Period of a Loaded Cantilever

Design

Aim: To investigate the relationship between the mass loaded on a cantilever and the period of oscillation of a loaded cantilever

Variables:

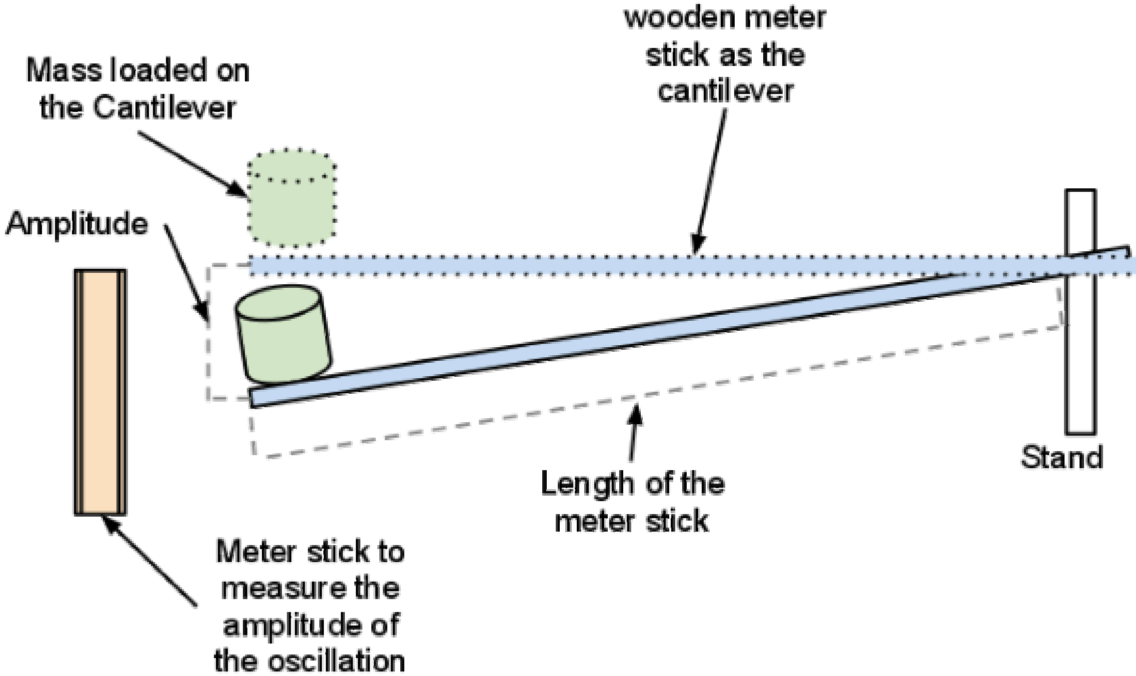
Independent variable: Mass loaded on a cantilever
Dependent variable: Period of oscillation of the cantilever
Control variables:

Amplitude	Maximum displacement of the end of the cantilever Can be kept constant by measuring the amplitude of each trial
Length of the cantilever	Can be kept constant by measuring the length of the cantilever or using the same model through the experiment
Air resistance	Different width of cantilever may have different air resistance. Kept constant by using a cantilever with the same cross-sectional area or simply use the same cantilever throughout the experiment. Carry out the experiment at room air density.
Stiffness of the cantilever	Kept constant by using the same cantilever of the same material throughout the experiment

Apparatus:

1	2 Meter sticks	One Meter stick as a cantilever and the other to measuring the amplitude of oscillations
2	Masses	Masses to be loaded on the cantilever as the independent variable in this experiment
3	Stopwatch	To measure the time needed for 10 number of oscillation with different mass loaded on the cantilever
4	Tapes	To stable the masses on the meter stick so that the masses will not fall during the oscillation
5	Scissor	To cut the tapes

Method



1. Set up apparatus as shown in the graph. Make sure that the meter stick is attached stably to the stand and the meter stick measuring the amplitude is attached vertically on the ground for higher accuracy of measurement.

2. Lay a mass of 50g at the end of the meter stick and stable it with plastic tapes.
3. Pull the end of the meter stick downwards with one finger with amplitude of 8 cm, measured by a meter stick. The oscillation of the cantilever should start with the same maximum displacement for all trials throughout the experiment.

4. Release the meter stick to allow the meter stick to oscillate. Start the stopwatch simultaneously when release.

5. Stop the stopwatch after 10 complete oscillations and record the time taken to complete 10 oscillations on a data sheet.

6. Repeat step 3 to 5 with different mass loaded on the end of the meter stick, each with at least 5 trials. Other variables should be kept constant, eg. amplitude of the oscillation, number of oscillations measured, length of meter stick, material of the meter stick etc. The same model set up in step 1 should be used throughout the experiment.

7. Calculate the average time taken for 1 oscillation/period at each

different mass loaded on the meter stick.

Data Collection

Length of meter stick: 60 cm ± 0.05 cm

Amplitude of the oscillation: 8 cm ± 0.05 cm

Raw Data

Mass loaded on the cantilever g ± 0.05g	Time taken for 10 oscillations ± 0.005s					Average time taken 10 period of oscillation ± 0.005s	Random uncertain ties ± s
	1st trial	2nd	3rd	4th	5th		
50.0	2.63	1.94	2.31	2.34	2.16	2.28	0.3
100.0	2.72	2.56	2.54	2.81	2.72	2.67	0.1
150.0	3.01	3.13	3.06	3.12	3.16	3.10	0.08
200.0	3.41	3.57	3.55	3.83	3.66	3.60	0.2
250.0	4.25	4.22	4.12	4.18	4.25	4.20	0.07
						Mean:	0.2

The uncertainty of the measurement is taken to half of the smallest division of the measuring instrument. For example, the uncertainty is 0.05g for an electronic balance, 0.005 s for the stopwatch and 0.05cm for the meter stick.

The random uncertainty of the average time taken for 10 oscillations was found by half of the range of the repeats. Eg. Random uncertainty at of time taken at mass of 50g ± 0.05g is (2.63-1.94)/2 = 0.3 (taken to the nearest 1 sig. fig.)

Data Process

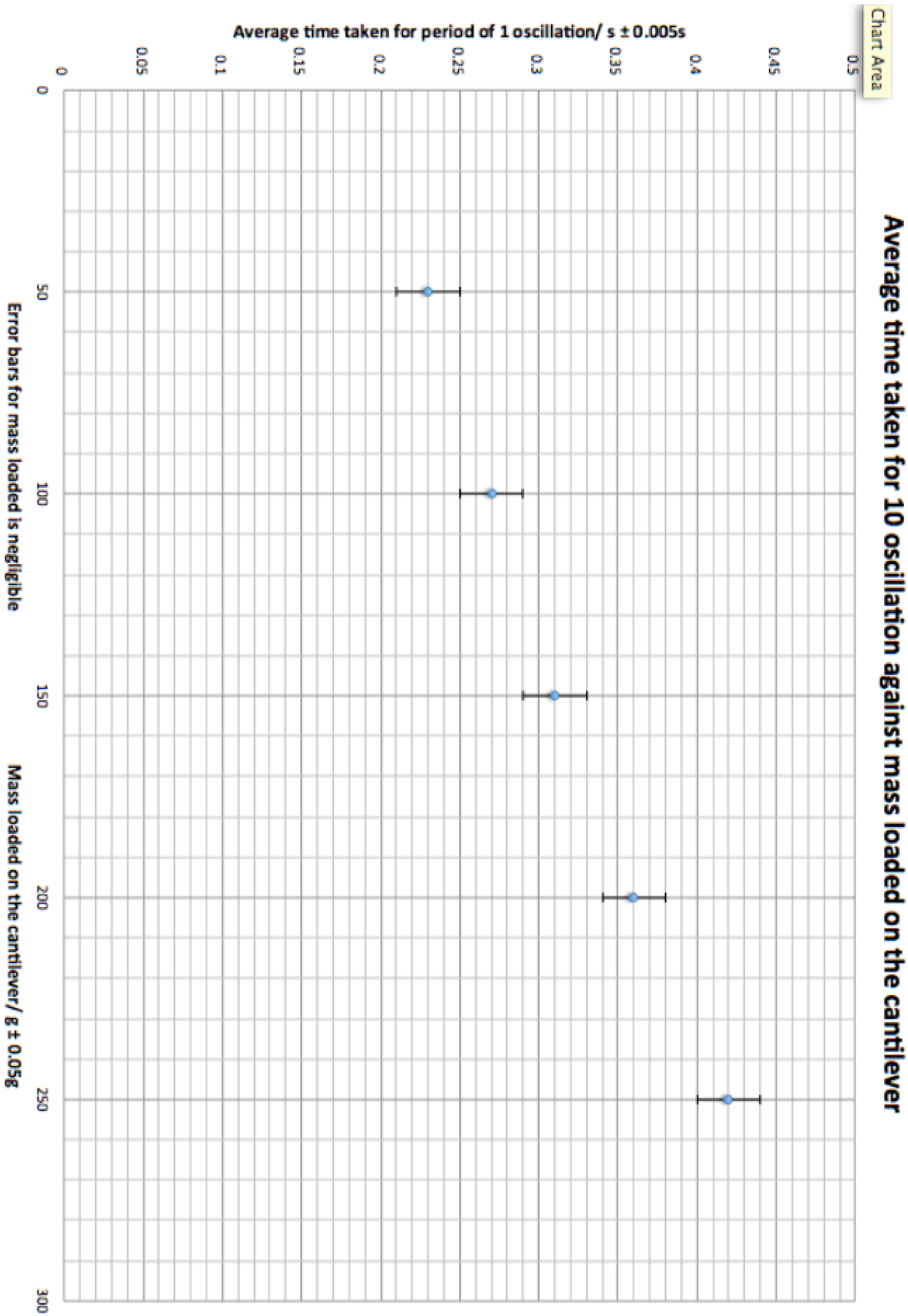
In this experiment, it is aimed to investigate the relationship between period and the mass loaded on a cantilever. Period is time taken for one oscillation. However, in this experiment, the time is taken to 10 oscillations, therefore, I have to divided the time taken by 10. The absolute uncertainty of the time taken by the stopwatch is still ± 0.005s. Data rounded to 2 decimal places.

Processed Data

Mass loaded on the cantilever g ± 0.05g	Period, Time taken for 1 oscillation ± 0.005s					Average time taken 10 period of oscillation ± 0.005s	Random uncertain ties ± s
	1st trial	2nd	3rd	4th	5th		
50.0	0.26	0.19	0.23	0.23	0.22	0.23	0.04
100.0	0.27	0.26	0.25	0.28	0.27	0.27	0.02
150.0	0.30	0.31	0.31	0.31	0.32	0.31	0.01
200.0	0.34	0.36	0.36	0.38	0.37	0.36	0.02
250.0	0.43	0.42	0.41	0.42	0.43	0.42	0.01
						Mean:	0.02

The random uncertainty in period was found by half of the range of the repeats. Eg. Random uncertainty at of time taken at mass of 50g ± 0.05g is (0.26-0.19)/2 = 0.04 (taken to the nearest 1 sig. fig.)

Processed Data graph



Maximum gradient: $\frac{\Delta y}{\Delta x} = \frac{(0.42-0.02)-(0.23-0.02)}{250-50} = 0.00115 = 1.15\times10^{-3}$ s g^{-1}

Minimum gradient: $\frac{\Delta y}{\Delta x} = \frac{(0.42-0.02)-(0.23-0.02)}{250-50} = 0.00075 = 7.5\times10^{-4}$ s g^{-1}

Best-fit gradient: (0.00115 + 0.00075) / 2 = 0.00095 s g^{-1} ± 0.0002 s g^{-1}
= 9.5×10^{-4} s g^{-1} ± 2×10^{-4} s g^{-1}

Conclusion

From the graph, the line of best fit is a positive straight line, which suggests a positive relationship between the period of the oscillation and the mass loaded on a cantilever. As the mass loaded on the cantilever increases, the period increases as well. It also means that as mass loaded on the cantilever increases, it oscillates slower and the frequency decreases. However, in the graph, the best-fit line does not pass through the origin, which may suggest that there are systematic uncertainties involved in this experiment.

The theoretical equation of natural frequency is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where f is the frequency, m is the mass loaded on the end of a beam and k is the stiffness of the beam. When the equation is rearranged, it suggests a positive linear relationship between T^2 and m with a gradient of $\frac{4\pi^2}{k^2}$, which does not support the results of this experiment. The main uncertainty in this experiment is damping.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\frac{1}{T^2} = \frac{k^2}{4\pi^2 m}$$

$$T^2 = \frac{4\pi^2}{k^2} \cdot m$$

Evaluation

Systematic uncertainties

Damping is one of the main uncertainties in this experiment. It may be cause by air friction or the stiffness of the meter stick. When a meter stick is deflected, it has the tendency to return to its original stage and the amplitude of the oscillation decreases. Period of the oscillation is increasing gradually and cause uncertainty to the data. There are also air resistance opposite the meter stick's movement and lead to the decrease in amplitude. An opposite force will act against the motion of the oscillation due to air resistance. This might cause uncertainty to the time measured and increase the period. To improve, a material with a larger damping ratio can be used to replace the meter stick so that the system return to equilibrium slower and damping in reduced.

The mass loaded on the cantilever measured does not include the mass of the tapes used for the attachment. This may lead to uncertainties to the mass and shift the best-fit curve slightly to the left. The mass of the tapes is not significant in comparison to the masses loaded and cannot cause a large uncertainty to the results. To improve, the mass of the tapes should be included into the mass loaded on the cantilever.

Random uncertainties
Stopwatch

Random uncertainty might be caused by the reaction time when using a stopwatch. There might be inconsistent delays due to eye-hand reaction time. This could cause random uncertainties to the time taken for 10 oscillations at different mass loaded. Improvements can be made by having the same person as the timer. Percentage uncertainty can also be reduced by timing for more oscillations (eg.20 oscillations) so that the random uncertainty caused by the stopwatch is less significant.

Oscillation

It was also difficult for our eyes to determine accurately the ending point of a period while the meter stick was oscillating at relatively small amplitude. To improve, the amplitude of the oscillation can be increased so that the amplitude can be measured more accurately.

Amplitude

There were random uncertainties involved in measuring the maximum amplitude of the oscillation with a meter stick. Before the oscillation, the end of the meter stick is pulled downwards with various displacements by the different weights/masses due to gravity. When oscillating, the upward and downward displacements may not be proportional, which means that meter stick is not at simple harmonic motion. The amplitude (maximum displacement) is difficult to be kept constant. For improvement, we can calculate the average amplitude by measuring the starting and ending amplitude. The amplitude should be kept constant throughout the experiment.

Bibliography

Natural Frequency and Resonance. (n.d.). *Home | College of Engineering and Computer Science*. Retrieved January 17, 2012, from http://www.cs.wright.edu/~jslater/SDTCOutreachWebsite/nat_frequency.htm