The solution to both problems is x*; this simple observation leads to 4 important identities.

1. e(p,v(p,m)) = m. the minimum expenditure necessary to reach utility v(p,m) is m.
2. v(p,e(p,u)) = u. the maximum utility from income e(p,u) is u.
3. xi(p,m) = hi(p,u(p,m)) the Marshallian demand function at income m is the same as the Hicksian demand function at utility v(p,m)
4. hi(p,u) = xi(p,e(p,u)) the Hicksian demand at utility u is the same as the Marshallian demand at income e(p,m)

expenditure Function : $v(p, e(p,u)) = \left(\frac{p1}{\alpha}\right)^{\alpha} \left(\frac{p2}{\beta}\right)^{\beta} M$

Hicksian Demand · via the shepherd's lemma

$x_1 h(p,u) = u\left(\frac{\alpha p2}{\beta p1}\right)^{\beta}$ $\quad x_2 hi(p,u) = u\left(\frac{\beta p1}{\alpha p2}\right)^{\alpha}$

Marshallian demand function: via the slutsky equation

$x_1(p,m) = \frac{\alpha m}{p1}$ $\quad x_2(p,m) = \frac{\beta m}{p2}$

iii. Explain the adding-up and symmetry properties of the rational agent that result from utility maximisation.

The mathematical requirements for a maximum imply something about the rational preferences; they are symmetric in the following sense:

$$\frac{\partial^2 U}{\partial x_1 \partial x_2} = \frac{\partial^2 U}{\partial x_2 \partial x_1}$$

This follows from Young's theorem, which refers formally to the cross partial derivatives are equal to each other as long as they are continuous. In utility maximisation this refers to the marginal utility of one good x1 being the same as x2. This implies no intuition why would x1 (apples) have the same utility as to x2 (pears) there is no reason but how people behave is an empirical question. Deaton and Meullbauer (1980) test whether behaviour reflects symmetric tendencies and they conclude that symmetry is only just rejected given homogeneity, and if some allowance were made for the asymptotic nature of the test the final results would suggest that the introduction of (arbitrary) time trends removes much of the conflict between the data and the