

Economic Growth

1. Explain the significance of assuming that the production function $Y = F(K, L)$ in the Solow model is *neoclassical*, i.e. it satisfies (i) positive and diminishing returns to private inputs, (ii) constant returns to scale and (iii) the INADA conditions $\lim_{K \rightarrow 0} F_K = \lim_{L \rightarrow 0} F_L = \infty$ and $\lim_{K \rightarrow \infty} F_K = \lim_{L \rightarrow \infty} F_L = 0$.

These assumptions have been the fundamental grounding on which most of the neoclassical results have been based on. Their implications are crucial to both an understanding of the neoclassical models and how the various new versions of growth theory which depart from these basic assumptions differ in the outcomes they predict.

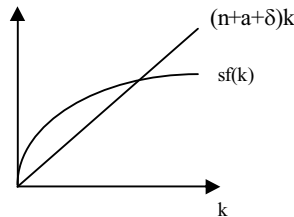
The steady state in the Solow model represents the capital to labour ratio and output to labour ratio which is constant and does not change, i.e. when $\Delta k = 0$, where $k = K/L$, and so $\Delta y = 0$, $y = Y/L$.

Thus using the capital accumulation equation and the production function, the following relationship is required for steady state:

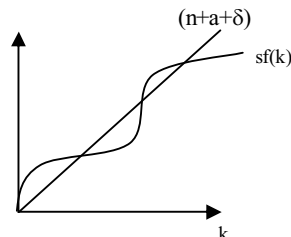
$$\Delta k = sf(k) - (n+a+\delta)k = 0 \rightarrow sf(k) = (n+a+\delta)k$$

s: savings rate ($I=S$), n: population growth rate, a: technological progress rate, δ : depreciation rate of capital.

In order to ensure that this condition occurs, i.e. a steady state occurs, it is necessary to have the Inada conditions, so that the eventually the level of investment equals the amount of depletion of capital per worker, so that there is no net change overall. The constant returns to scale requirement is also necessary since the equation is in terms of capital per worker and output per worker, and thus the Inada conditions will still hold for the variables in per worker terms if one has constant returns to scale.



While the Inada conditions satisfies the condition that eventually there will be a level at which there is not net change in capital per worker, this does not guarantee that there will only be one steady state. In order to ensure this, one needs to have the first assumption, i.e. that of diminishing marginal returns, which implies that output per worker rises smoothly with any increase in capital and therefore only one level of capital is consistent with the steady state level. If this condition was not satisfied, then could have a condition such as this, which would lead to multiple steady states:



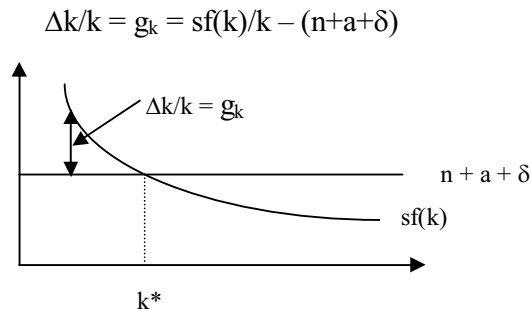
Finally for convergence, what is necessary is that the following condition is satisfied, with k^* representing the steady state level., if $k < (>) k^*$, then $\Delta k > (<) 0$. Using the equation above, we can rearrange it to show the condition required for the above to be satisfied:

$$\begin{aligned}\Delta k &= sf(k) - (n+a+\delta)k \\ \rightarrow \Delta k/k &= g_k = sf(k)/k - (n+a+\delta) \\ \rightarrow &\text{ need } sf(k)/k \text{ to be decreasing with } k, \text{ which is satisfied given the condition} \\ &\text{ of decreasing marginal returns.}\end{aligned}$$

2. For 98 countries from 1960 to 1997, the growth rate of GDP per capita shows little relation to the level of initial GDP per capita. Does this finding contradict with the neoclassical model of economic growth of Solow and Swan? Explain.

The answer to the question above has to be an unequivocal no. Whether the Solow-Swan model is disproved will depend on further analysis of the data. The question is basically stating that absolute convergence does not occur according to the data, and yet the Solow-Swan model only predicts conditional convergence.

According to the Solow model, the further one is below the steady state, the faster its rate of growth should be. This is based on the assumption that the growth rate in the economy is proportional to the growth rate of capital. Using the following equation, derived from the previous question, one can see that the further one is away from the steady state, the greater the growth rate of capital (g_k) and hence income:



Thus the further one is from the steady state is (k^*) the faster the growth rate will be. One common mistake people make is that this implies that countries that are poor relative to their steady state will grow faster rather than in general poor countries will grow faster. Hence the result stated in the question should not come as any surprise, since it is unlikely that all the countries in the sample of 98 will have the same steady state or even near the steady state. This becomes more obvious if one considers what the steady state value depends on. In order to do this, I will assume that the production function will take on a Cobb-Douglas form ($Y=K^\alpha AL^{1-\alpha}$, where A is the technology level, implying that instead of y being output per labour, it is output per effective labour, and the same for k). Therefore the steady state can be obtained as follows:

$$\begin{aligned}\Delta k &= sk^\alpha - (n+g+\delta)k = 0 \\ \rightarrow k &= (s/(n+g+\delta))^{1/(1-\alpha)} \\ \rightarrow y &= (s/(n+g+\delta))^{\alpha/(1-\alpha)}\end{aligned}$$

Therefore as this equation shows the steady state will depend on savings, population growth rate, technological growth rate and depreciation. While it is reasonable to assume that the last two are constant across countries, the same cannot be said for population growth rate and savings, which can easily differ between countries and therefore the absolute convergence should not be expected. Thus the results stated in the question do not run counter to the Solow-Swan model.

4. ‘Any adequate theory of growth must also explain the relative differences in growth rates between nations.’ Assess the ability of modern growth theories to address this issue.

Since the groundbreaking paper by Robert Solow in 1956, the study of economic growth has been at the forefront of many macroeconomic studies. Its importance is obvious, with its significance for living standards in a country, as well as distribution of income in the world. Generating a formal model for it though had not been done before Solow, and since then there has been a flurry of models that have been suggested to account for long-term growth, that have tried to deal with the shortcomings of Solow’s model. In this paper it is my intention to look at these various models and assess their usefulness and consistency with the data.

Solow’s model was based on neoclassical foundations, which may appear to be logical if one is considering the long run. He assumed a production function with constant returns to scale that had properties such as diminishing marginal returns to the various inputs and various Inada conditions mentioned above. Thus the production function is generally represented as being a Cobb Douglas Production function with inputs capital and labour. It is also assumed that there is technology in the production function, which has is augmented with the labour term as a Harrod-Neutral term:

$$Y = F(K, L)$$

$$Y = K^{\alpha} (AL)^{1-\alpha}$$

$$y = k^{\alpha}$$

Where k and y are the levels of capital and output per effective worker respectively. Using these properties as assuming that there is constant ratio between labour and capital and labour and output in steady state, then one can use the capital accumulation equation to derive the growth rate of capital per effective

$$\Delta k = sf(k) - (n+a+\delta)k$$

$$\rightarrow \Delta k/k = sf(k)/k - (n+a+\delta)$$

n : population growth rate, a : rate of technological progress, δ : rate of depreciation of capital.

In steady state this growth rate should equal zero, so that the level of capital is not changing and therefore the economy is stable. Using this one can obtain the steady state level of capital per effective worker and hence also the steady state level of output:

$$\Delta k/k = sf(k)/k - (n+a+\delta) = 0$$

$$\rightarrow k^* = [s/(n+a+\delta)]^{1/1-\alpha}$$

$$\rightarrow y^* = [s/(n+a+\delta)]^{\alpha/1-\alpha}$$

$$(Y/L)^* = A[s/(n+a+\delta)]^{\alpha/1-\alpha}$$

Hence Solow was able to show that the differences in levels of GDP per capita between countries can be explained by the differences in their savings rate and their population growth rate, assuming that depreciation and technological progress would be the same. This general conclusion seemed consistent with the data, at least qualitatively if not necessarily quantitatively. However the results of his analysis had an even more important implication, which was that countries with lower levels of GDP should grow faster than countries that were richer, hence resulting in convergence in their incomes. This however is implied not in the absolute sense but in the conditional sense, since countries rate of growth will depend on their level of capital (and hence income) relative to *their* steady state. This is because as the equation above showed the level of capital growth can be expressed as follows:

$$\Delta k/k = sf(k)/k - (n+a+\delta)$$

Therefore assuming, as we have done, that production function exhibits decreasing returns to scale, then the smaller k is the higher the growth rate is.

In steady state though, the growth rate of output will only be equal to the rate at which effective labour increases, since output to effective labour ratio has to remain constant, thereby implying that the growth rate of the economy in terms of just GDP would be equal to population growth and technological progress, while GDP per capita will grow at technological progress rate.

As I have shown, from this fairly simple model, one can have a lot of interesting implication, and a lot early empirical work was focused on testing its implications, in terms of its effects on the level of GDP and growth rates. One of the common errors carried out by these early researchers was that they all took evidence of the lack of convergence in the world as a whole to represent evidence against the model. Hence people had confused its implication of *conditional* convergence with that of *absolute* convergence. Fortunately in recent years a more comprehensive set of data has become available of macroeconomic aggregates in the world due to Heston and Summers, and thus a more adequate test of the theory has become available.

One of the best tests of this model in recent years has been carried out by Mankiw, Romer and Weil, who carry out the following regression, by simply rearranging the equation for output to labour ratio obtained above:

$$\ln(Y/L) = \ln(A) + (\alpha/1-\alpha)\ln(s) - (\alpha/1-\alpha)\ln(n+a+\delta)$$

This equation is regressed with tests on the coefficient. The results prove very encouraging in three respects: all the coefficients have the expected signs (qualitatively good results), test for the two gradient coefficients being equal were not rejected, and high R^2 was obtained (0.69 for 98 non-oil producing countries), meaning that the model explained a lot of actual behaviour. However there were two predictions that were very inconsistent with the data, which was that it predicted a value for α of 2/3, which means that capital accounts for 2/3 of total output, which is at odds with the general finding in data that this ratio is in fact 1/3. The models prediction about the rate of convergence is also inconsistent with data, predicting a rate which is more than double that one would expect (half life of 17.5 years)

Apart from this empirical problem with the model, this model has been criticised by many others for being too simplistic for not accounting for many aspects of growth. This gave rise to many of the new growth models that tried to be “more realistic”. However before I move onto looking at some, it is important to point out

that despite its simplicity it is very impressive how this model can explain so much of actual long term growth. The even more interesting observation is that this model was able to do so without requiring government policy to have any effect on long term growth, which has obvious big implications for society.

The next extension of this model was to introduce to human capital. This was based on obvious apparent differences in labour quality between countries and thus it was necessary to account for this. According to Kendrick, around half of total US capital stock in 1969 was in the form of human capital and is likely to have increases since then. Many different ways of modelling it are possible, but here I will adopt that used by Mankiw, Romer and Weil, who use the following equation:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta} (AL^{1-\alpha})$$

Since H is human capital, one can now obtain two capital accumulation equations:

$$\Delta k = s_k y - (n+a+\delta)k$$

$$\Delta h = s_h y - (n+a+\delta)h$$

Where $h = H/AL$, s_k (s_h) is the share of income spent on physical (human) capital goods. Assume that $\alpha + \beta < 1$, i.e. decreasing returns to all capital, rather than constant returns as implied by the endogenous growth models examined later. Again by the process before, the steady state can be obtained for both types of capital can be obtained and thus one can also be obtained for income in the following form, analogous to the Solow model before:

$$\ln(Y/L) = \ln(A) + (\alpha/1-\alpha-\beta)\ln(s) - (\alpha/1-\alpha-\beta)\ln(n+a+\delta) + \beta/(1-\alpha) \ln(h^*)$$

The only major difference with this model is the inclusion of the human capital term and thus this will indicate that the model before would have suffered from omitted variable bias and therefore its results were wrong, if this human capital model is correct. This equation allows one to see what the inclusion of human capital implies for the predictions. Firstly it is obvious to see that the effects of savings will be more important because the coefficient will now be bigger and therefore this will mean that the effects of physical capital is more important on the level of income. The model also shows that the more a country invests in its human capital, the higher the income it can expect, and thus emphasises the importance of education for attainment of higher levels of income. However the model still emphasises that in terms of long-term growth the most important factors are still population growth and technological progress.