

1. Consumer Behaviour and Duality:

- i. Explain what is meant by 'duality' in the context of the standard consumer utility maximisation problem. What assumptions on preferences are required in order for duality theory to hold?

A) The duality problem refers to the mathematical principle that any constrained maximisation also has associated with it a dual problem in constrained minimisation this focuses on the constraints in the original primal problem. The primal consumer problem is maximising utility subject to a budget constraint. The dual problem is to minimise the expenditure needed to achieve a given level of utility.

The Marshallian demand function expressed as a function of prices and income takes the demand bundle as a solution to utility maximisation (it presents the primal problem). However the Hicksian demand function expressed as a function of price and utility illustrates the utility maximisation in terms of expenditure minimisation (duality). This latter approach leads to "expenditure functions", in which the spending required to reach a given utility target depends on goods market prices.

Important to note that they both give equivalent when the desired level of utility is attained by a certain level of income Marshall expresses the problem in terms of maximising utility whereas Hicks looks at it from minimising expenditure. Hicksian is the same as Marshallian as long as consumer income is compensated so to keep utility constant (compensated) so it only incorporates the substitution effect whereas Marshallian demand incorporates both income and substitution effects (uncompensated).

B) The assumptions on preferences that are required for the duality to hold is local non-satiation, which means that one can always do better even if one is restricted to only small changes in the consumer bundle. So that it gives the possibility of doing better by maximising utility by minimising expenditure so being more selective of what you purchase to increase utility. If this did not hold then X_i (the consumer bundle) would not solve both utility maximisation and expenditure minimisation problems. The assumption of transitivity is also required this is just the rational ordering of bundles so that one bundle is chosen out of all the rest.

- ii. Suppose the indirect utility function is given by: $v(p, m) = \left(\frac{\alpha}{p_1}\right)^{\alpha} \left(\frac{\beta}{p_2}\right)^{\beta} m$. Show how one can derive the Marshallian and Hicksian demand curves.

Given the following 4 identities that link the Expenditure function, the indirect utility function, the Marshallian demand function and the Hicksian demand function.

1. Duality problem expressed formally:
The utility maximisation problem: $V(p, m^*) = \max p x$
Such that $p x \leq m^*$

Let x^* be the solution to the this problem and let $u^* = u(x^*)$.

2. Consider the expenditure function minimisation problem:
 $E(p, u^*) = \min p x$
Such that $u(x) \geq u^*$