

FIR Project

Statement of the Problem:

The objective of this project is to use three different design methods to design a low-pass filter that meets specifications given, and then compare these three different methods through different parameters. In this project, seven filters should be designed using Matlab. And we compare them on worst case gain, largest tap weight coefficient, maximal passband and stopband error, magnitude frequency response, impulse response, group delay and zeros/poles location. Finally, use these filters to do filtering, and then compare their responses to the predicted one.

Discussion of Results:

➤ Part 1: Window Method

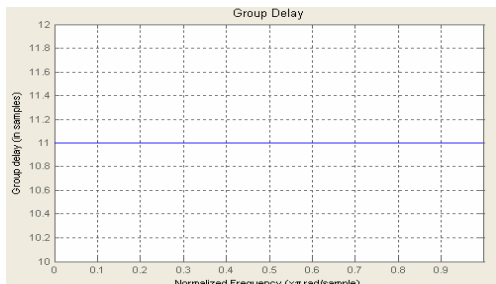
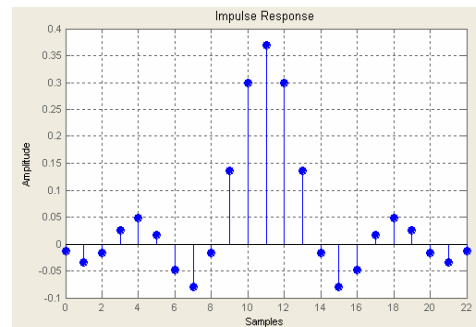
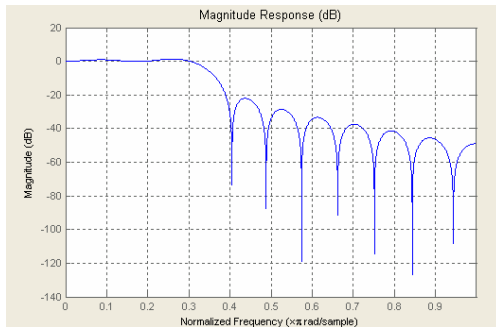
(a) Use fir1 function to synthesize an FIR that meets specifications using a boxcar window.

Worst gain = 1.8372

Largest tap weight coefficient = 0.3694

Maximal passband error = 0.1678

Maximal stopband error = 0.0795



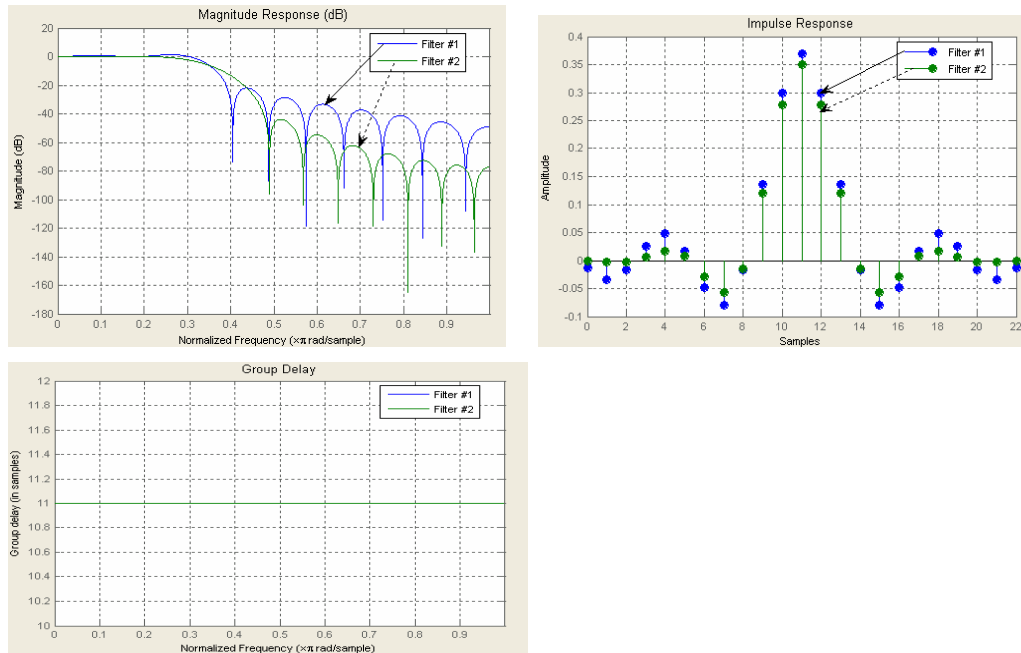
(b) Use Hann window to synthesize an FIR that meets specifications.

Worst gain = 1.4154

Largest tap weight coefficient = 0.3496

Maximal passband error = 0.0052

Maximal stopband error = 0.2385



**Filter #1 is the unwindowed design, and Filter #2 is the windowed design.

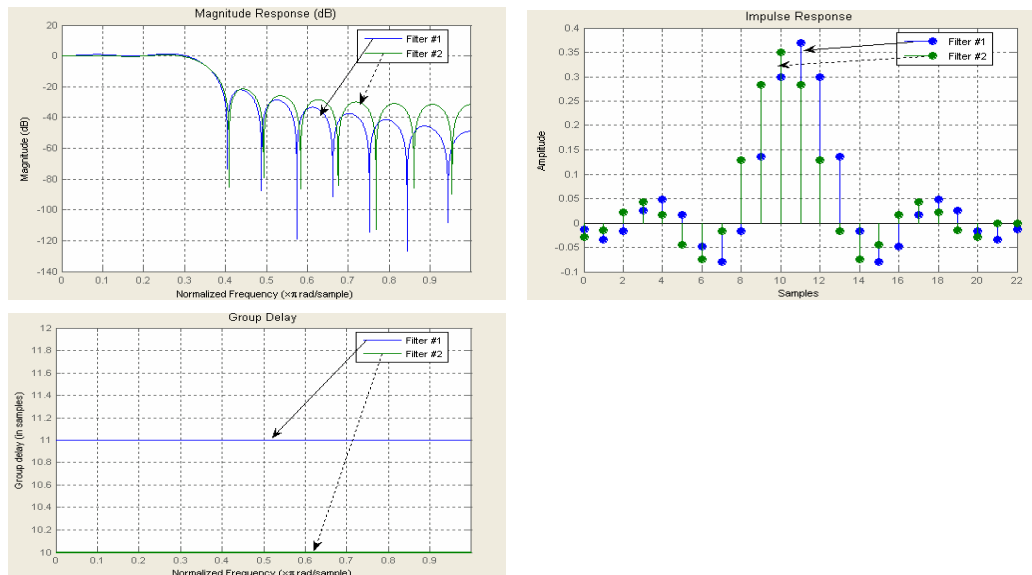
From the comparison above, we can see that the unwindowed design has a more critical passband and stopband edge, but the windowed one has a smaller maximal passband error as we expected. Also, the windowed one has a larger attenuation on stopband than the unwindowed one. The group delay responses of two designs are the same.

(c) Use Kaiser window to synthesize an FIR that meets specifications

Worst gain = 1.6900 Largest tap weight coefficient = 0.3500 $N = 21$ (which is 20 in matlab)

Maximal passband error = 0.0706

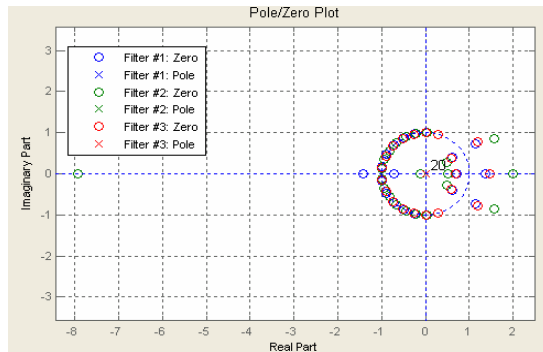
Maximal stopband error = 0.0852



** Filter #1 is the unwindowed design, and Filter #2 is the kaiser design.

From the comparison above, we can see that both designs have critical passband and stopband edges, but the kaiser one has a smaller maximal passband error as we expected. Also, the kaiser one has a smaller attenuation on stopband compare with the unwindowed one. The group delay responses of two designs are different, the Kaiser one only has 20th order, so the group delay is 10, not 11 as the unwindowed one.

(d) The zeros of the three windowed designs



** Filter #1 is the “boxcar” design, and Filter #2 is the Hann design, Filter #3 is the Kaiser design.

From figure above, we can see that Hann design has a zero far from unit circle, which is corresponding to the slower attenuation compared to the other two designs. The zeros of “boxcar” design are similar to the Kaiser design.

➤ Part 2: LMS Method

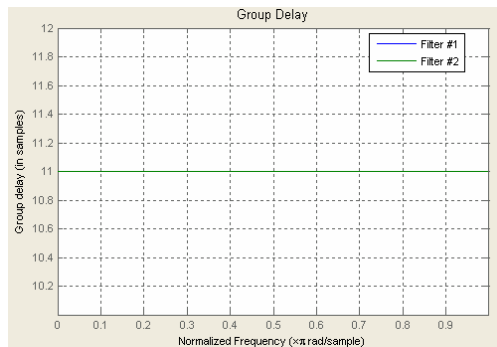
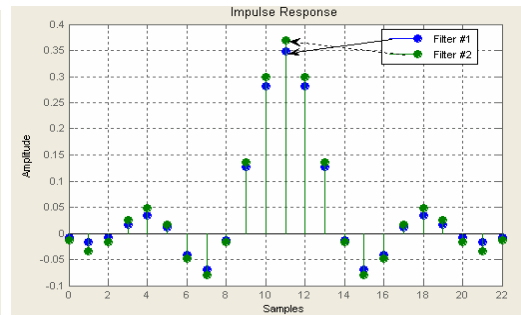
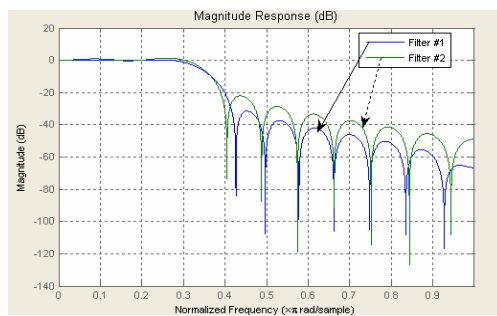
(a) Using Matlab’s `firls` function to meet the original design specification.

Worst gain = 1.5990

Largest tap weight coefficient = 0.3477

Maximal passband error = 0.0403

Maximal stopband error = 0.1137



** Filter #1 is the 2(a) design, and Filter #2 is the “boxcar” design.

From the comparison above, we can see that the “boxcar” design has a more critical passband and stopband edge, but the LMS one has a smaller maximal passband error as we expected. Also, the LMS one has a larger attenuation on stopband than the “boxcar” one. The group delay responses of two designs are the same.

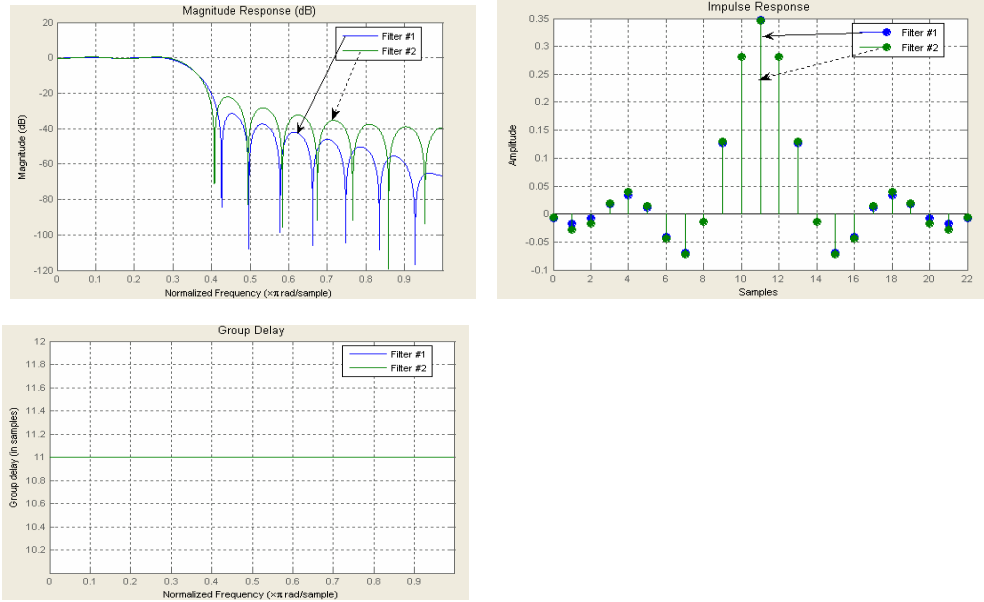
(b) Using Matlab’s `fircls1` function to meet the original design specification.

Worst gain = 1.6771

Largest tap weight coefficient = 0.3464

Maximal passband error = 0.0516

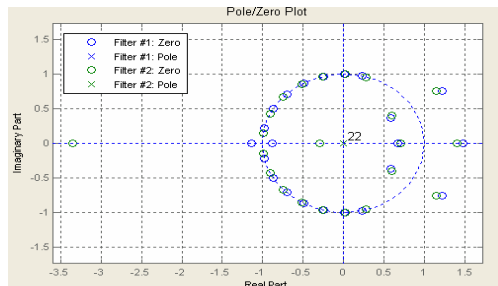
Maximal stopband error = 0.0782



** Filter #1 is the 2(a) design, and Filter #2 is the 2(b) design.

From the comparison above, we can see that the 2(b) design has a more critical passband and stopband edge, but the 2(a) one has a smaller maximal passband error. Also, the 2(a) one has a larger attenuation on stopband than the 2(b) one. The group delay responses of two designs are the same.

(c) The zeros of the two LMS designs



** Filter #1 is the 2(a) design, and Filter #2 is the 2(b) design.

From figure above, we can see that 2(b) design has a zero far from unit circle, which is corresponding to the slower attenuation compared to the other design. The zeros around the unit circle are similar to each other.

➤ Part 3: Equiripple Method

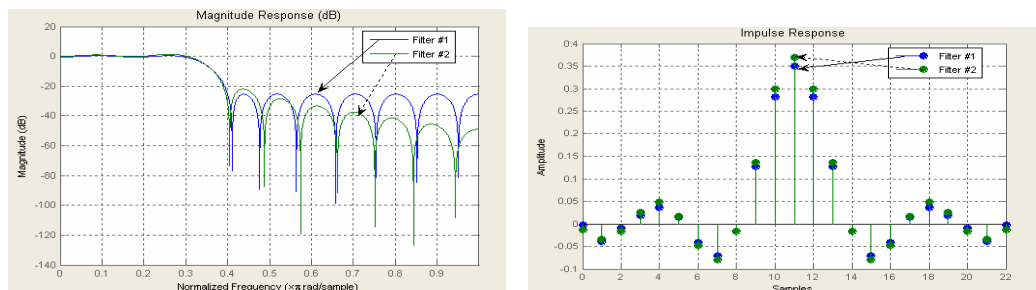
(a) Using Matlab's `firg` function to meet the original design specification (uniform error weight)

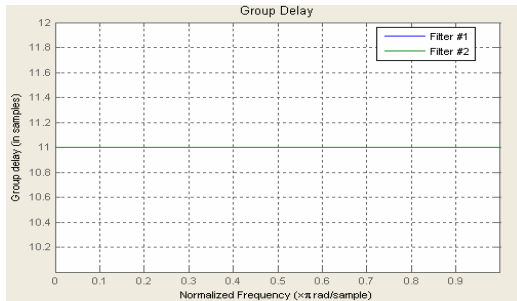
Worst gain = 1.6646

Largest tap weight coefficient = 0.3500

Maximal passband error = 0.0538

Maximal stopband error = 0.0538





** Filter #1 is the 3(a) design, and Filter #2 is the “boxcar” design.

From the comparison above, we can see that the “boxcar” design has a more critical passband and stopband edge, but the 3(a) one has a smaller maximal passband error. Also, the “boxcar” one has a larger attenuation on stopband than the 3(a) one. The group delay responses of two designs are the same.

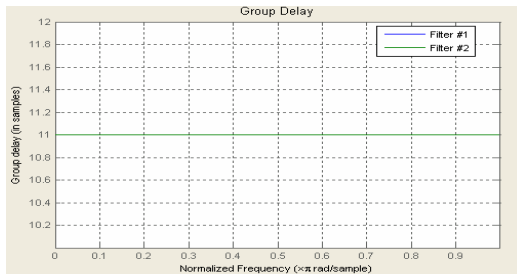
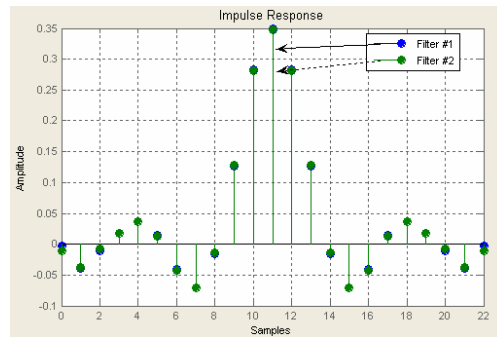
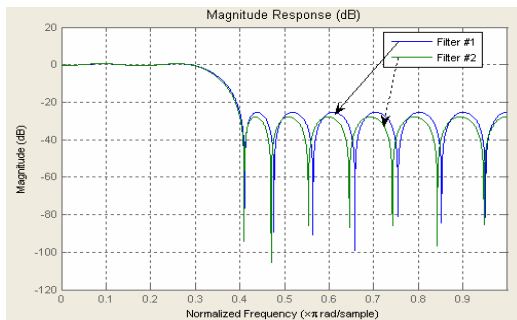
(b) Using Matlab’s firpm function to meet the original design specification

Worst gain = 1.6639

Largest tap weight coefficient = 0.3476

Maximal passband error = 0.0638

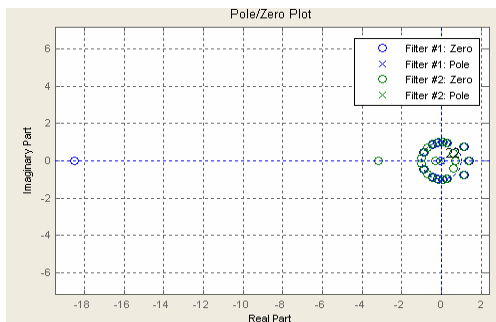
Maximal stopband error = 0.0594



** Filter #1 is the 3(a) design, and Filter #2 is the 3(b) design.

From the comparison above, we can see that the 3(b) design has a more critical passband and stopband edge. And the stopband error is 0.0488 (which is consistent with $0.0538 \times (1 - 20\%) = 0.04304$), the passband error is 0.0639 (which is consistent with $0.0538 / (1 - 20\%) = 0.06725$). The group delay responses of two designs are the same.

(c) The zeros of the two equiripple designs



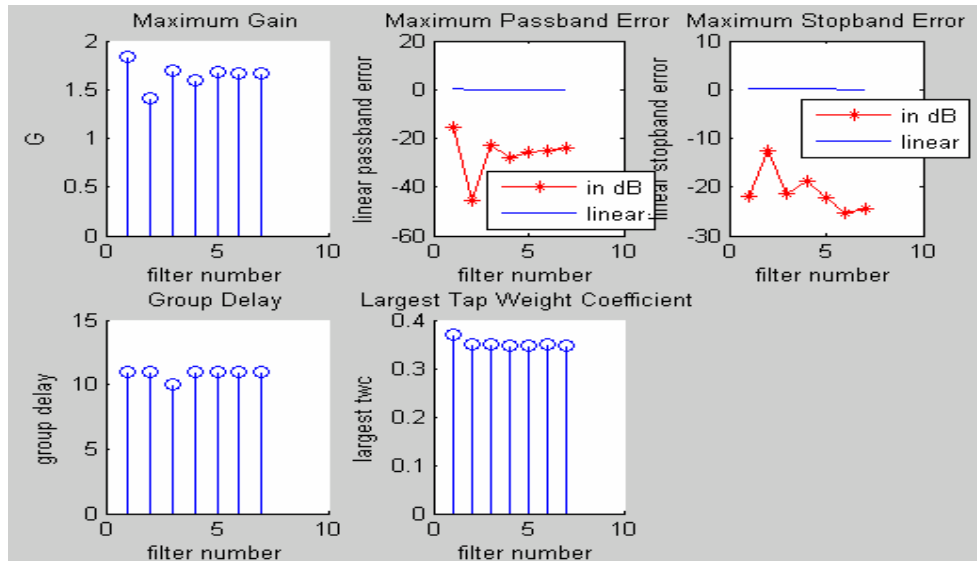
** Filter #1 is the 3(a) design, and Filter #2 is the 3(b) design.

From figure above, we can see that 3(a) design has a zero far from unit circle, which is corresponding to the slower attenuation compared to the other design (almost no attenuation on the figure shown). There is only one zero stays outside the unit circle for 3(b) design, which is the minimum phase design.

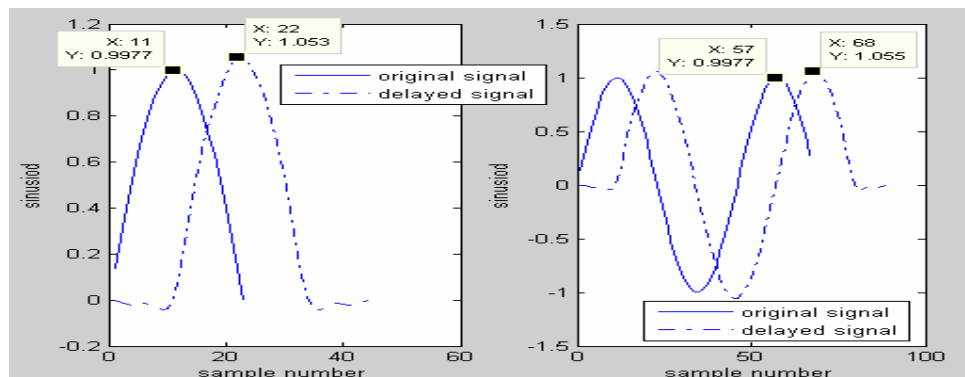
➤ Part 4: Testing

(a) Table the features for the 7 designed FIRs:

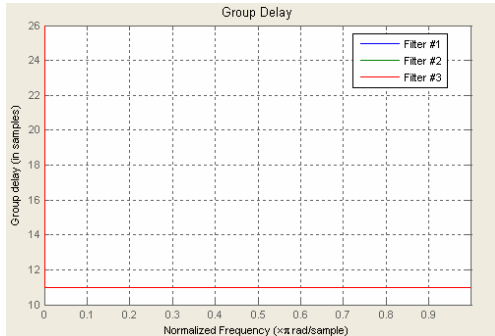
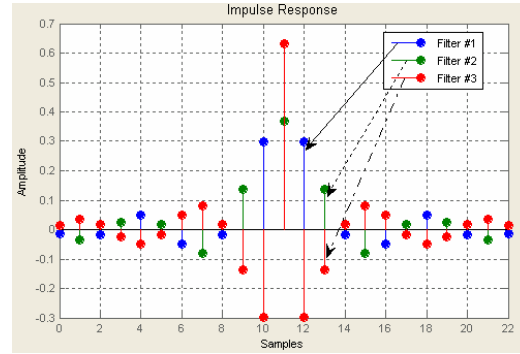
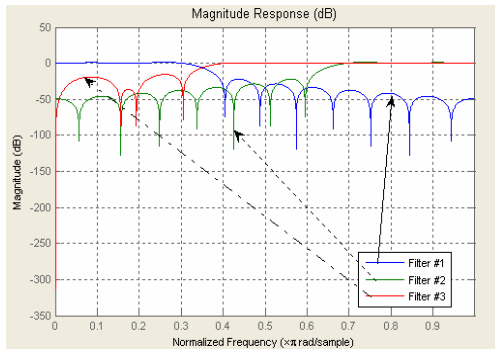
Features	Filter #1	Filter #2	Filter #3	Filter #4	Filter #5	Filter #6	Filter #7
Maximum gain	1.8372	1.4154	1.6900	1.5990	1.6771	1.6646	1.6639
Maximum passband linear	0.1678	0.0052	0.0706	0.0403	0.0516	0.0538	0.0638
Maximum passband error(dB)	-15.5052	-45.7568	-23.0266	-27.8855	-25.7472	-25.3838	-23.9007
Maximum stopband linear	0.0795	0.2385	0.0852	0.1137	0.0782	0.0538	0.0594
Maximum stopband error(dB)	-21.9886	-12.4495	-21.3913	-18.8858	-22.1339	-25.3838	-24.5274
Group delay	11	11	10	11	11	11	11
Largest tap weight coefficient	0.3694	0.3496	0.3500	0.3477	0.3464	0.3500	0.3476



(b) From the figure followed, we can figure out that the group delay is $22-11=11$ samples regardless of the input frequency.



(c) Compare the original, mirror, and complement FIR's impulse, magnitude frequency, and group delay response



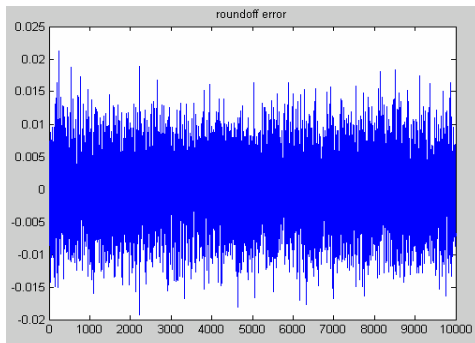
**Filter #1 is the original filter, Filter #2 is the mirror filter, and Filter #3 is the complement filter.

(d) Maximal output is 1.8372, which equals to the worst gain prediction of this filter.

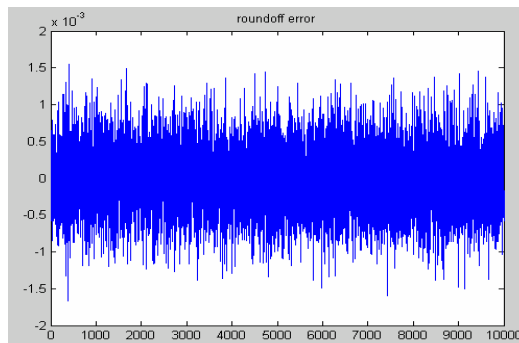
➤ Part 5: Run-time Architecture

(a) $N = 8, M=1$; $N = 12, M=1$; $N = 16, M=1$;

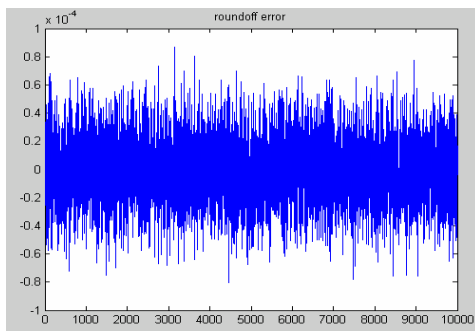
Round off error



N=8



N=12

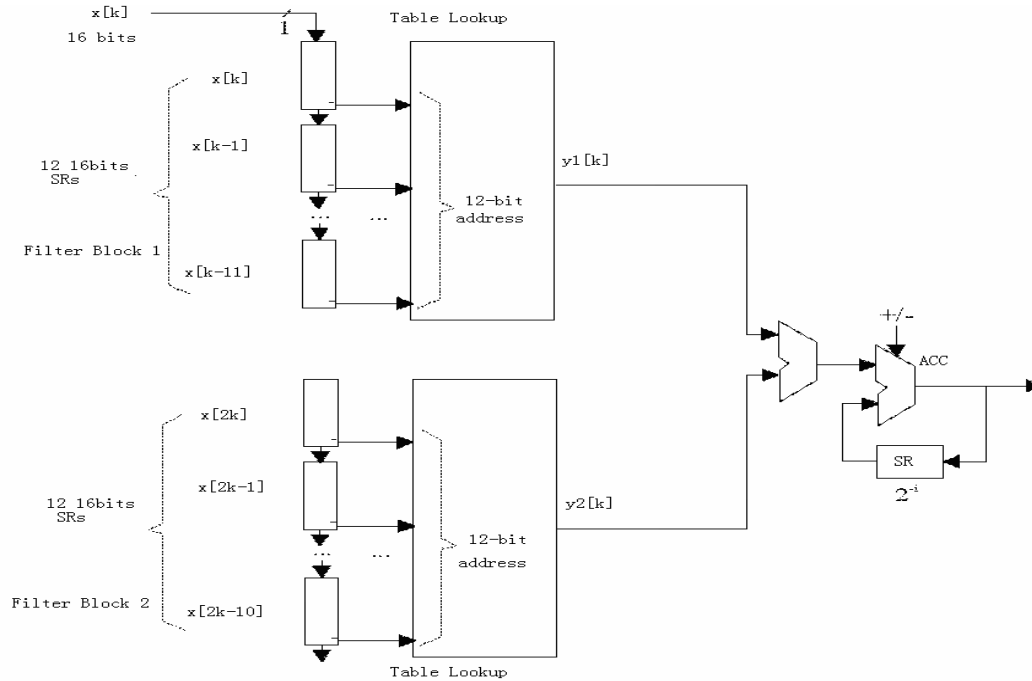


N=16

From the comparison above, we can see clearly that as the value of N increases, the round-off error decreases.

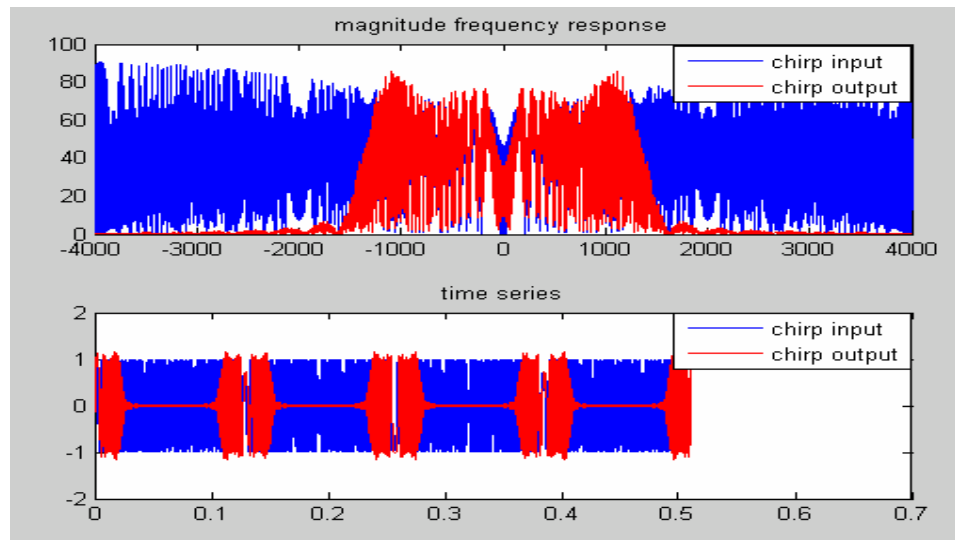
Bits of precision is $N-1-1=N-2$

(b) Choose two 12-bit address space which has memory cycle time of 12 ns, so the maximum run-time filter speed is $1/(12\text{ns/cycle} \times 16 \text{ bits}) = 1/(192 \text{ ns/filter cycle}) = 5.21 \times 10^6 \text{ filter cycles/sec}$



➤ Part 6: Experimentation

- The maximal of the output time-series is 1.1341. It is reasonable, because it is smaller than the worst case gain which is 1.8372. So this agrees with the predicted filter response.
- The “chirp” function makes a short, high-pitched sound, and it sounds four times, which is corresponding to the $4 \times f_s$. When all the .wav files are played, we can hear obviously that the frequency of output sound is much lower than the frequency of input sound, which means that the filter did filter high-frequency components out.



From the figure above, we can see the high-frequency components are gone, which agrees with the predicted filter response, a low-pass filter.

Summary:

Through this project, the detailed processes of designing a filter by three different methods have been understood. And we know more about all the parameters which would affect properties of the filters, and how to use different methods to design them and make best trade-off between each other.