1.4214			1.0000	**7th**	1.4168			1.0000		rad/s	Hz
	1.1490		1.4214	0.8084		1.1447		1.4168	0.8080		84.0124	13.3710
	0.5851			0.4117		0.5826			0.4112			
4th	1.4172			1.0000	**8th**	1.4168			1.0000			
	1.1451		1.4172	0.8080		1.1447		1.4168	0.8080			
	0.5828			0.4113		0.5826			0.4112			

Appendix-2

[M]^-1	4.1929	0.0000	0.0000	**[K]**	1.2068	-1.2594	-0.0507		1.2068	-1.2594	-0.0507		
	0.0000	3.6630	0.0000		-1.2594	2.4769	-1.4252		-1.2594	2.4769	-1.4252		
	0.0000	0.0000	3.6630		-0.0507	-1.4252	3.1164		-0.0507	-1.4252	3.1164		
[A]	5.0600	-5.2805	-0.2126		1.0000								
	-4.6132	9.0729	-5.2205		-2.0000								
	-0.1857	-5.2205	11.4154		2.0000								
1st	15.1959			1.0000		**10th**	16.4493			1.0000			
	-33.2000		15.1959	-2.1848			-37.0004		16.4493	-2.2494			
	33.0860			2.1773			37.7529			2.2951			
2nd	16.1340			1.0000		**11th**	16.4499			1.0000			
	-35.8022		16.1340	-2.2190			-37.0030		16.4499	-2.2494			
	36.0747			2.2359			37.7566			2.2952			
3rd	16.3024			1.0000		**12th**	16.4503			1.0000			
	-36.4191		16.3024	-2.2340			-37.0043		16.4503	-2.2495			
	36.9229			2.2649			37.7585			2.2953			
4th	16.3751			1.0000		**13th**	16.4504			1.0000			
	-36.7055		16.3751	-2.2415			-37.0050		16.4504	-2.2495			
	37.3311			2.2798			37.7595			2.2953			
5th	16.4119			1.0000		**14th**	16.4505			1.0000			
	-36.8519		16.4119	-2.2454			-37.0054		16.4505	-2.2495			
	37.5405			2.2874			37.7600			2.2954			
6th	16.4308			1.0000		**15th**	16.4506			1.0000			
	-36.9271		16.4308	-2.2474			-37.0055		16.4506	-2.2495			
	37.6480			2.2913			37.7602			2.2954			
7th	16.4405			1.0000		**16th**	16.4506			1.0000			
	-36.9656		16.4405	-2.2484			-37.0056		16.4506	-2.2495			
	37.7030			2.2933			37.7604			2.2954			
8th	16.4455			1.0000		**17th**	16.4506			1.0000		**Rad/s**	**Hz**
	-36.9852		16.4455	-2.2490			-37.0057		16.4506	-2.2495		405.5935	64.5522
	37.7312			2.2943			37.7605			2.2954			
9th	16.4480			1.0000		**18th**	16.4506			1.0000			
	-36.9953		16.4480	-2.2492			-37.0058		16.4506	-2.2495			
	37.7455			2.2948			37.7605			2.2954			