

$$x = y + (z \times t)$$

$$x - z \times t = y$$

$$E(P_x, P_y, U)$$

$$y = x - z \times t$$

For any chosen level of utility  $U$  the following identity will hold:  
 $H_x(P_x, P_y, U) = d_x(P_x, P_y, E(P_x, P_y, U))$

- In words the quantity demanded is the same for the compensated and uncompensated demand functions as long as the income is exactly adequate to attain the required utility level this is point  $P_{x1}$  on the above graph.
- However they do not respond the same to a change in price in particular differentiating this equality with respect to  $P_x$  yields the following equation:

$$\frac{\partial h_x}{\partial P_x} = \frac{\partial d_x}{\partial P_x} + \frac{\partial d_x}{\partial E} \times \frac{\partial E}{\partial P_x}$$

Rearranging gives:

$$\frac{\partial d_x}{\partial P_x} = \frac{\partial h_x}{\partial P_x} - \frac{\partial d_x}{\partial E} \times \frac{\partial E}{\partial P_x}$$

- The uncompensated demand response to a price change is equal to the compensated demand response minus another term.
- The first term is the slope of the compensated demand curve, which is the substitution effect that comes about from a change in  $P_x$ .
- The second term reflects the way in which changes in  $P_x$  affect the demand for  $X$  through changes in expenditure levels. Thus this term reflects the income effect. The negative sign shows the direction of the effect. An  $P_x$  increases the expenditure level necessary to keep utility constant also has to increase mathematically:

$$\frac{\partial E}{\partial P_x} > 0$$

- But because nominal income is kept the same the extra expenditure is not available so the consumption of  $X$  has to be reduced to meet this shortfall the extent of the reduction is given by: