Signals And Systems Coursework 1

Q1) for system B where y (n) = 2x (n)-3x (n-1) +x (n-2)

i. The impulse response function h(n) for system A is ;
h (0) =2, h (1) =-3 , h (2)=1

ii. Let x(n)=exp($2\pi jnfT_s$)
y(n)=2exp($2\pi jnfT_s$) - 3 exp($2\pi j(n-1)fT_s$) + exp($2\pi j(n-2)fT_s$)
=2 exp ($2\pi jnfT_s$) -3 [exp ($2\pi jnfT_s$). exp ($-2\pi jfT_s$)] + [exp ($2\pi jnfT_s$). exp ($-4\pi jfT_s$)]
= exp ($2\pi jnfT_s$) [2-3 exp ($-2\pi jfT_s$) + exp ($-4\pi jfT_s$)]
= H (f) exp ($2\pi jnfT_s$)
The frequency response function H (f) = 2 - 3 exp ($-2\pi jfT_s$) + exp ($-4\pi jfT_s$)

iii. A (f) = abs (H (f)) = |H (f)|
Recall that H (f) = 2 - 3 exp ($-2\pi jfT_s$) + exp ($-4\pi jfT_s$)
Treating H (f) as a complex number,
H (f) = 2 – 3[cos ($-2\pi fT_s$) + jsin ($-2\pi fT_s$)] + [cos ($-4\pi fT_s$) + jsin ($-4\pi fT_s$)]
= 2 – 3[cos ($2\pi fT_s$) – jsin ($2\pi fTs$)] + [cos ($4\pi fT_s$) – jsin ($4\pi fT_s$)]
= 2 – 3cos ($2\pi fT_s$) + 3jsin ($2\pi fT_s$) + cos ($4\pi fT_s$) – jsin ($4\pi fT_s$)
Collecting like terms,
[2 – 3cos ($2\pi fT_s$) + cos ($4\pi fT_s$)] + j [3sin($2\pi fT_s$) – sin($4\pi fT_s$)]

The amplitude response function is
A (f) = $\sqrt{[2 - 3\cos(2\pi fT_s) + \cos(4\pi fT_s)]^2 + [3\sin(2\pi fT_s) - \sin(4\pi fT_s)]^2}$

iv. Recall that - □ (f) = angle H (f)

Therefore, □ (f) = - angle H (f)

□ (f) = - $\tan^{-1}[3\sin(2\pi f T_s) - \sin(4\pi f T_s)/[2 - 3\cos(2\pi f T_s) + \cos(4\pi f T_s)]$

For system B where $y(n) = 1/3[x(n) + x(n-1) + x(n-2)]$

i. The impulse response function h (n) is given by;

$h(0) = 1/3$

$h(1) = 1/3$

$h(2) = 1/3$

ii. To find the frequency response function H (f)

Let $x(n) = \exp(2\pi njfT_s)$

$y(n) = 1/3[\exp(2\pi njfT_s) + \exp(2\pi(n-1)jfT_s) + \exp(2\pi(n-2)jfT_s)]$

$= 1/3[\exp(2\pi njfT_s) + \exp(2\pi njfT_s).\exp(-2\pi jfT_s) + \exp(2\pi njfT_s).\exp(-4\pi jfT_s)]$

$= 1/3 \exp(2\pi njfT_s)[1 + \exp(-2\pi jfT_s) + \exp(-4\pi jfT_s)]$

$= H(f).\exp(2\pi njfT_s)$

Therefore the frequency response function is

$H(f) = 1/3[1 + \exp(-2\pi jfT_s) + \exp(-4\pi jfT_s)]$

iii. $H(f) = 1/3[1 + \exp(-2\pi jfT_s) + \exp(-4\pi jfT_s)]$

Treating H (f) as a complex number,

$H(f) = 1/3[1 + (\cos(-2\pi fT_s) + j\sin(-2\pi fT_s)) + (\cos(-4\pi fT_s) + j\sin(-4\pi fT_s))]$

$= 1/3[1 + (\cos(2\pi fT_s) - j\sin(2\pi fT_s)) + (\cos(4\pi fT_s) - j\sin(4\pi fT_s))]$

$= 1/3[1 + \cos(2\pi fT_s) - j\sin(2\pi fT_s) + \cos(4\pi fT_s) - j\sin(4\pi fT_s)]$

Collecting like terms;

$H(f) = 1/3[1 + \cos(2\pi fT_s) + \cos(4\pi fT_s) - j\sin(2\pi fT_s) - j\sin(4\pi fT_s)]$

$= 1/3[(1 + \cos(2\pi fT_s) + \cos(4\pi fT_s)) + j(-\sin(2\pi fT_s) - \sin(4\pi fT_s))]$

$$A(f) = \sqrt{\left(\frac{1 + \cos(2\pi fT_s) + \cos(2\pi fT_s)}{3}\right)^2 + \left(\frac{-\sin(2\pi fT_s) - \sin(4\pi fT_s)}{3}\right)^2}$$

iv. $$\square(f) = \left(\frac{-\tan^{-1}\left(\frac{-\sin(2\pi f T_s)}{3}\right) - \sin(4\pi f T_s)}{\left(\frac{1+\cos(2\pi f T_s)}{3}\right) + \cos(4\pi f T_s)} \right)$$

For System C, to find the equation, we find the product of the frequency response functions H (f) for systems A and B. Also, note that because the linear systems scale, if the input is scaled, the output will also be scaled.

Therefore,

$H(f)_C = H(f)_A * H(f)_B * \exp(2\pi jfnT_s)$

$= \{[2 - 3\exp(-2\pi jfT_s) + \exp(-4\pi jfT_s)] * 1/3[1 + \exp(-2\pi jfT_s) + \exp(-4\pi jfT_s)]\} * \exp(2\pi jfnT_s)$

$= \{1/3[2 + 2\exp(-2\pi jfT_s) + 2\exp(-4\pi jfT_s) - 3\exp(-2\pi jfT_s) - 3\exp(-4\pi jfT_s) - 3\exp(-6\pi jfT_s) + \exp(-4\pi jfT_s) + \exp(-6\pi jfT_s) + \exp(-8\pi jfT_s)]\} * \exp(2\pi jfnT_s)$

$= \{1/3[2 - \exp(-2\pi jfT_s) - 2\exp(-6\pi jfT_s) + \exp(-8\pi jfT_s)]\} * \exp(2\pi jfnT_s)$

$= 1/3[2\exp(2\pi jfnT_s) - \exp(2\pi j(n-1)fT_s) - 2\exp(2\pi j(n-3)fT_s) + \exp(2\pi j(n-4)fT_s)]$

Recall that $x(n) = \exp(2\pi jfnT_s)$

Thus, $y(n) = 1/3[2x(n) - x(n-1) - 2x(n-3) + x(n-4)]$

$= 2/3x(n) - 1/3x(n-1) - 2/3x(n-3) + 1/3x(x-4)$

i. The impulse response function h(n) is given by :
   h (0)= 2/3
   h (1)= -1/3
   h (2)= 0
   h (3)= -2/3
   h (4)= 1/3

ii. The frequency response function is

$H(f)= 1/3[2 - \exp(-2\pi jfT_s) - 2\exp(-6\pi jfT_s) + \exp(-8\pi jfT_s)]$

iii. To find the amplitude response function A(f),

We recall that, $A(f) = abs(H(f)) = |H(f)|$

And $H(f)= 1/3[2 - \exp(-2\pi jfT_s) - 2\exp(-6\pi jfT_s) + \exp(-8\pi jfT_s)]$

Treating H(f) as a complex number,

$H(f)=1/3[2 - [\cos(-2\pi fT_s) + j\sin(-2\pi fT_s)] -2[\cos(-6\pi fT_s) + j\sin(-6\pi fT_s)] + [\cos(-8\pi fT_s) + j\sin(-8\pi fT_s)]$

$=1/3[2 - [\cos(2\pi fT_s) - j\sin(2\pi fT_s)] - 2[\cos(6\pi fT_s) - j\sin(6\pi fT_s)] + [\cos(8\pi fT_s) - j\sin(8\pi fT_s)]$

Rearranging and collecting like terms,

$H(f) = 1/3\ [2 - \cos(2\pi fT_s) - 2\cos(6\pi fT_s) + \cos(8\pi fT_s)] + 1/3j\ [\sin(2\pi T_s) + 2\sin(6\pi fT_s) - \sin(8\pi fT_s)]$

$A(f)=\sqrt{(1/3[2 - \cos(2\pi fT_s) - 2\cos(6\pi fT_s) + \cos(8\pi fT_s)])^2 + (1/3[\sin(2\pi T_s) + 2\sin(6\pi fT_s) - \sin(8\pi fT_s)])^2}$

iv. To find the phase response function □(f)

We recall that, -□(f) = angle H(f)

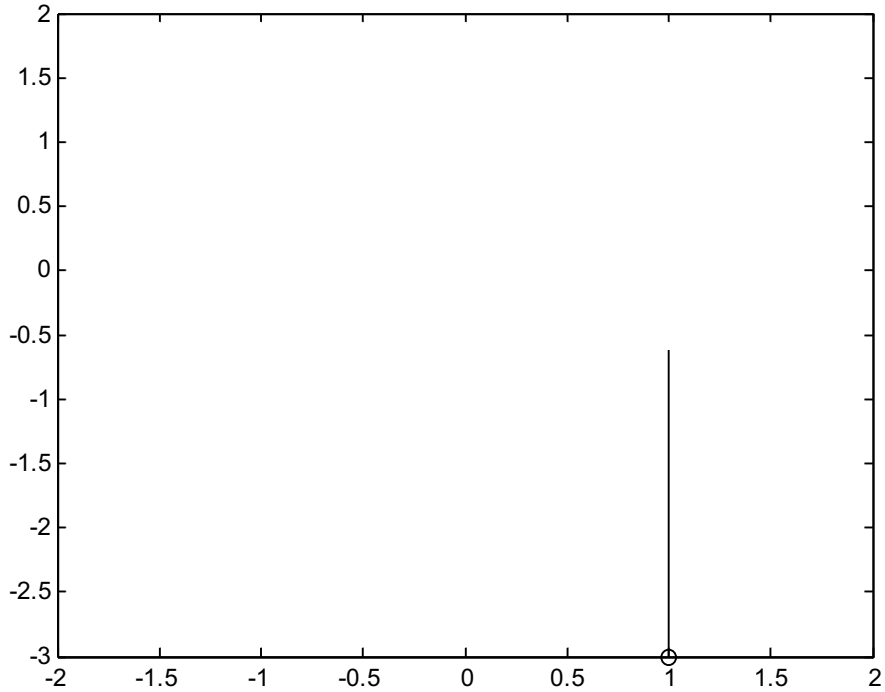
Therefore, □(f) = - angle H(f)

$$\square(f) = -\tan^{-1}\left(\frac{1/3[\sin(2\pi T_s) + 2\sin(6\pi fT_s) - \sin(8\pi fT_s)]}{1/3[2 - \cos(2\pi fT_s) - 2\cos(6\pi fT_s) + \cos(8\pi fT_s)]}\right)$$

$$= -\tan^{-1}\left(\frac{[\sin(2\pi T_s) + 2\sin(6\pi f T_s) - \sin(8\pi f T_s)]}{[2 - \cos(2\pi f T_s) - 2\cos(6\pi f T_s) + \cos(8\pi f T_s)]}\right)$$

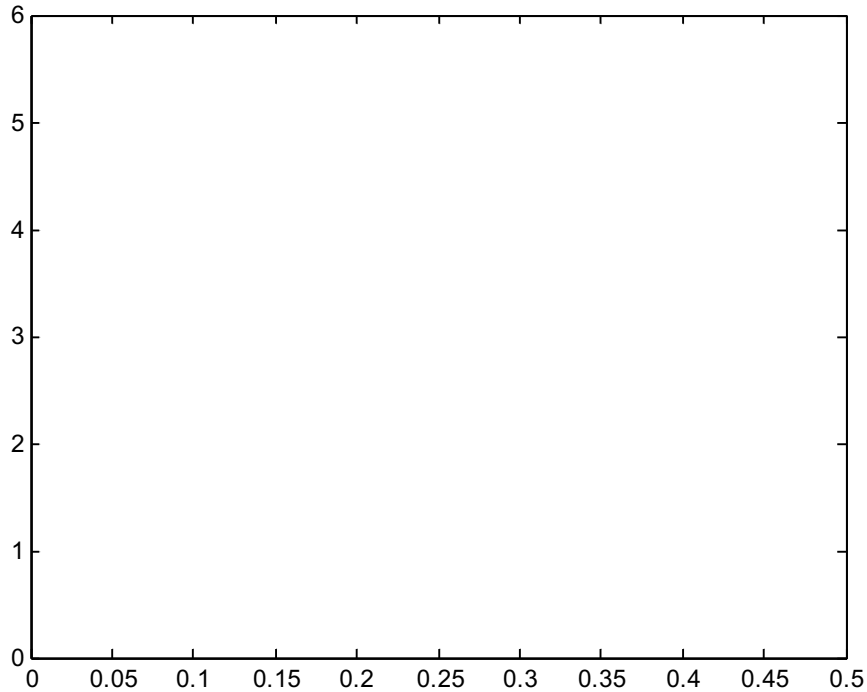
MATLAB CODE USED IN GENERATING RESULTS FOR Q1

```
function [y]=LTI_sys_A(x,a,b,c)
%function assumes that the system equation is
%y(n)= a*x(n)+ b*x(n-1)+ c*x(n-2)
y(1)=0;%for y(1) and y(2) I would need x(0) and
y(2)=0;%x(-1) which I don't have so assume y(1)
% and y(2) are 0.
lx= length(x);
for n=3:lx,
   y(n)=a*x(n)+ b*x(n-1)+ c*x(n-2);
end;

 t=-2:1:2;

 x=0*t;

 pulse=find(t==0);%I could use x(3)=2 here

 x(pulse)=1;  %This is just a useful command

 title('Impulse Response Function For System A ')

 y=LTI_sys_A(x,2,-3,1);

stem(t,y,'k');xlabel('Time');ylabel('Output')
```




```
Ts=1;

 f=0:0.001:0.5;

 H=2- 3.*exp(-2*pi*j*f*Ts)+ exp(-4*pi*j*f*Ts);

 A=abs(H);

 plot(f,A,'k')

 title('Amplitude Response For System A')
```

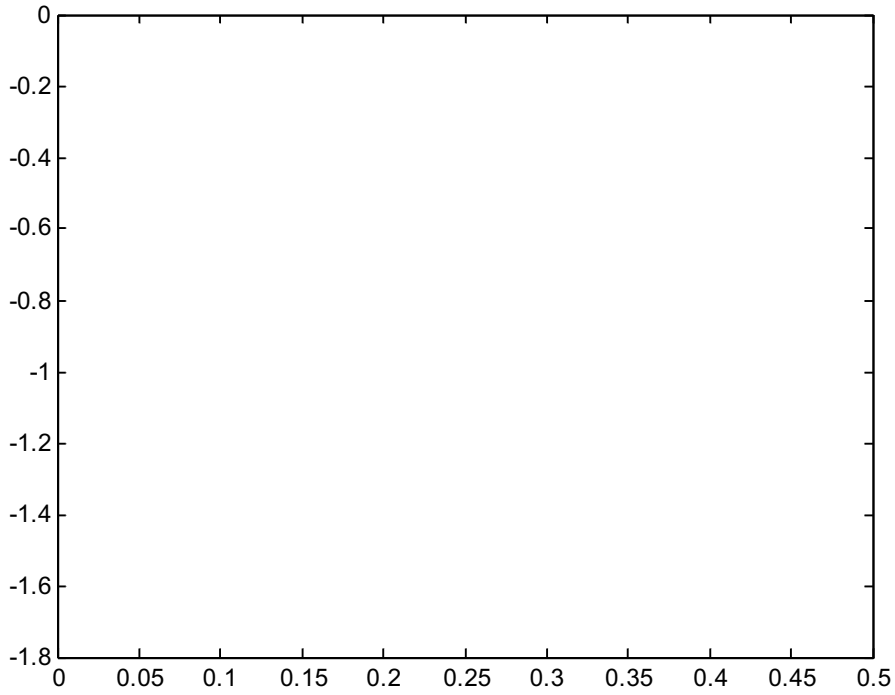
```
ylabel('Amplitude');xlabel('Frequency
```



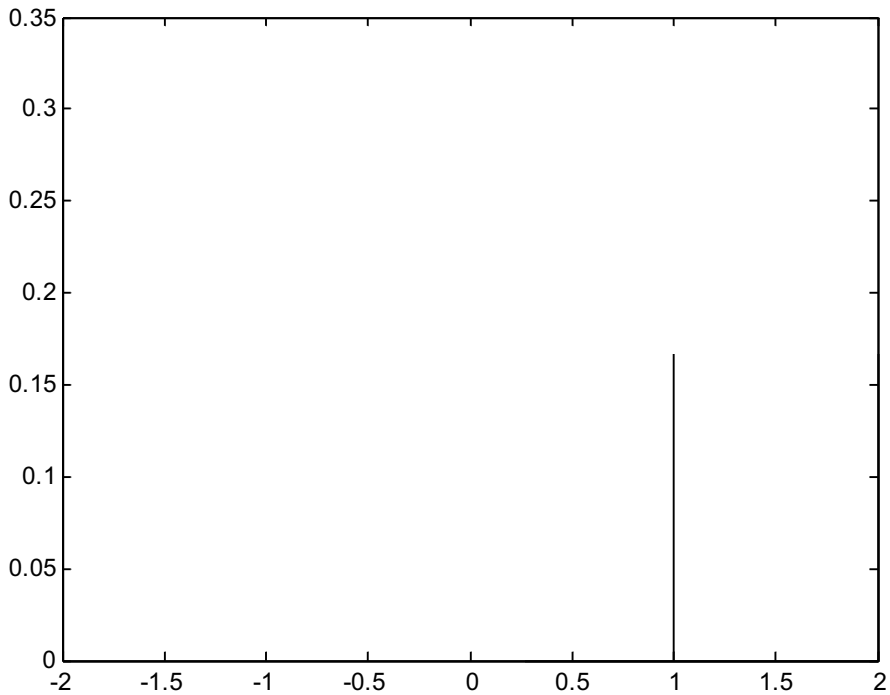

```
(Hz)')
```

```
Ts=1;

f=0:0.001:0.5;

H=2-3.*exp(-2*pi*j*f*Ts)+ exp(-4*pi*j*f*Ts);

Theta= -angle(H);% the negative sign is to indicate that

%what would be a positive shift is actually a time delay.

plot(f,Theta,'k')
```

```
title('Phase Response For System A')

xlabel('Frequency (Hz)');ylabel('Phase Response (Radians)')
```

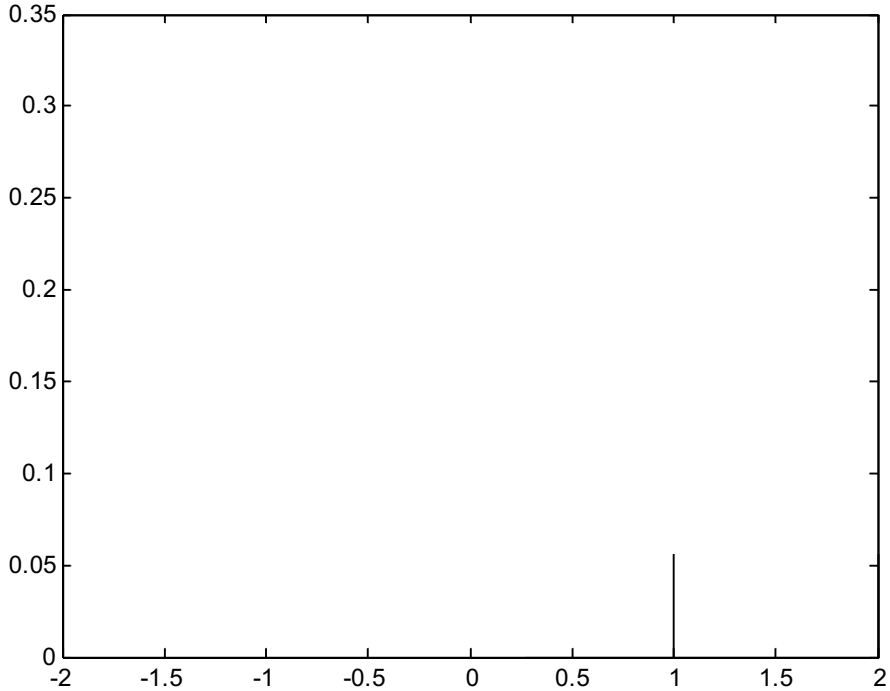


```
function [y]=LTI_sys_B(x,a,b,c)
%function assumes that the system equation is
%y(n)= a*x(n)+ b*x(n-1)+ c*x(n-2)
y(1)=0;%for y(1) and y(2) I would need x(0) and
y(2)=0;%x(-1) which I don't have so assume y(1)
% and y(2) are 0.
lx= length(x);
for n=3:lx,
   y(n)=a*x(n)+ b*x(n-1)+ c*x(n-2);
end;


 t=-2:1:2;

 x=0*t;

 pulse=find(t==0);%I could use x(4)=1/3 here

 x(pulse)=1;      %This is just a useful command

 y=LTI_sys_B(x,0.3333,0.3333,0.3333);

 stem(t,y,'k');xlabel('Time');ylabel('Output')

 title('Impulse Response Function For System B')
```

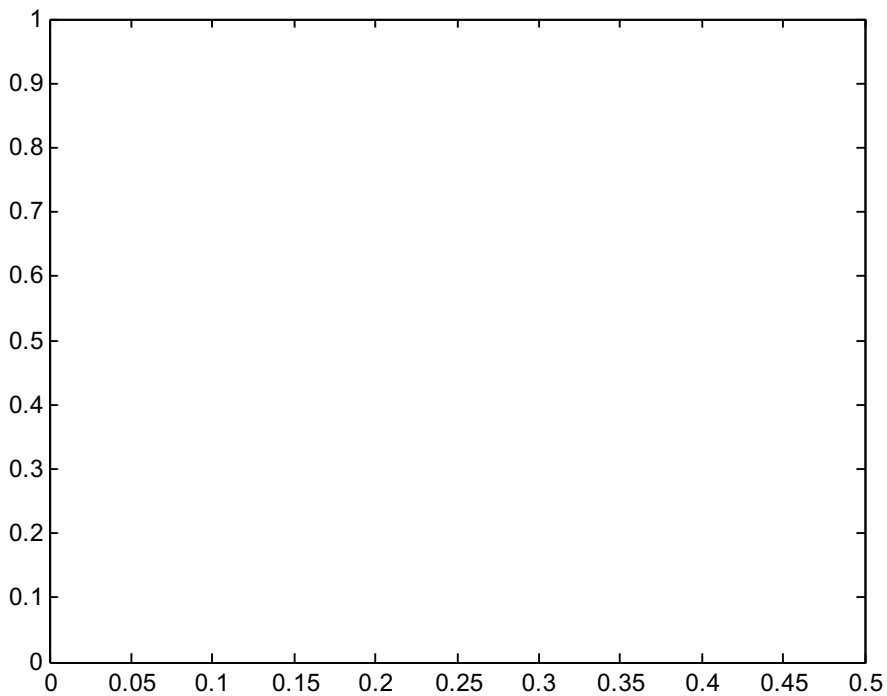



```
function [y]=LTI_sys_B(x,a,b,c)
%function assumes that the system equation is
%y(n)= a*x(n)+ b*x(n-1)+ c*x(n-2)
y(1)=0;%for y(1) and y(2) I would need x(0) and
y(2)=0;%x(-1) which I don't have so assume y(1)
% and y(2) are 0.
lx= length(x);
for n=3:lx,
   y(n)=a*x(n)+ b*x(n-1)+ c*x(n-2);
end;


t=-2:1:2;

x=0*t;

pulse=find(t==0);%I could use x(4)=1/3 here

x(pulse)=1;      %This is just a useful command

y=LTI_sys_B(x,0.3333,0.3333,0.3333);

stem(t,y,'k');xlabel('Time');ylabel('Output')
```

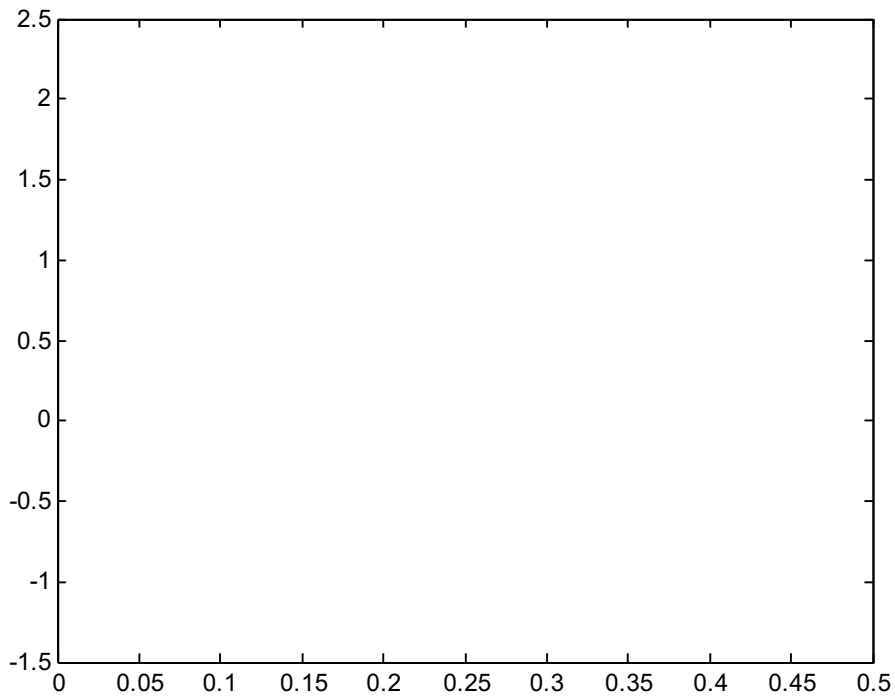
```
title('Impulse Response Function For System B')
```



```
Ts=1;

f=0:0.001:0.5;

H=0.3333+ 0.3333.*exp(-2*pi*j*f*Ts)+ 0.3333.*exp(-4*pi*j*f*Ts);

A=abs(H);

plot(f,A,'k')

xlabel('Frequency (Hz)');ylabel('Amplitdue')

title('Amplitude Response For System B')
```

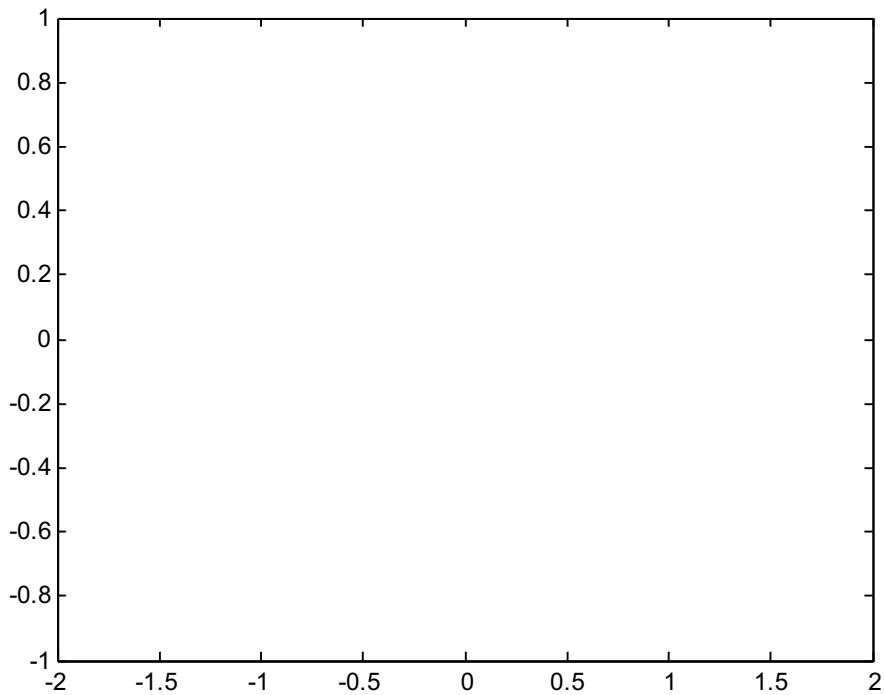


```
Ts=1;

f=0:0.001:0.5;

H=0.3333+ 0.3333.*exp(-2*pi*j*f*Ts)+ 0.3333.*exp(-4*pi*j*f*Ts);

Theta=-angle(H);

plot(f,Theta,'k')

xlabel('Frequency (Hz)');ylabel('Phase Response (Radian)')

title('Phase Response For System B')
```



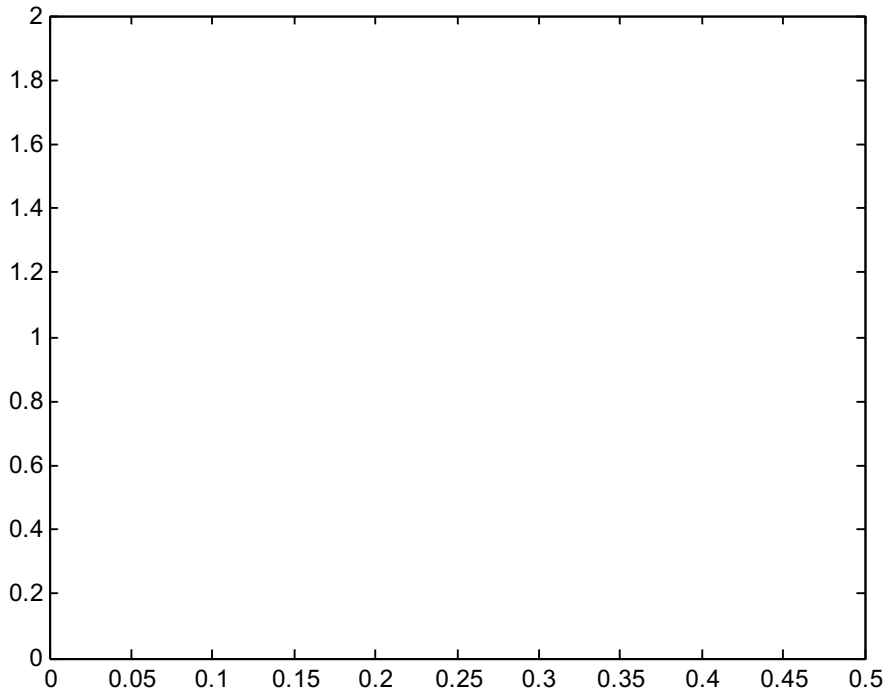

```
function [y]=LTI_sys_C(x,a,b,c,d,e)
%function assumes that the system equation is
%y(n)= a*x(n)+ b*x(n-1)+ c*x(n-2) + d*x(n-3)+ e*x(n-4)
y(1)=0;%for y(1) and y(2) I would need x(0) and
y(2)=0;%x(-1) which I don't have so assume y(1)
% and y(2) are 0.
lx= length(x);
for n=5:lx,
   y(n)=a*x(n)+ b*x(n-1)+ c*x(n-2)+ d*x(n-3) + e*x(n-4);
end;
```

```
t= -2:1:2;

x= 0*t;

pulse= find(t==0);

x(pulse)=1;

y= LTI_sys_C(x,0.6667,0.3333,0,-0.6667,0.3333);

stem(t,y,'k');xlabel('Time');ylabel('Output')

title('Impulse Response Function For System C')
```

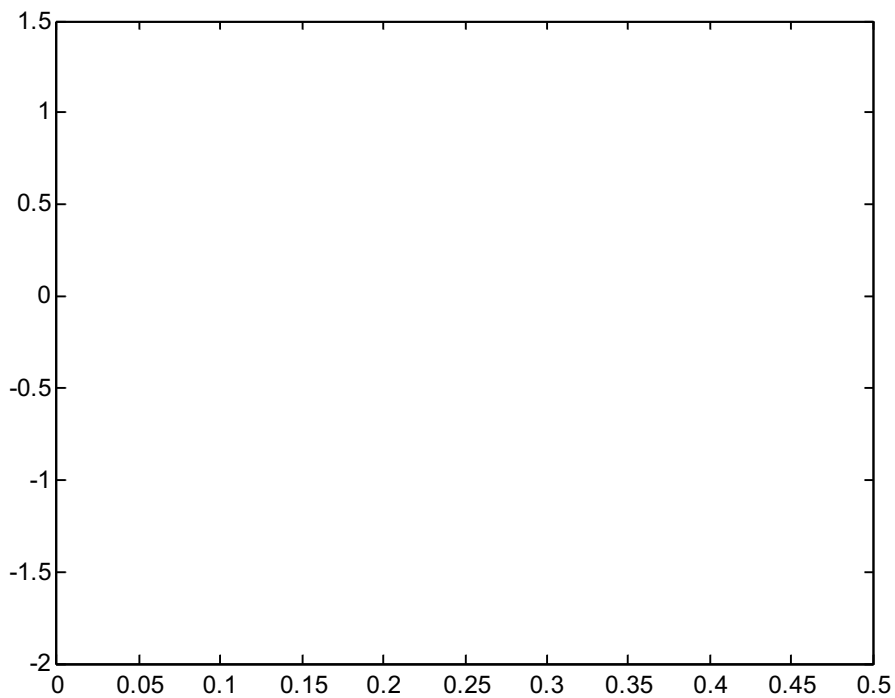



```
Ts=1;

f=0:0.001:0.5;

H=0.6667-0.3333.*exp(-2*pi*j*f*Ts)-0.6667.*exp(-6*pi*j*f*Ts)+0.3333.*exp(-
8*pi*j*f*Ts);
```

```
A=abs(H);

plot(f,A,'k')

xlabel('Frequency (Hz)');ylabel('Amplitude')

title('Amplitude Response For System C')
```



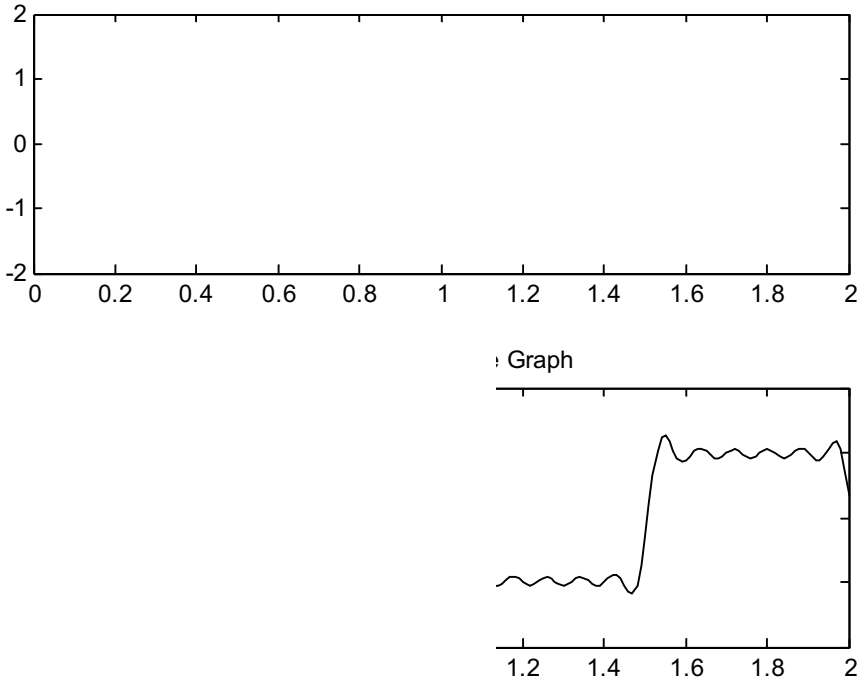
```
Ts=1;

f=0:0.001:0.5;

H=0.6667-0.3333.*exp(-2*pi*j*f*Ts)-0.6667.*exp(-6*pi*j*f*Ts)+0.3333.*exp(-
8*pi*j*f*Ts);

Theta=-angle(H);

plot(f,Theta,'k')

xlabel('Frequency (Hz)');ylabel('Phase Response(Radian)')

title('Phase Response For System C')
```




```
c=(1/((2*n)+1)^2)*cos((2*n)+1)*(w*t);
c=(1/((2*n)+1)^2)*cos((2*n)+1)*(w*t);
   b=(1/((2*n)+1)^2)*cos((2*n)+1)*(w*t);
```

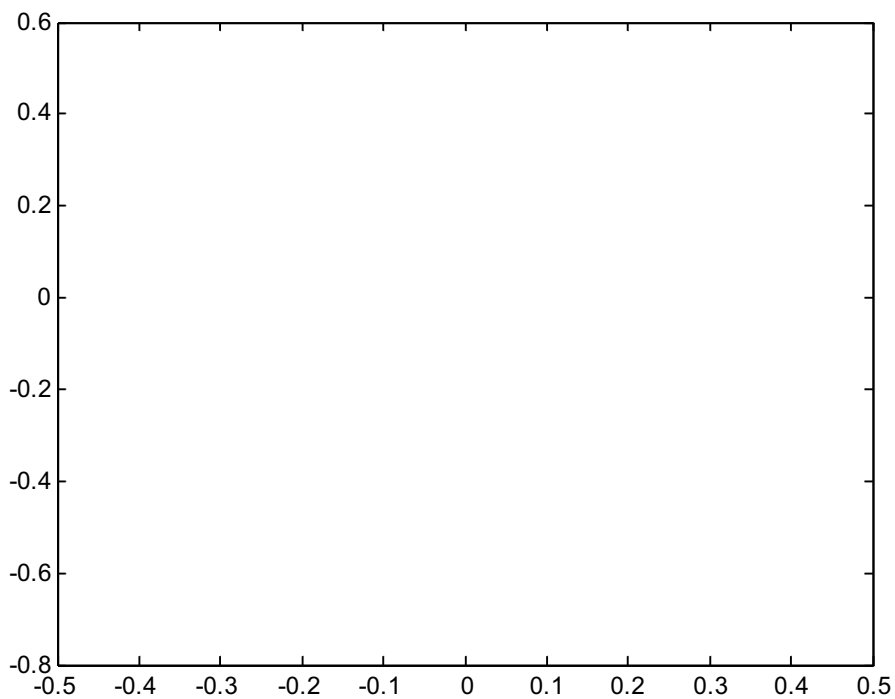
Q2) **MATLAB CODE FOR Q2**

```
T=1;
Ts=T/100;
w0=2*pi/T;
t=0:Ts:2;
x=0*t;
y=0*t;
for n= 0:1:5,
   x=x+(1/(2*n+1).^2) * cos((2*n+1)*w0*t);
   f=(2*n+1)/T;
   HA=2-3*exp(-2*pi*j*f*Ts)+ exp(-4*pi*j*f*Ts);
   HB=(1+exp(-2*pi*j*f*Ts)+exp(-4*pi*j*f*Ts))/3;
   HC=HA*HB;
   AC=abs(HC);
   thetaC=-angle(HC);
   y=y+AC.*(1/(2*n+1).^2)*cos((2*n+1)*w0*t-thetaC);
end;
xlabel('Time(s)')
ylabel('Amplitude')
title('Amplitude response Graph')
subplot(211)
plot(t,x,'k')
subplot(212)
xlabel('Time(s)')
ylabel('Phase(radians)')
title('Phase Response Graph')
plot(t,y,'k')
```


2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Graph
1.2
1.4
1.6
1.8
2

Q3) **MATLAB CODE FOR Q3**

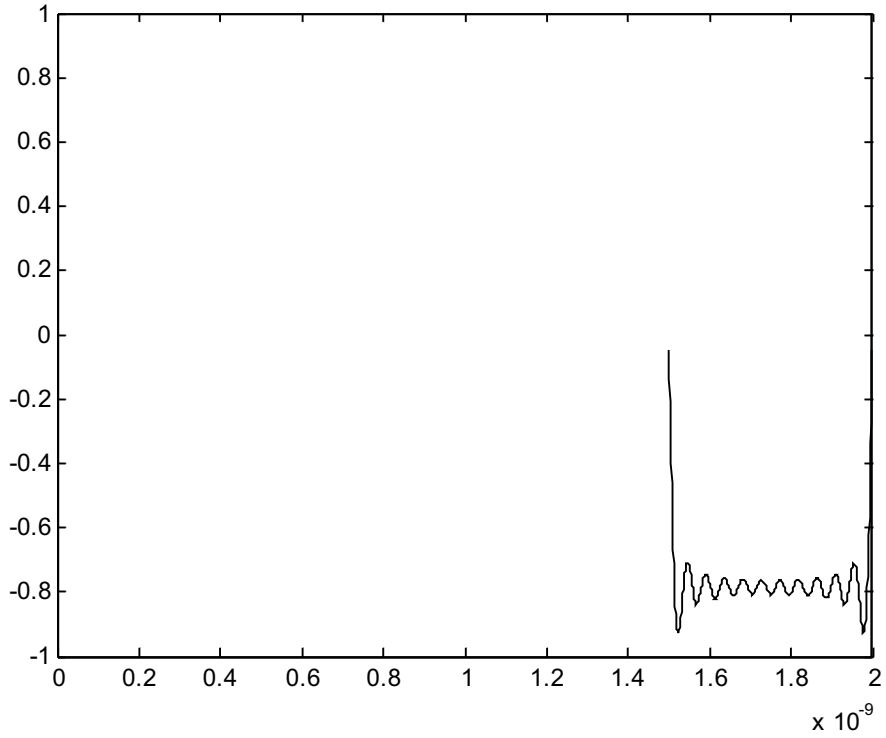
```
T=1;
f0=1/T;
Ts=0.001;
t=-T/2:Ts:T/2;
A0=integ(x,t)*2/T;
x=t;
xapprox=0*t+A0/2;
for n=1:50,
   A(n)=integ(x.*cos(2*pi*n*f0*t),t)*2/T;
   B(n)=integ(x.*sin(2*pi*n*f0*t),t)*2/T;
   xapprox=xapprox+A(n)*cos(2*pi*n*f0*t)+B(n)*sin(2*pi*n*f0*t);
end;
plot(t,x,'k',t,xapprox,'k--')
xlabel('Time(s)')
ylabel('x(t)')
```

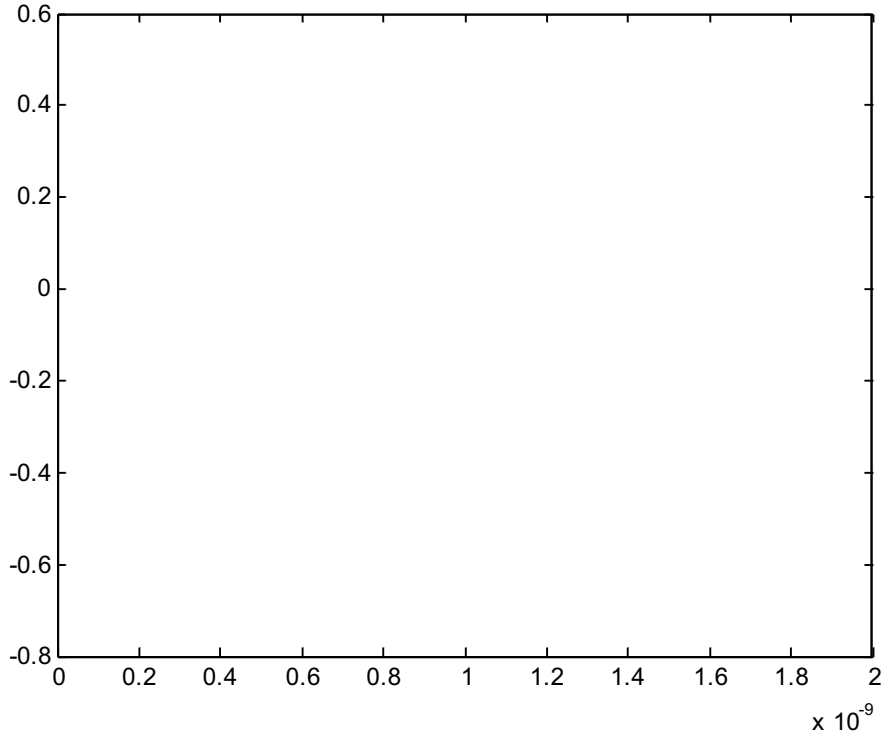


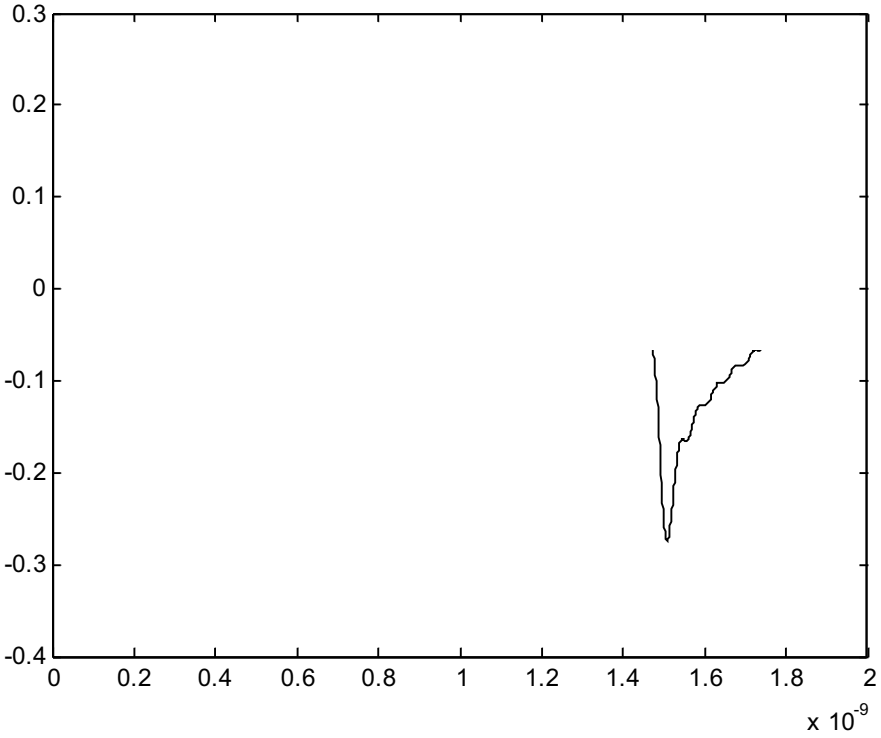
Therefor, the number of terms needed to get a reasonable approximation to the saw-tooth wave x(t) is 50

Q4) **MATLAB CODE FOR Q4.**

```
R1=1;R2=0.5;C1=50*10^-12;C2=C1;T=10^-9;
w0=2*pi/T;
t=0:10^-12:2*10^-9;
x=0*t; i=0*t;Vc=0*t;
N=10; % Series runs from n=0 to N
for n=0:N,
   f=(2*n+1)/T;
   x=x+sin(2*pi*f*t)/(2*n+1);
   ZC1=-j/(w0*C1);
   ZC2=-j/(w0*C2);
   Z=R1+ZC1+R2*ZC2/(R2+ZC2);
   H=1/Z;
   A=abs(H);
   theta=-angle(H)
   i=i+A*sin(2*pi*f*t-theta)/(2*n+1);
   Vco=H*R2*ZC2/(R2+ZC2);
   A=abs(Vco);
   theta=-angle(Vco);
   Vc=Vc+A*sin(2*pi*f*t-theta)/(2*n+1);
end;
plot(t,x,'k')
plot(t,i,'k')
plot(t,Vc,'k')
```


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^-9


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^-9


0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^-9