Design of a Thermal/Fluid/Control System

I. INTRODUCTION & CONCEPTS

The wide use of thermal/fluid systems in a variety of applications has made them invaluable to many engineering disciplines. Their unique, flowing, and non-linear nature has caused scientists to both characterize and control them by means of systems of differential equations. Through the case study of a warming bed, this project will focus first on simulating and observing a steady-state heat-transfer system and the interrelation of its variables, and second on the control of that system through proportional control and "on/off" control methods.

In a steady-state system, conditions of objects subject to the system do not change. Specifically for the warming bed, any heat provided by the bed is lost by the patient. By examining one small heating element from the bed, the following energy-balance equation is developed:

$$\frac{dT_w}{dx} = \frac{h_A P^*}{\dot{m}_w c_w}\left(T_w - T_P\right) \tag{1}$$

Equation 1 can be separated and integrated, resulting in an equation for T(x):

$$T(x) = T_P + (T_{in} - T_P)e^{-\frac{h_A P^*}{m_w c_w}x} \tag{2}$$

Furthermore, the heat transfer from the bed to the patient is given as:

$$\dot{Q} = \dot{m}_w c_w (T_{in} - T_{out}) \tag{3}$$

while the heat lost by the patient to the surroundings (due to convection and radiation) is:

$$\dot{Q} = h_B A(T_P - T_\infty) + \varepsilon\sigma(T_P^4 - T_{wall}^4) \tag{4}$$

The second half of the project focuses on time-dependent analysis and feedback control: systems whose behavior and status is dependent on time and whose control is based as a response to the system's performance. At the foundation of these systems is *Equation 5*:

$$M_w c_w \frac{dT_w(t)}{dt} = \dot{m}_w c_w (T_{in} - T_{out}) - \frac{T_w(t) - T_P}{R} \tag{5}$$

From this equation and the average of Tin and Tout, *Equation 6* can easily be derived for later use:

$$\frac{dT_w(t)}{dt} = -\frac{2\dot{m}_w}{M_w}(T_w(t) - T_{in}) - \frac{1}{M_w c_w R}(T_w(t) - T_P) \tag{6}$$

Two control methods are employed in the second phase of this project. The first is proportional control, in which the initial water temperature is increased or decreased, based upon how great a temperature difference exists between the average water temperature and a user-defined target temperature. The change in initial temperature is directly proportional to that error, as is illustrated in the following equation:

$$T_{in}(t) = K_P(T_d - T_w) \tag{7}$$

The second method, "on/off" control, focuses on the extremes of desired performance. The initial water temperature has two settings: the low or "off" setting and the high or "on" setting. When the average temperature becomes higher than desired, the initial temperature is set to the "off" setting. When the average temperature drops too low, the initial temperature is increased to the "on" setting.

ε: Emissivity	m_w: Mass flow rate	T_∞: Air temperature
σ: Stefan-Boltzmann	P^*: Equivalent channel width	T_{out}: Out-going temperature
c_w: Specific heat of water	Q: Total heat rate	T_P: Patient temperature
h_A: Water-skin heat transfer coef	R: Thermal resistance	T_w: Water temperature
h_B: Skin-air heat transfer coef	T: Time	T_{wall}: Room temperature
K_P: Proportionality constant	T_d: Target temperature	x: Distance
M_w: Total mass	T_{in}: Initial Temperature	

II. PROBLEM STATEMENT & RESULTS

Part I: Steady-State Simulation

The first half of this project simulated the heat transfer of a hypothermia bed using a steady-state simulation, a scenario in which all heat transferred to the patient is in turn transferred to the environment through convection and radiation. First, a 3D-plot of the mass velocity and initial temperature vs. change in temperature was programmed. The resulting *Figure 1* shows how ΔT increases linearly with m_v and exponentially with T_{in}. From the figure, values for m_v and T_{in} can be estimated at 0.02 kg/s and 40°C, respectively.

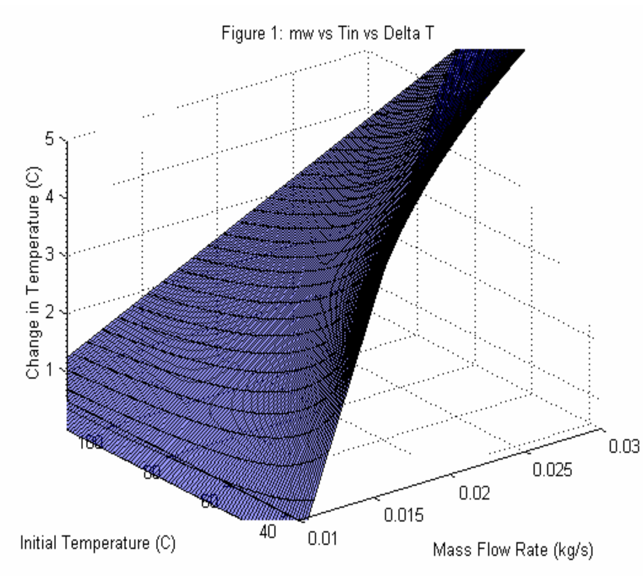


Figure 1: mw vs Tin vs Delta T

A plot of T_w vs. position was then made using a simple Euler progression and *Equation 1*. This plot is shown in *Figure 2*. From this method, the temperature of water leaving the system is found to be 313.5 K.

The mean temperature of the water was found using the equation

$$\overline{T}_w = \frac{1}{L}\int_0^L T(x)dx \qquad (8)$$

where T(x) is from *Equation 2*. Evaluating this integral from L=0 to L=2m, T_w is found to be 314.85 K.

The heat transfer rate was calculated directly from *Equation 3*. Here, T_{out} was determined using the Euler's method employed in plotting T vs. position. The resulting MATLAB script produced an output of 302.72 J/s.

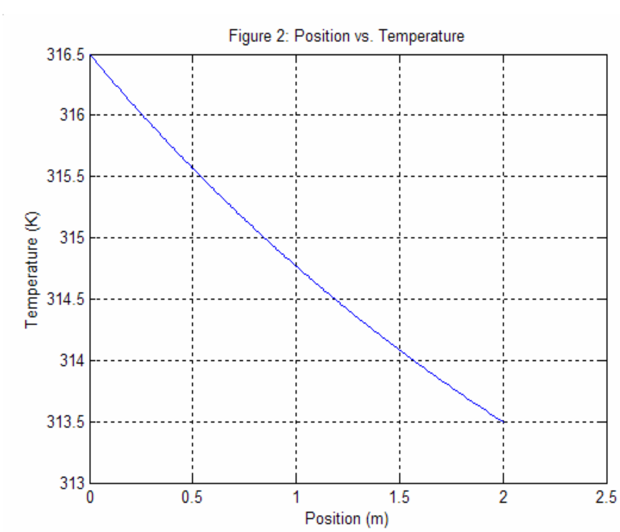


Figure 2: Position vs. Temperature

The steady-state quality of this simulation is due to the equal transfer of heat from the bed to the patient and from the patient to the surroundings through convection, conduction, and radiation. Thus, *Equation 3* can be set equal to *Equation 4*, resulting in a fourth-degree equation for the surrounding temperature. Using the "roots" command, MATLAB was used to solve this resulting polynomial, producing one real solution of 296 K. (See Appendix A)

Finally, with a T_{in} of 42°C, m_v is to be determined in order to maintain the patient at 37°C. By substituting *Equation 2* into *Equation 3*, and maintaining the previously found value of Q, a plot of m_v vs. ΔT is made. From this plot, the value of m_v is estimated as 0.0337 kg/s. Evaluating *Equation 3* for this m_v, the corresponding T_{in} is calculated to be 313.85 K (See Appendix A)

Part II: Time-Dependent Control

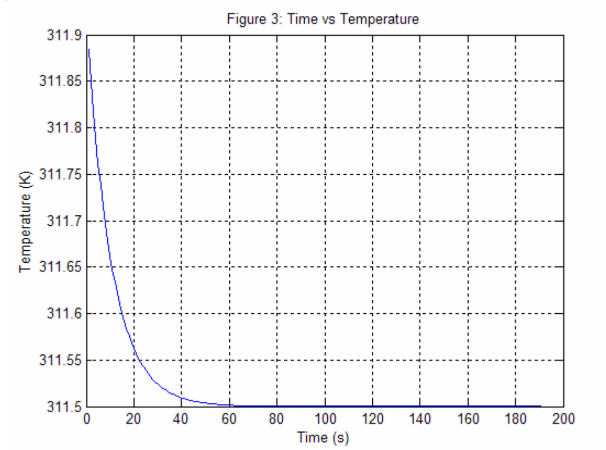
The time-dependent control portion of this project dealt extensively with basic, first-order differential equations. The first problem asked that, for a given initial temperature $T_w(0)$=39.5°C and T_{in}=40°C, the steady state value of T_w be determined. *Equation 6* was solved through separation and integration, resulting in the following equation of $T_w(t)$:

$$T_w(t) = \frac{AT_{in} + BT_P}{A+B} + \left[T_0 - \frac{AT_{in} + BT_P}{A+B}\right]e^{-(A+B)t} \qquad (9)$$

Applying this equation to an array of T_{in} and plotting vs. t results in *Figure 3*. From this chart, T_w can be seen to approach 311.5 K asymptotically, making this value the steady state value of T_w. The time constant, τ, can be determined from the exponential component of *Equation 9*. Since

$$e^{-t/\tau} = e^{-(A+B)t} \qquad (8)$$

then τ=1/(A+B).

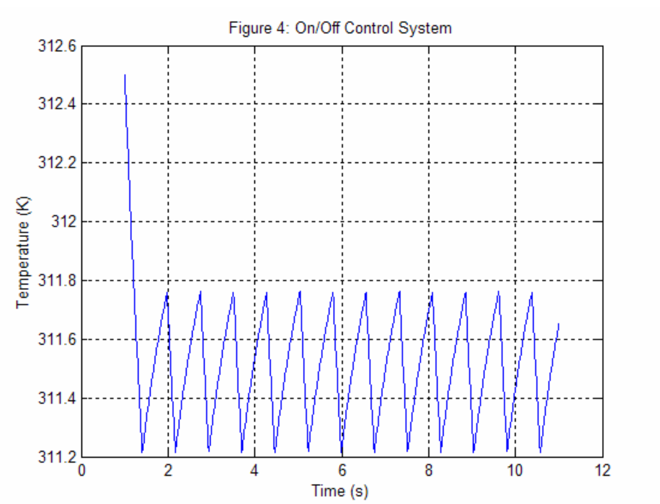


The second portion dealt with a proportional control system, asking for a proportionality constant, K_P, for a system with time constant τ/10 and target value T_d=38.5°C. Substituting *Equation 7* for *Tin* in *Equation 9*, separating the variables, and integrating as before, the following solution for T_w(t) results:

$$Tw(t) = \frac{AK_P T_d + BT_P}{A + AK_P + B} + \left[T_0 - \frac{AK_P T_d - BT_P}{A + AK_P + B}\right]e^{-(A+AK_P+B)t} \qquad (10)$$

Setting τ_1 from the previous portion equal to the new τ_2, K_P can be calculated as 17.999. (See Appendix B) It should also be noted that as t approaches infinity, the second half of the sum in T_w(t) approaches zero and thus T_w(t) approaches 295.8495 K. For this reason, T_w will not reach T_d under the specified conditions.

The final portion of the time-dependent control phase employed an "on/off" control system. Here, if T_w dropped below 38.25°C, T_{in} was switched to 42°C, while if T_w rose above 38.75°C, T_{in} switched to 34C. This function was coded in MATLAB using a series of "if" statements, as well as Euler's method. (See Appendix B) The resulting oscillating plot is shown in *Figure 4*. From this plot, the number of oscillations (i.e., number of peaks or troughs in the wave pattern) may be simply counted over a measured length of time, resulting in the frequency of oscillation of T_w. In this graph, thirteen oscillations occurred over a 10 second interval, illustrating the function's frequency to be 1.3 Hz.



III. CONCLUSION

Through this project, the nature of fluid/thermal systems has become apparent and their association with differential equations made obvious. In addition, several methods for controlling these fluid systems: proportional control and on/off control, have been investigated. Tying all these concepts together was their interrelated use in a real-life application, namely a hypothermia bed.

APPENDIX A:

SELECTED STEADY-STATE PROBLEM CODES

```
clear
Tp = 310;          %Temperature of person (C)
P = 0.12;          %Total width (m)
hA = 260;          %Water-Skin heat transfer coeffecient (W/m^2*K)
hB = 6;            %Skin-Air heat transfer coefficient (W/m^2*K)
cw = 4180;         %Specific heat of water (J/kg*K)
epsilon = 0.95;    %Emissivity
sigma = 5.67E-8;   %Stefan-Boltzmann constant (W/m^2*K^4)
e = 2.71828;       %Natural base
mw = 0.02411;      %Mass flow rate (kg/s)
Tin = 316.5;       %Flow in temperature (K)
L=2;               %Length of pipe (m)
i=1;               %Initialize counter
x(1)=0;            %Set initial position (m)
Told(1)=Tin;       %Set initial temperature (K)
dx = .01;          %Set change in position
A=1.8;             %Surface are of heat transfer (m^2)
while x(i) <= 2.01
   dtdx=(-(hA*P)/(mw*cw))*(Told(i)-Tp); %Calculate change temperature/change position
   Tnew(i) = Told(i) + dtdx*dx;      %Calculate temperature after position change
   Told(i+1)=Tnew(i);                %Set final temperature equal to initial temperature of next stage
   x(i+1)=x(i)+dx;                   %Phase shift position
   i = i + 1;                        %Increase counter
end
Q = (mw*cw*(Tin-Told(i)))            %Total heat rate (J/s)
C = [-epsilon*sigma*A 0 0 -hB*A epsilon*sigma*A*(Tp^4)+hB*A*Tp-Q]    %Create polynomial
matrix
roots(C)                             %Find zeros of polynomial matrix
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```
for mw=(.001:.00001:.1)
   F(i)= ((mw*cw*(Tin-(Tp+(Tin-Tp)*e.^((((-hA*P)/(mw*cw)))*2))))-Q);   %Write to heat rate matrix
   p(i)=mw;      %Write to mw matrix
   i = i + 1;    %Increase counter
end
plot(p,F)        %Plot
xlabel('mw (kg/s)');
ylabel('Delta T (K)');
axis([.03 .035 -5 5])
grid on
mw=0.0337        %mw determined by inspection
Tout = Tp + (Tin-Tp)*e.^((-2*hA*P)/(mw*cw)) %Calculate corresponding Tout
```

APPENDIX B:
SELECTED TIME-DEPENDENT PROBLEM CODES

```
clear
Tp=310;        %Temperature of person (K)
L=2;           %Length of Bed (m)
cw=4180;       %Specific heat of water (J/kg*K)
mw=0.4785;     %Mass flow rate (kg/s)
Mw=2;          %Total mass (kg)
```

```
R=2.5e-4;       %Thermal resistance
T0=312.5;       %Initial temperature (K)
Tin=313;        %Initial temperature (K)
Td=311.5;       %Target temperature (K)
e = 2.71828;    %Natural base
i=1;            %Initialize counter
A=2*mw/Mw;      %Calculate A
B=1/(Mw*cw*R);  %Calculate B
Kp=(9*A + 9*B)/A    %Calculate proportionality factor
for t=[1:.1:20]
   Tw(i)=(A*Kp*Td+B*Tp)/(A+A*Kp+B)           +           (T0-(A*Kp*Td+B*Td)/(A+A*Kp+B))*e.^(-
(A+A*Kp+B)*t);
   ti(i)=t;
   i=i+1;
end
plot(ti,Tw)
grid on
```

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```
for t=[0:.01:10]
   dTw(i) = (-A*(Tw(i)-Tin)-B*(Tw(i)-Tp))*dt;
   Tw(i+1)= Tw(i) + dTw(i);
   if Tw(i)>=311.75
      Tin=TL;
      ti(i+1)=ti(i)+dt;
      i=i+1;
   elseif Tw(i)<=311.25
      Tin=TU;
      ti(i+1)=ti(i)+dt;
      i=i+1;
   else
      Tin=Tin;
      ti(i+1)=ti(i)+dt;
      i=i+1;
   end
end
plot(ti,Tw)
```