How does the mass on the spring affect its time period of oscillation?

Plan:

This investigation will be on how changing the mass attached to a spring affects the time it takes for it to oscillate twenty times. Since there is simple harmonic motion being executed by the mass, the time it takes to complete an oscillation can be described as the time period, therefor we will try and answer the question of weather there is a correlation between the increase in the mass on the spring and a larger time period. In this investigation, the **independent variable** will be the mass which will be increased throughout the experiment; the **controlled variables** will be the distance the mass was pulled down, the spring used, the position of the ruler behind the oscillating spring, the surface area of the masses and keeping the oscillations in a vertical line, while the **dependant variable** is the time period of oscillation.

Research Question:

How does the increase of mass attached to a spring affect its time period of oscillation?

Dependant variable:

- Time period of oscillation: masses on a spring produce simple harmonic motion, where time period is found by timing how long it takes for twenty complete oscillations to take place (since it is quite difficult to time a single oscillation with precision) and dividing this time by twenty. The result is the time period.

Independent variable:

- The mass attached to the spring: the masses will be placed on a mass hanger attached to the spring. With each test, another fifty grams will be added to the mass hanger. This will cause the force on the spring to vary and consequently its time period. The number of oscillations to be counted in twenty every time since timing the movement of more than one oscillation makes the experiment more reliable by reducing the human error of reaction time in starting and stopping the stop watch. Furthermore, the masses to be measured will have to begin at 200 grams and go up at 50 grams intervals. This range was chosen because at testing with masses smaller that 200 grams, the oscillations produced were not in a vertical straight line but rather at angles sideways. This interval was preferred because it enables the taking of seven readings without exceeding the spring's elastic limit.

Controlled variables:

- Vertical distance that the mass is pulled down: this will be measured from the bottom of the mass hanger and will be used as another method of ensuring that a constant force is being applied in each repeat. The fact that the larger mass will cause the spring to be more stretched means that the vertical distance pulled down has to be a constant value rather than trying to pull down to a constant mark.

- Spring constant: this constant will be kept by using the same spring throughout the experiment. Here we have to be careful in not exceeding the elastic limit of the spring otherwise the spring constant is no longer valid.

- The position of the ruler: as will be explained in more detail further on in the method, in order to create a point of reference to count the complete oscillations as well as to measure the extension of the sting, a ruler will be placed behind the spring and masses. Ideally, this ruler should be completely vertical therefore a wall will be used as reference to it. Furthermore, in order to take accurate measurements, the reading off of the ruler should always be done with the eyes in the same level of the readings.

- Surface area of masses: There were no masses of 50grams, only of 10 and 100, therefore to keep a constant surface area the masses being added should be of a regular shape. All of the ones to be used in this experiment are circles of regular surface area.

- Keeping the oscillations is a vertical line: if this constant was not kept, the distance which the mass would move would be different but not, the horizontal oscillations become a variable too. By testing with the masses, I found that with masses of less than 200 grams, the oscillations would not be vertical. To keep this variable constant, the first reading will be taken with a 200 grams mass.

Amount of Data:

Three repeats will be recorded. The measurements will be taken at 50 grams difference starting at 200 grams mass (plus the mass on the mass hanger) until 500 grams. This was the point chosen to record the first reading because with smaller masses, the oscillations would be irregular and sideways rather than a vertical simple harmonic motion.

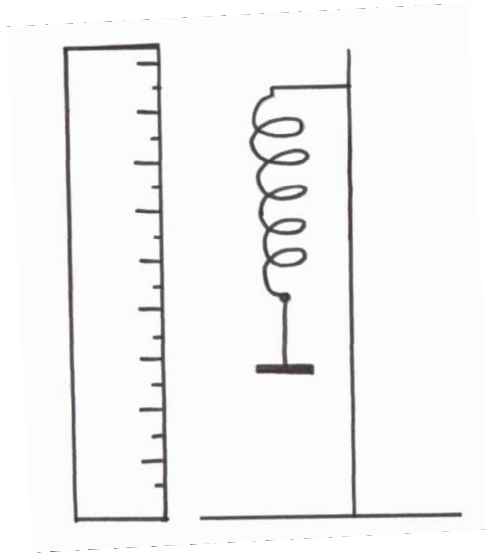
Qualitative Observations:

Despite the efforts of measuring accurately in order to produce reliable data, these were some unexpected factors that have influenced on the modifications of the original plan or adding extra uncertainties to the results.

The first one was that mentioned in the independent variables section of having to alter the range of masses to be investigated. Instead of starting from 0 grams as would be expected, our first measurement was taken at 200 grams as to ensure that the oscillations were kept in a vertical line rather than the sideways oscillations that took place when the masses tested were inferior to this value.

Secondly, we have the fact that the experiment was greatly based on the human reflex of starting and stopping the stopwatch which adds to the uncertainties on the experiment but due to equipment restrictions, not much could have been done.

Method:



Apparatus:

- A spring (careful when choosing the spring and consider whether it will be strong enough to hold up to 500 grams without exceeding its elastic limit)
- A ruler (to me used as a reference when pulling down the mass to begin each experiment)
- A clamp stand and a clamp
- A mass hanger
- A stop watch

1. Set up the apparatus as shown in the diagram above
2. Starting with a mass of 200 grams, pull the mass hanger down two centimetres and let go at the same time that the stop watch begins timing.
3. Choose a point of reference as to where a full oscillation will be. As soon as 20 complete oscillations have taken place, stop the stop watch.
4. Repeat steps 2 and 3 with different masses up to the seventh reading with a 500 grams mass. Repeat each reading thrice in order to improve reliability of results.

Mass on spring (g) ±1%	Time taken for 20 complete oscillations (seconds) ±0.01			Number of oscillations	Average Time taken (seconds)	Uncertainty of Average Time taken ±
	1st repeat	2nd repeat	3rd repeat			
200	11.34	11.37	11.22	20	11.31	0.08
250	12.75	12.69	12.75	20	12.73	0.03
300	13.91	13.69	13.87	20	13.82	0.11
350	14.85	14.94	14.79	20	14.86	0.08
400	15.94	15.96	15.87	20	15.92	0.05
450	16.85	16.91	16.81	20	16.86	0.05
500	17.66	17.69	17.75	20	17.70	0.05

Uncertainties and other calculations:

Most of the calculations here were done through the use of software, an example of each type is shown below.

For the uncertainties of the time taken as well as that of mass on spring, I have used the smallest scale of the reading (in the stop watch and in the mass). In order to measure the mass, the unit used in calculations and in the recording of results was grams, but in the graph the standard units will be used.

For the % uncertainty of the mass, even though they were labelled with their masses, at measuring some there was some error of about 1g each time therefore we could open space for an error of about ±1%. This error was taken as a percentage because through adding extra masses with the same error, we would have a growing absolute error. In this case, and error of ±1 would mean a constant value throughout the experiment, which is not the case.

The uncertainty for mass will be calculated through the percentage value of ±1% because if we had an absolute value, the error would be constant rather than increasing. Therefore, the absolute uncertainty can be calculated through taking the 1% ((1/100)* the value of mass) of each value of mass, for example, as done below when mass = 200 grams.

(1/100)*200= ±2 grams

For the average time taken, the results of the three repeats were added and then divided by three. An example of this calculation was done using the reading with a 200 grams mass:

(11.34 + 11.37 + 11.22)/3 = 11.31 seconds

The absolute uncertainty of the average time taken was calculated by taking half the value of the range of the time taken readings. For example, using the reading taken with a 450 grams mass, the smallest reading was of 15.87 and the largest reading of 15.96, therefore:

(15.97- 15.87)/2 = ±0.05 seconds

Using the reading taken at m= 300 grams, on the other hand do not have the same value since the range is different:

(13.91-13.69)/2 = ± 0.11 seconds

The % uncertainty for average time taken was calculated by using the absolute value found, dividing it by the value of average time taken and multiply the result by 100, therefore:

(0.05/16.86) * 100 = ±0.30%

Again, this will not be a constant value for all masses:

(0.11/13.82)* 100= ±0.80%

The time period was calculated by dividing the results of average time taken by twenty. Since the time recorded as that of twenty oscillations, by doing this calculation we should find a more reliable value or the time taken for one complete oscillation to take place:

16.86/ 20 = 0.84 seconds

The absolute uncertainty of time period, will be the same of that of average time taken since no calculations involving uncertainties were made

The % uncertainty for time period will be found by using the absolute error found, dividing it by the value of time period and multiplying that by 100% as done below with the data collected with the 300 grams mass:

(±0.11/0.69)* 100 = ±15.92%

Mass on spring (g)		Time taken for 20 complete oscillations (seconds) ±0,01			Average Time taken (seconds)			Time period (seconds)		
±1%	abs. uncertainty ±	1st repeat	2nd repeat	3rd repeat		abs. uncertainty ±	%uncertainty ±		abs. uncertainty ±	%uncertainty ±
200	2.00	11.34	11.37	11.22	11.31	0.07	0.66	0.57	0.07	13.26
250	2.50	12.75	12.69	12.75	12.73	0.03	0.24	0.64	0.03	4.71
300	3.00	13.91	13.69	13.87	13.82	0.11	0.80	0.69	0.11	15.92
350	3.50	14.85	14.94	14.79	14.86	0.08	0.50	0.74	0.08	10.09
400	4.00	15.94	15.96	15.87	15.92	0.05	0.28	0.80	0.05	5.65
450	4.50	16.85	16.91	16.81	16.86	0.05	0.30	0.84	0.05	5.93
500	5.00	17.66	17.69	17.75	17.70	0.04	0.25	0.89	0.04	5.08

This table shows the results with the processed data.

Having researched that the time period of a spring is proportional to the squared root of the mass, I decided to investigate the spring constant (k) using the Hooke's Law experiment.

This law states that "for relatively small deformations of an object, the displacement or size of the deformation is directly proportional to the deforming force or load[1]." Below is the data collected.

Mass on spring (g)		Force (N)		Extension (m)	
±1%	abs. uncertainty ±	±1%	abs. uncertainty ±	± 0,01	%uncertainty ±
200	2,00	2,0	0,02	0,08	13,75
250	2,50	2,5	0,03	0,10	11,00
300	3,00	3,0	0,03	0,12	9,17
350	3,50	3,5	0,04	0,15	7,33
400	4,00	4,0	0,04	0,16	6,88
450	4,50	4,5	0,05	0,18	6,11
500	5,00	5,0	0,05	0,20	5,50

The uncertainty of mass on spring has already been calculated for previous results by multiplying the value by (1/100) since the percentage uncertainty is the absolute constant value of 1%.

The force of the spring has been calculated by dividing the value of mass by 100. Aware that the conversion rate is of 1kg to 10N, in order to turn the mass collected into kilograms they would have to be divided by 1000 and then multiplied by 10 to find the force. This cal be simplified by dividing the mass by 100 like done in the example below where mass = 300 grams:

(300/100)= 3N

The % error of force will be the same value as that for mass (±1%) as to keep the growing error quality.

The absolute error of force will be calculated through the same method used for the mass of spring, by multiplying the value by (1/100) as shown below:

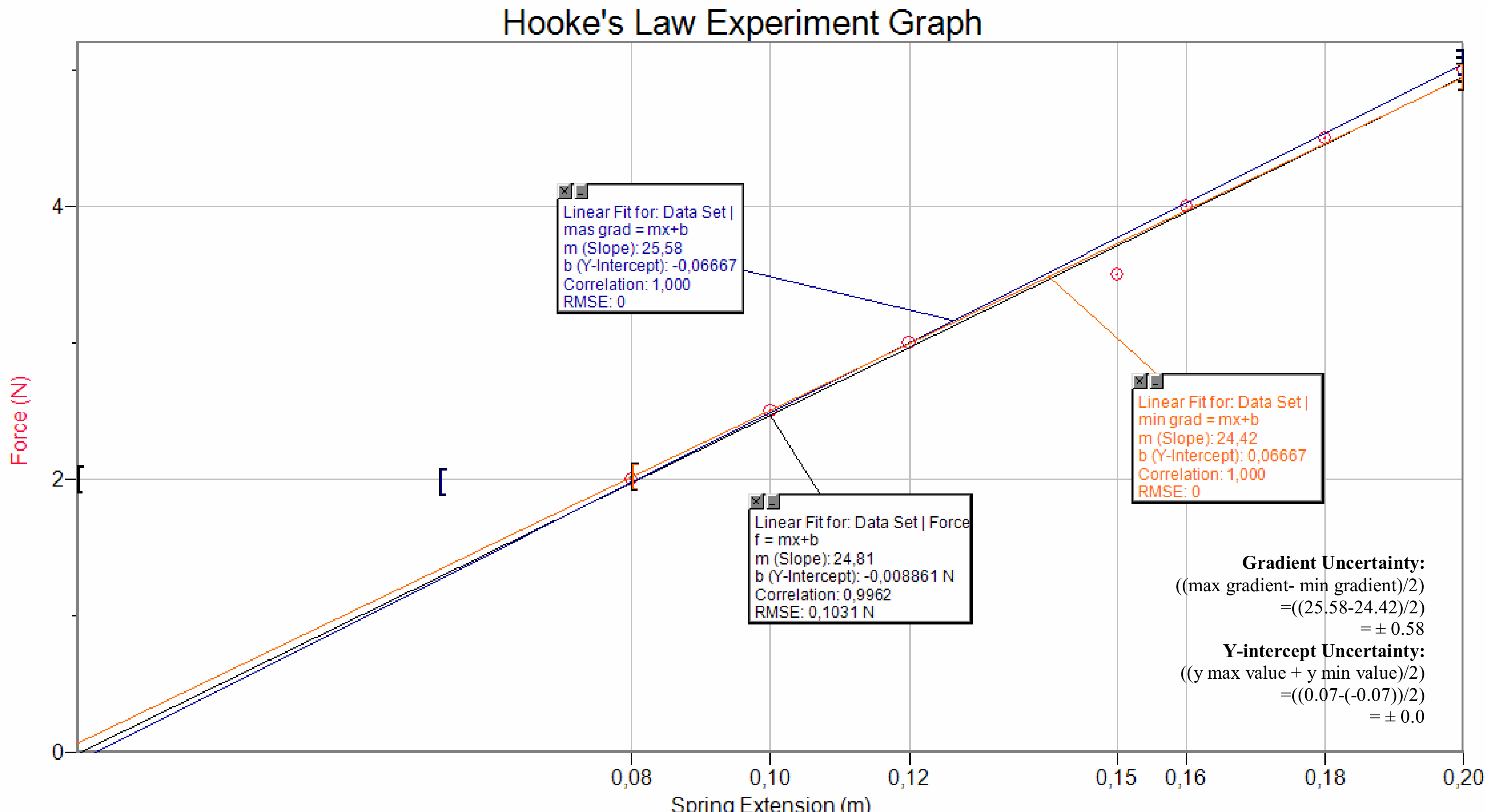
(1/100)*4= ±0.04N

The absolute error of extension has been decided by using the smallest reading on the ruler which was 0,01m.

The % uncertainty of extension has been found by dividing the absolute error by the value of extension and multiplying the result by 100, for example, in the first reading (extension of 0,08m):

(0,01/0,08)*100= ±12.5%

[1] http://www.britannica.com/EBchecked/topic/271336/Hookes-law



This is the graph produced from the results from the Hooke's Law experiment

This graph and experiment are relevant because the gradient here is also the spring constant (k) which can be used when calculating the time period of a spring. As well as using the method previously shown, to find how long it takes for a full oscillation to be completed and its relation to the mass attached we can derive the equation for Hooke's Law.

This reads:

$F = kx$ Where x = extension

$a = \frac{f}{m} = \frac{kx}{m}, \therefore \frac{a}{x} = \frac{k}{m}$ Where a = acceleration

From Simple Harmonic Motion, we know that:

$a = -w^2x$

$-w^2 = \frac{a}{x} \quad \therefore \quad -w^2 = \frac{k}{m}$

$= \frac{2\pi}{T}$

$$T = \frac{2\pi}{w} = 2\pi\sqrt{\frac{m}{k}}$$

Therefore, from deriving this equation, we find a way of testing the values of time period previously calculated but also, from this new equation, drawing a graph becomes easier. It is clear why the graph displayed below was drawn in the shape of a Time Period^2 (squared) – mass graph.

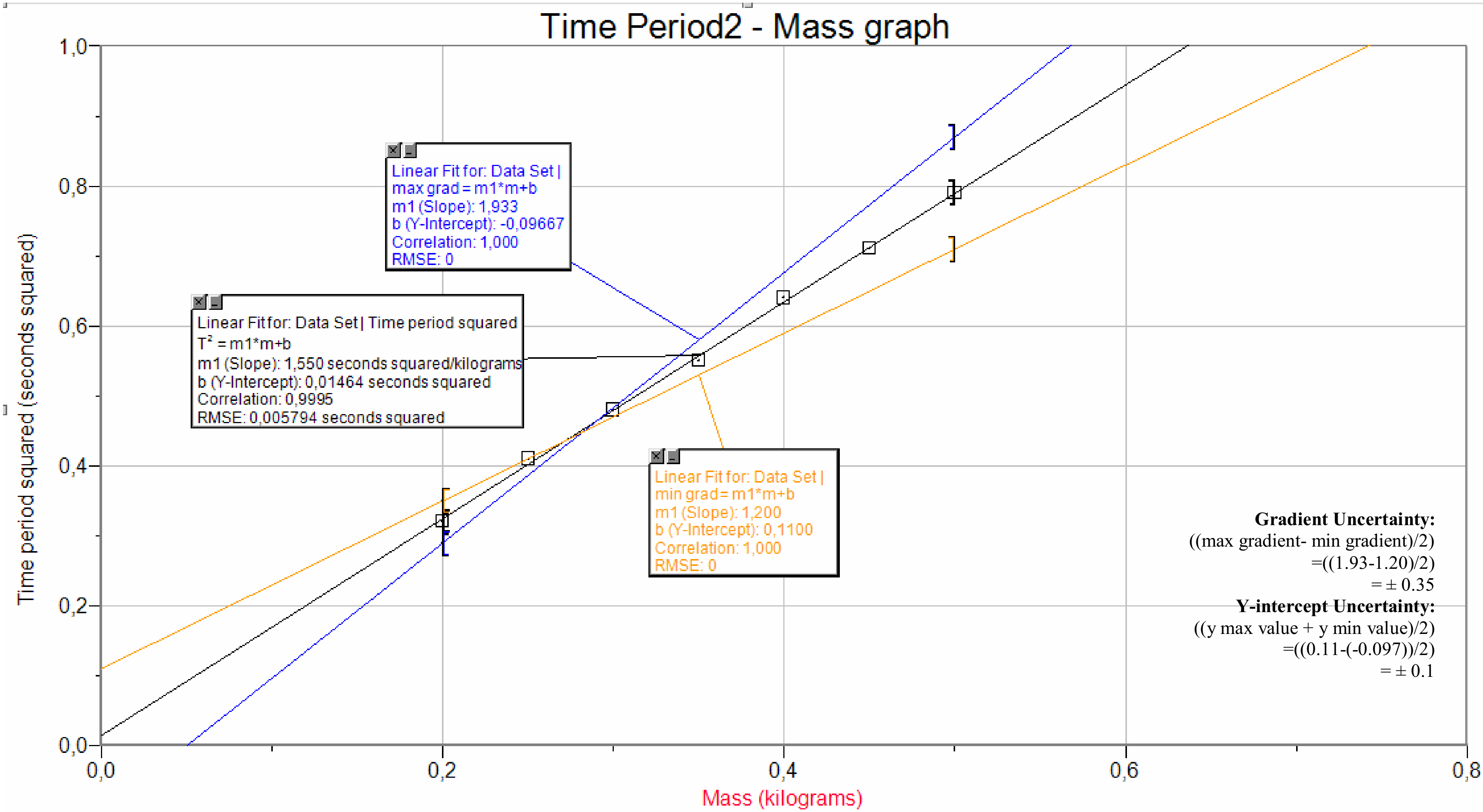
<u>In order to calculate the % uncertainty of Time Period squared</u>, I have to multiply the percentage uncertainty of time period by two, for example, using the time period squared of 0.32:

±13.26*2 = ±26.53%

<u>From these results we can calculate the absolute error</u> by dividing the percentage by 100 and multiplying the result by the value for Time period squared

(±26.53/100)*0.32= ±0.08

Time period (seconds)			Time period squared (seconds^2)		
	abs. uncertainty ±	%uncertainty ±		abs. uncertainty ±	%uncertainty ±
0,57	0,07	13,26	0,32	0,08	26,53
0,64	0,03	4,71	0,41	0,04	9,43
0,69	0,11	15,92	0,48	0,15	31,83
0,74	0,08	10,09	0,55	0,11	20,19
0,80	0,05	5,65	0,63	0,07	11,30
0,84	0,05	5,93	0,71	0,08	11,86
0,89	0,04	5,08	0,78	0,08	10,17


Time Period2 - Mass graph
Time period squared (seconds squared)
Mass (kilograms)
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
Linear Fit for: Data Set | max grad = m1*m+b
m1 (Slope): 1,933
b (Y-Intercept): -0,09667
Correlation: 1,000
RMSE: 0
Linear Fit for: Data Set | Time period squared
T^2 = m1*m+b
m1 (Slope): 1,550 seconds squared/kilograms
b (Y-Intercept): 0,01464 seconds squared
Correlation: 0,9995
RMSE: 0,005794 seconds squared
Linear Fit for: Data Set | min grad = m1*m+b
m1 (Slope): 1,200
b (Y-Intercept): 0,1100
Correlation: 1,000
RMSE: 0
Gradient Uncertainty:
((max gradient- min gradient)/2)
=((1.93-1.20)/2)
= ± 0.35
Y-intercept Uncertainty:
((y max value + y min value)/2)
=((0.11-(-0.097))/2)
= ± 0.1

Conclusion:

From the results collected and displayed above, it could be stated that within experimental error, the time period squared is proportional to the period of oscillation of the spring, taken that the controlled variables remain constant. In other words, as the mass attached increases, the oscillations take a longer time to be completed, which answers the question being investigated. This is obvious when observed in the graph above. The data points are all within the allowed error from the best fit line and the origin (0,0) lies between the y-intercept of maximum gradient and that of minimum gradient when taken the uncertainty for gradient into consideration, as shown in the calculations below.

Y intercept calculations:

Minimum Value: -0.01 + 0.1 = - 0.09 (below the origin)

Maximum value: 0.01 -0.1 = 0.10 (above the origin)

Furthermore, according the Hooke's Law theory investigated, $T^2 \propto m$. This establishes the relationship between the two graphs chosen to be drawn, since for the graph of time period^2 against mass, the gradient would be given by $\frac{4\pi^2}{k}$ where k is the gradient of the graph drawn for Hooke's Law (24.81 ± 0.58). To test this theory, $4\pi^2$ will be divided by the value found for k from the first graph and compared to the gradient in the second graph:

$\frac{4\pi^2}{24.81} = 1.59$ (using data from Hooke's Law experiment)

The uncertainty here will be the same as that for the gradient in the Hooke's Law experiment. The theoretical gradient for the graph T^2 - m is 1.59 ± 0.58 while the gradient found through the plotting of the data is 1.55, value which lies within experimental error of the theoretical value. This is even more evidence to the strong positive correlation between the time period of a spring and the mass attached to it and a positive answer to the research question.

Limitations and Improvements

Many of the limitations to this experiment are related to what the lab could offer since we did not dispose of unlimited equipment. These include: the fact that a large portion of the experiment depended on human sensitivity, for example, when it came to the measuring of time or counting the number of full oscillations. Other limitations might include the surface area of masses, and the movement of other parts of the set up (such as the clamp stand) while, in theory, when testing simple harmonic motion, only the mass should move.

Concerning these related to human error, we have the number of oscillations counted as well as the time these took to take place. Even though it would have been a significantly more complex set up, these errors could have been annulled or smoothened by using data loggers and light gates. The latter would be responsible for counting the number of oscillations by recording the number of times the mass went through the gate. When doing this manually, there was not only concern for confusing the number of oscillations taken place but also counting them incorrectly based on the reference point. The light gate would be set in a fixed position so that this reference point was constant. It would also be set connected to the stopwatch so that when twenty oscillations were counted, the time would be stopped. This connection would eliminate the reaction time which would have to be taken into consideration in this experiment. Even if the counting was absolutely accurate, there would have to be time for the reflex to take place but not if we would use a mechanical set up rather than a human one. Since this set up was not possible in the lab, the repeats were taken as to diminish the chance of error.

Another limitation that could have affected the results is the fact that not all masses had the same surface area. The ones measuring 100 grams were much bigger than those measuring 10 grams but both were used, though not at the same time. Even if small, this causes the air resistance, which is the opposing force to gravity in this case, to vary and could mean a change in the time taken for oscillations. If I were to avoid this factor, a mass of constant weight and surface area would have to used and could not be replaced. For example, there would only be 5 grams masses rather than 10 and 100 grams which would be placed and replaces as so to produce the desired mass.

As previously mentioned, when measuring the simple harmonic motion of a body, we consider that only the body is moving while the rest of the system is completely still. If this is not the case, the rest of the system would also be doing simple harmonic motion which could interfere with the oscillation of the body. Unfortunately, there was no way in which we could be assured that the clamp as well as the clamp stand would not move, even though some measures were taken so that this effect was reduced. For example, the clamp's screw was as tight as possible on the clamp stand and the base of the stand was turned inwards as to offer more stability. A larger base would have increased further this stability and using a clamp stand which did not have a removable clamp already or fixing the stand to the floor or table with screws are all measures which could have increased the immobility of the system around the oscillating masses.

Finally, there is the process of pulling the masses down to the same height before the oscillations which could have been improved. This is a very important part to the experiment since it affects the force applied to the masses and therefore the time period. In order to improve it, the readings could simply have been taken with the eyes at the same level as the ruler and using a constant point of reference all the way through, for example, the bottom of the mass hanger. Also, a straight piece of wood or a smaller ruler could have been used at the bottom or the mass hanger when measuring the distance to serve as a straight base for it and avoid mistakes due to the round base.