IB Physics Investigation – Mechanics

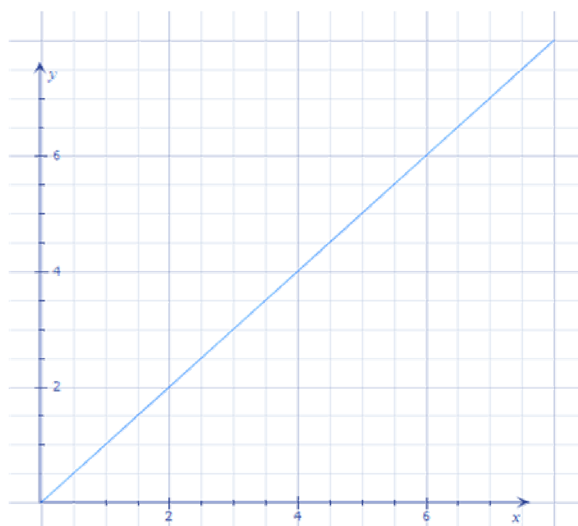
Title: Hooke's Law- to determine the spring constant of a metal spring

1. BACKGROUND/ INTRODUCTION

A branch of the mechanical energy, elastic potential energy is one of the fundamental base energy concepts that is very commonly used in the modern technology. Elastic potential energy deals with the elasticity of an object (e.g. to what extent a string can be stretched?), and it's used in a wide range of everyday life situations; such as in the process of decelerating an airplane landing on an aircraft ship. And the type of strings used there, is regarded to the spring constant of that string, in which it may or may not be able to stop an airplane within a certain distance.

Under investigation of Hook's Law, the aim of this experiment is to determine different spring constants of some springs, and that is done by measuring how far a spring stretches when a certain amount of mass added. In order to find the spring constant, we investigate it by rearranging energy formulas and basic knowledge:

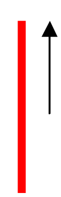
- The general formula for Energy is $E = Fs$, where F is force and s is displacement.



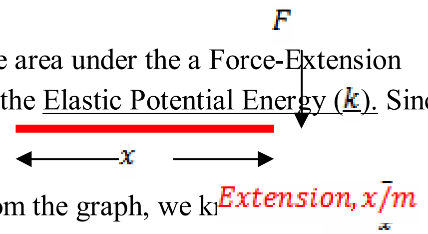
- The formula for Elastic Potential Energy is $E_k = {}^{1}/_{2}kx^2$, where k is

Force, F/N

- the **spring constant** and x is the extension of the spring.



- The relation between the two formulas is that s (displacement) is equal to x (extension) since both have the same units (measured in meters). Therefore, we can state that $E = Fs = Fx$
- The area under the a Force-Extension ($F - x$) graph gives the *work done* on the spring and therefore, the <u>Elastic Potential Energy (k).</u> Since it's the area of a triangle, then $E = {}^1/_2 Fx$



- From the graph, we k then $F = kx$. Therefore if we substitute this into the first formula ($E = {}^1/_2 Fx$) we get $E = {}^1/_2 (kx)x$ and that is equal to $E = {}^1/_2 kx^2$

From this information we can extract that k, the spring constant can be found as follows:

1. Measure the force and extension of the spring
2. Plot a $F - x$ graph
3. Find the slop of the graph, which is k, the spring constant

2. <u>QUESTION</u>

What is the role of the spring constant in the relationship of force and extension and how can it be determined?

3. <u>HYPOTHESIS</u>

The independent variable of this experiment is Mass (the number of masses added) and the dependent variable is Extension (the extension of the spring). The control variables are the same ruler used (the same scale) with the same initial positions (each spring was measured by the ruler sat on 4 cm), as well as the same masses are used for each spring.

The expected findings of this experiment is that the spring with the largest spring constant will extended least with the same amount of force applied. Therefore, the variation of the spring constant causes variation of extensions of different springs at the same amount of added-force.

4. <u>EQUIPMENT & MATERIALS</u>

The equipment used to conduct this experiment are:

- Rulers
- Masses
- Springs
- Retort Stand

5. PROCEDURES

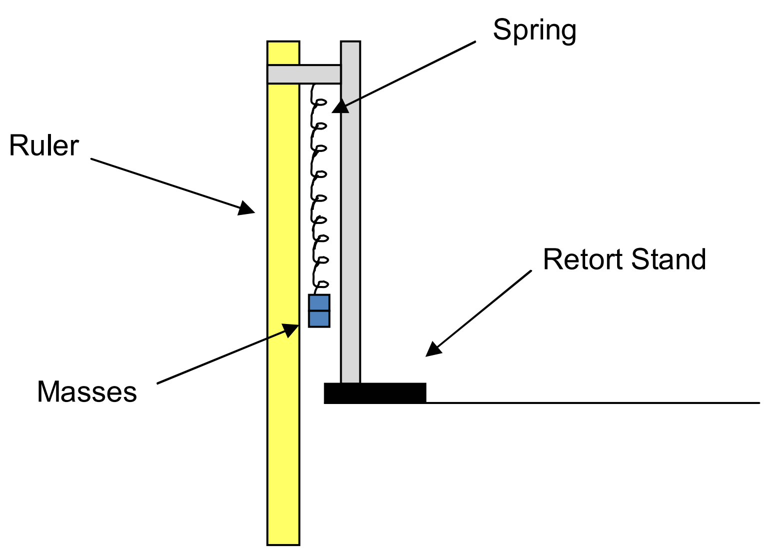
The procedure of the experiment consists of few steps that are listed below.

1. Label each spring by a letter (A, B, C, …) to make the process of determining them simpler.
2. Fix a 100 cm ruler on the Retort Stand and hang the first spring (spring A) parallel to the ruler, but not touching it to avoid any frictional forces that form a source of error.
3. Measure the length of the spring with no mass added.
4. Add a mass to the hanging spring, and record the new length (record your measurements by the tip of the spring's pointer) .

   *The difference between the lengths is the extension of the spring due to the weight force (mass acted by gravity) on the spring.

5. Keep adding masses until the spring reaches the end of the ruler. Make sure to add a constant mass each time, to make the spring constant calculations easier.
6. Repeat the experiment (steps) on the other springs. Make sure that you use the same masses used for spring A.

a) Experimental Diagram:



6. RESULTS:

a) Data Collection and Presentation:

Raw Data:

We started our graphs with 4 cm. (Subtract 4)

Spring A (Green)

Mass, m (g)	Length, L_1 (cm)	Length, L_2 (cm)	Length, L_3 (cm)
0	9.8	9.8	9.8
50	19.3	19.2	19.3
100	29.2	29.3	29.3
150	39.2	39.1	39.2
200	49.1	48.9	49.1
250	58.9	58.8	58.8
300	68.5	68.4	68.4
350	78.2	78.1	78.2
400	87.6	87.6	87.6
450	97.2	97.2	97.2

Spring B (Pink)

Mass, m (g)	Length, L_1 (cm)	Length, L_2 (cm)	Length, L_3 (cm)
0	12.3	12.3	12.2
50	17.3	17.3	17.3
100	22.3	22.2	22.3
150	27.2	27.1	27.2
200	32.1	32.1	32.1
250	37.0	37.1	37.1
300	42.0	42.0	42.1
350	47.0	46.9	47.0
400	51.9	51.8	51.9
450	56.7	56.6	56.7
500	61.6	61.6	61.6
550	66.5	66.4	66.5
600	71.4	71.4	71.3

Spring C (Silver)

Mass, m (g)	Length, L_1 (cm)	Length, L_2 (cm)	Length, L_3 (cm)
0	12.3	12.3	12.4
50	15.8	15.7	15.8
100	19.1	19.1	19.1
150	22.5	22.6	22.5
200	25.8	25.8	25.8
250	29.1	29.1	29.1
300	32.4	32.4	32.4
350	35.7	35.7	35.7
400	40.0	40.0	40.0
450	42.4	42.4	42.4
500	45.6	45.7	45.7
550	49.0	49.1	49.1
600	52.3	52.3	52.3

Spring D (Gold)

Mass, m (g)	**Length, L_1 (cm)**	**Length, L_2 (cm)**	**Length, L_3 (cm)**
0	11.8	11.8	11.8
50	13.6	13.6	13.6
100	15.4	15.4	15.4
150	17.3	17.3	17.3
200	19.2	19.2	19.2
250	21.0	21.0	21.0
300	22.9	22.9	22.9
350	24.8	24.8	24.8
400	26.7	26.7	26.7
450	28.5	28.5	28.5
500	30.4	30.4	30.4
550	32.3	32.3	32.3
600	34.1	34.1	34.1

The uncertainty in the Masses Measurement was $\pm$ 1 g
The uncertainty in the Ruler Measurement was $\pm$ 0.05 cm

b) Data Processing and Presentation:

DATA PROCESSING : use $g = 10\ ms^{-2}$

Spring A :

Mass m (g)	Force F (N)	Extension x_1 (cm)	Extension x_2 (cm)	Extension x_3 (cm)	Average Extension x (cm)
50	0.5	9.5	9.4	9.5	9.5
100	1.0	19.4	19.5	19.5	19.5
150	1.5	29.4	29.3	29.4	29.4
200	2.0	39.3	39.1	39.3	39.3
250	2.5	49.1	49.0	49.0	49.0
300	3.0	58.7	58.6	58.6	58.6
350	3.5	68.4	68.3	68.4	68.4
400	4.0	77.8	77.8	77.8	77.8
450	4.5	87.4	87.4	87.4	87.4

Spring B :

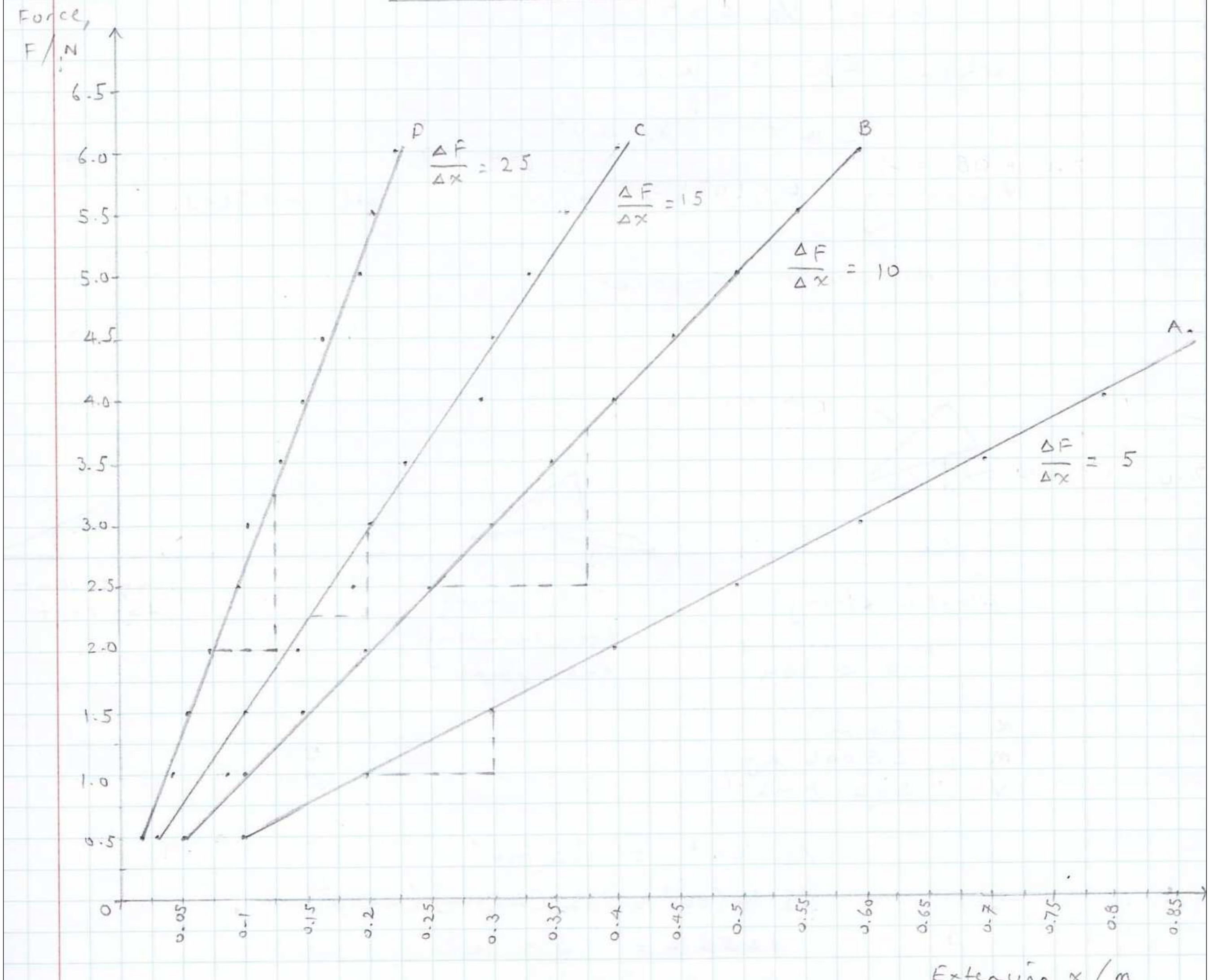
Mass m (g)	Force F (N)	Extension x_1 (cm)	Extension x_2 (cm)	Extension x_3 (cm)	Average Extension x (cm)
50	0.5	5.0	5.0	5.1	5.0
100	1.0	10.0	9.9	10.1	10.0
150	1.5	14.9	14.8	15.0	14.9
200	2.0	19.8	19.8	19.9	19.8
250	2.5	24.7	24.8	24.9	24.8
300	3.0	29.7	29.7	29.9	29.8
350	3.5	34.7	34.6	34.8	34.7
400	4.0	39.6	39.5	39.7	39.6
450	4.5	44.5	44.3	44.5	44.4
500	5.0	49.4	49.3	49.4	49.4
550	5.5	54.3	54.1	54.3	54.2
600	6.0	59.2	59.1	59.1	59.1

Spring C :

Mass m (g)	Force F (N)	Extension x_1 (cm)	Extension x_2 (cm)	Extension x_3 (cm)	Average Extension x (cm)
50	0.5	3.5	3.4	3.4	3.4
100	1.0	6.8	6.8	6.7	6.8
150	1.5	10.2	10.3	10.1	10.2
200	2.0	13.5	13.5	13.4	13.5
250	2.5	16.8	16.8	16.7	16.8
300	3.0	20.1	20.1	20.0	20.1
350	3.5	23.4	23.4	23.3	23.4
400	4.0	27.7	27.7	27.6	27.7
450	4.5	30.1	30.1	30.0	30.1
500	5.0	33.3	33.4	33.3	33.3
550	5.5	36.7	36.8	36.7	36.7
600	6.0	40.0	40.0	39.9	40.0

Mass m (g)	Force F (N)	Extension x_1 (cm)	Extension x_2 (cm)	Extension x_3 (cm)	Average Extension x (cm)
50	0.5	1.8	1.7	1.8	1.8
100	1.0	3.6	3.5	3.6	3.6
150	1.5	5.5	5.4	5.5	5.5
200	2.0	7.4	7.3	7.4	7.4
250	2.5	9.2	9.1	9.2	9.2
300	3.0	11.1	11.0	11.1	11.1
350	3.5	13.0	12.9	13.0	13.0
400	4.0	14.9	14.8	14.9	14.9
450	4.5	16.7	16.6	16.7	16.7
500	5.0	18.6	18.5	18.6	18.6
550	5.5	20.5	20.4	20.5	20.5
600	6.0	22.3	22.2	22.3	22.3

Force - Extension Graph



* In the graph, extension was converted to meters.

7. **CONCLUSIONS**

The results support my hypothesis that states that the spring with the least spring constant will extend most. And that's shown in the graphs, the most extended spring, which is spring A has the smallest slope (i.e. the spring constant). Whereas spring D, which is the steepest linear graph extended least among all 4 springs.

Possible sources of errors that may have limited the certainty of the results could be:

- Human errors (in recording measurements)
- Accuracy of the masses (do the masses exactly weigh as they're labeled?)
- Each spring sat on an initial position on the ruler (at 4 cm), which may has slightly moved by the time.
- The springs possibly touched the ruler very rarely, which may caused less extension due to friction.

Calculation percentage error (Spring Constants):

The spring constant of each spring is represented by the gradient of its graph.

The uncertainty of the force, F:

$$F = W = mg$$

The uncertainty of the mass is ± 1 g, then $F = \frac{(m \pm 1)}{1000}$ kg $\times 10\ \text{ms}^{-2}$

$$= \frac{m}{100} \pm \frac{1}{100}\ \text{N}$$

Therefore, the uncertainty of F is ± 0.01N

The uncertainty of the extension, x:

The ruler's uncertainty is ± 0.05cm which is equal to ± 0.0005m; which is also the uncertainty of x

Spring A:

The gradient of spring A's graph: $\frac{\Delta F}{\Delta x} = \frac{(1.5-1.0) \pm 0.01}{(0.3-0.2) \pm 0.0005}$

$$= \frac{0.5 \pm 0.001}{0.1 \pm 0.0005}$$

$$= \frac{0.5 \pm 0.2\%}{0.1 \pm 0.5\%}$$

$$= \left(\frac{0.5}{0.1}\right) + (0.2\% + 0.5\%)$$

$= 5 \pm 0.7\%$

$k = 5 \pm 0.035$

The percentage uncertainty of spring A is 0.7%

Spring B:

The gradient of spring B's graph: $\frac{\Delta F}{\Delta x} = \frac{(3.75-2.5) \pm 0.01}{(0.375-0.25) \pm 0.0005}$

$$= \frac{1.25 \pm 0.001}{0.125 \pm 0.0005}$$

$$= \frac{1.25 \pm 0.08\%}{0.125 \pm 0.4\%}$$

$$= \left(\frac{1.25}{0.125}\right) + (0.08\% + 0.4\%)$$

$$= 10 \pm 0.48\%$$

$$k = 10 \pm 0.048$$

The percentage uncertainty of spring B is 0.48%

Spring C:

The gradient of spring C's graph: $\frac{\Delta F}{\Delta x} = \frac{(3.0-2.25) \pm 0.01}{(0.2-0.15) \pm 0.0005}$

$$= \frac{0.75 \pm 0.001}{0.05 \pm 0.0005}$$

$$= \frac{0.75 \pm 0.13\%}{0.05 \pm 1\%}$$

$$= \left(\frac{0.75}{0.05}\right) + (0.13\% + 1\%)$$

$$= 15 \pm 1.13\%$$

$$k = 15 \pm 0.17$$

The percentage uncertainty of spring C is 1.13%

Spring D:

The gradient of spring D's graph: $\frac{\Delta F}{\Delta x} = \frac{(3.25-2.0) \pm 0.01}{(0.125-0.075) \pm 0.0005}$

$$= \frac{1.25 \pm 0.001}{0.05 \pm 0.0005}$$

$$= \frac{1.25 \pm 0.08\%}{0.05 \pm 1\%}$$

$$= \left(\frac{1.25}{0.05}\right) + (0.08\% + 1\%)$$

$$= 25 \pm 1.08\%$$

$$k = 25 \pm 0.27$$

The percentage uncertainty of spring D is 1.08%

8. EVALUATION

The followed procedure achieved a good level of accuracy which was demonstrated in the results that were quite reasonable in comparison to reality. The equipment used in the procedure are not very sensitive however, hence the results weren't very accurate because the range of uncertainty is not negligible. But the range could be narrowed by developing the design of the procedure, using some more sensitive equipments/better method.

Possible solutions to reduce sources of errors:

- Hanging the ruler and the spring into two separate Retort Stands, so we can more clearly see and measure both.
- Using light for example to determine the level between the spring and the ruler, or any better pointer for the spring. Because the current spring pointer keeps rotating which makes it hard for the observer to record accurately.
- Using bars for the Retort Stand instead of the current handles that hold the ruler; so the ruler will be attached/hanged to it. Because the handle is made to hold beakers often (it has a rounded shape which makes the ruler not very stable).