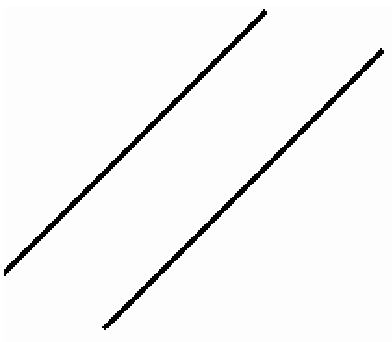
Parallels and Parallelograms Math Portfolio

Introduction:

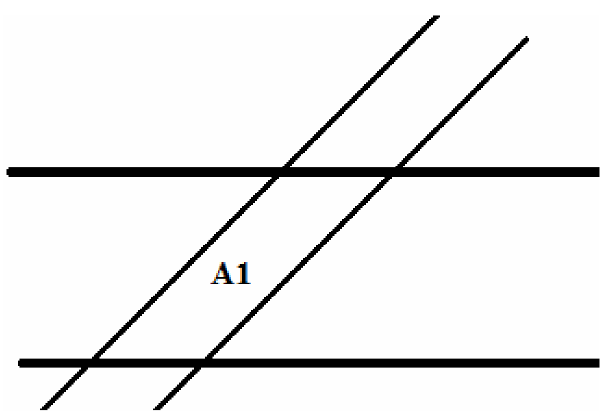
This investigation aims at finding a relationship between the numbers of horizontal parallel lines and the transversals. When these lines intersect they form parallelograms. The aim of this investigation is examine and determine a general statement for transversals and horizontal lines and how they affect the number of parallelograms formed within the figure. A diagram of a parallelogram and a transversal is shown below.



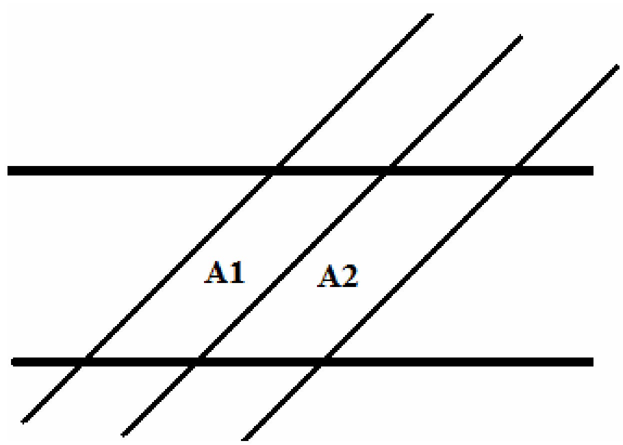
These lines represent two transversals; however they are supposed to intersect with a horizontal parallel line.

These lines represent two horizontal parallel lines. They are intersected with a number of transversals.

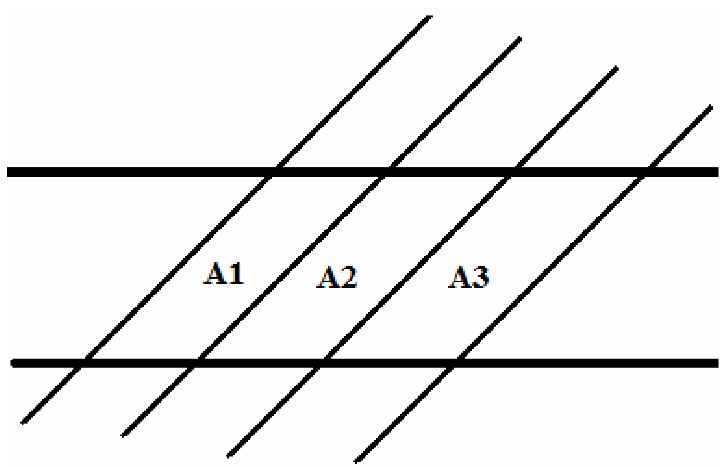
When there is one horizontal line and two transversals, this makes one parallelogram, A_1, as shown below:



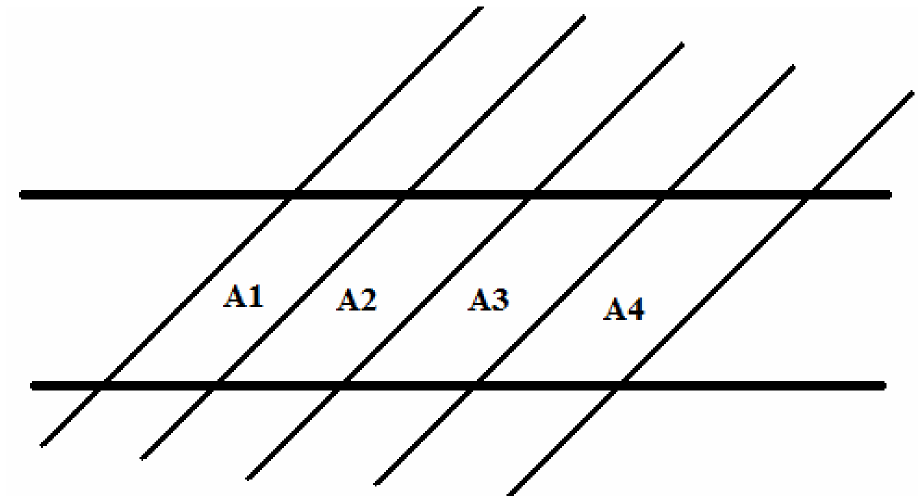
This diagram shows that when there are two horizontal lines and two transversals, one parallelogram is formed. This parallelogram is called A_1. However, when another transversal is added to the same diagram and same pair of horizontal lines, it looks like this:



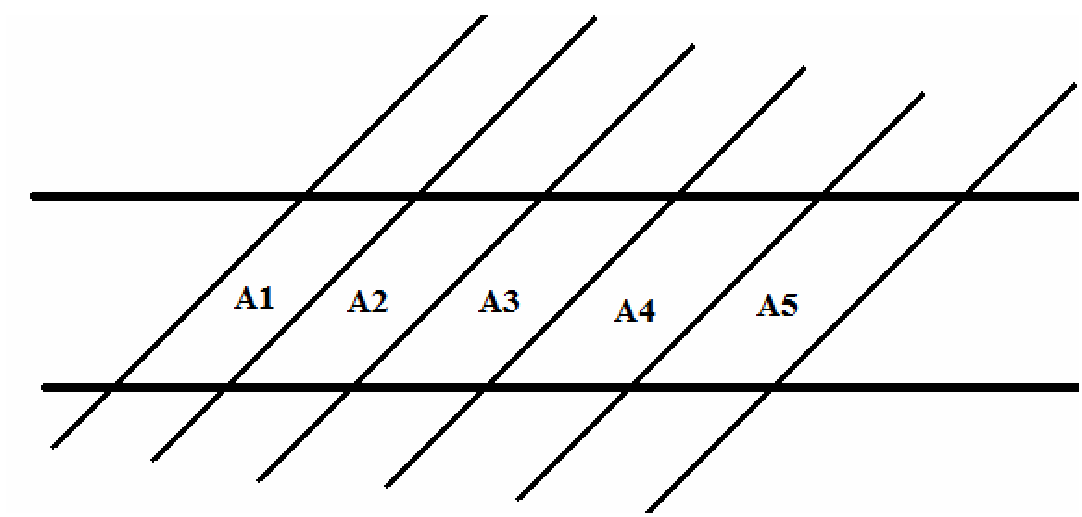
This figure shows that when another transversal is added to a pair of horizontal parallel lines, three parallelograms are formed. Although the third parallelogram is not labeled, it is clear that the total of $A_1 \cup A_2$ makes a big parallelogram, A_3. So therefore, there is A_1, A_2, and A_3. Another transversal added to this figure is shown below:



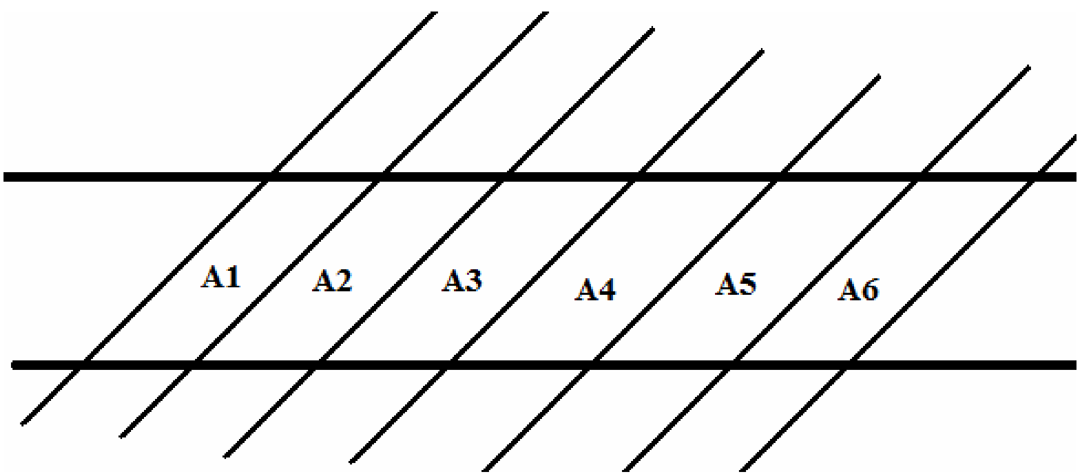
In this figure, since a fourth transversal was added to the pair of horizontal parallel lines, this contains six different parallelograms within the diagram. Firstly you have A_1, A_2, and A_3. However, $A_1 \cup A_2$ form another parallelogram, which makes A_4. Also, $A_2 \cup A_3$ form another parallelogram, which creates A_5. Lastly, $A_1 \cup A_2 \cup A_3$ create a final parallelogram, A_6. Therefore, there are six parallelograms formed when a fourth transversal is added to the pair of horizontal parallel lines. When five transversals are added to this figure, the figure appears as:



As a fifth transversal is added to the same figure, this contains much more parallelograms. In this diagram there are ten parallelograms. This diagram firstly contains A_1, A_2, A_3, and A_4. Also A_5 is formed when A_1 and A_2 are combined, $A_1 \cup A_2$. A_6 is formed because $A_2 \cup A_3$ can be combined to form another parallelogram. Also $A_3 \cup A_4$ make another parallelogram, forming A_7. When $A_1 \cup A_2 \cup A_3$ are combined this forms another parallelogram, making A_8. Then $A_2 \cup A_3 \cup A_4$ make a parallelogram which is A_9. Then lastly combining all of these together, $A_1 \cup A_2 \cup A_3 \cup A_4$, this makes the last parallelogram in this figure, A_{10}. The figure below represents six transversals added to the same figure.



In this figure, there are six transversals added to the pair of horizontal parallel lines. With six transversals, there are fifteen parallelograms within this diagram. The figure firstly obviously shows five parallelograms, A_1, A_2, A_3, A_4, and A_5. A_6 is formed when you combine A_1 with A_2, $A_1 \cup A_2$. Then when you combine A_1 with A_2 and A_3 ($A_1 \cup A_2 \cup A_3$), A_7 is formed. Also $A_1 \cup A_2 \cup A_3 \cup A_4$ create another parallelogram, A_8. Another parallelogram is formed when $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ are combined. This makes A_9. When $A_2 \cup A_3$ are combined this makes A_{10}. Also when $A_2 \cup A_3 \cup A_4$ are combined, this makes another parallelogram, therefore A_{11} is created. $A_2 \cup A_3 \cup A_4 \cup A_5$, when these are combined, this makes A_{12}. Then when, A_3 combines with A_4, $A_3 \cup A_4$, this makes A_{13}. A_{14} is shown when $A_3 \cup A_4 \cup A_5$. Lastly, the fifteenth parallelogram formed is when A_4 is combined with A_5, $A_4 \cup A_5$.

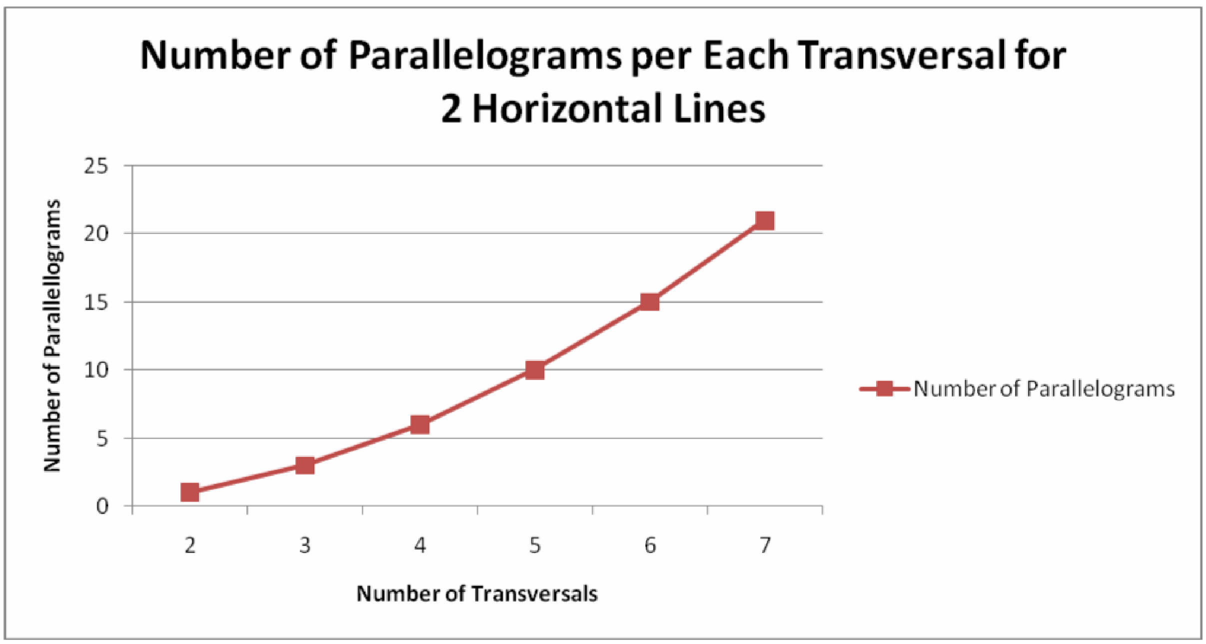


With seven transversals, there are twenty one parallelograms within in this figure. Firstly there are six obvious parallelograms listed, A_1, A_2, A_3, A_4, A_5 and A_6. A_7 is shown because A_1 can combine with A_2, $A_1 \cup A_2$. Then $A_1 \cup A_2 \cup A_3$ show another parallelogram, A_8. After this $A_1 \cup A_2 \cup A_3 \cup A_4$ show that there is another parallelogram, A_9. A_{10} is shown by the combination of $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$. Then A_{11} is shown because the combination of $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$. The twelfth parallelogram, A_{12}, is shown by combining $A_2 \cup A_3$. A_{13} is shown by combining $A_2 \cup A_3 \cup A_4$. A_{14} is shown by combining $A_2 \cup A_3 \cup A_4 \cup A_5$. A_{15} is then shown when combining, $A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$. A_{16} is formed because of the combination of, $A_3 \cup A_4$. A_{17} is therefore shown by the combination of $A_3 \cup A_4 \cup A_5$. Then after this, A_{18} is shown when combining, $A_3 \cup A_4 \cup A_5 \cup A_6$. By combining $A_4 \cup A_5$, this shows another parallelogram, A_{19}. A_{20} is shown when combining $A_4 \cup A_5 \cup A_6$. Lastly, the twenty first parallelogram is shown when combining $A_5 \cup A_6$.

The table below is the results found when adding a transversal each time to a pair of horizontal parallel lines, resulting in an amount of parallelograms.

Number of Transversals	Number of Parallelograms
2	1
3	3
4	6
5	10
6	15
7	21

Using technology, a graph of this table was made, to find a relationship between the number of transversals and the number of parallelograms made. The graph is shown below.



This graph shows that the general function has to be exponential or quadratic, because of the curve. However, when graphing this on the calculator, an exponential formula does not fit the graph as well as a quadratic formula, using regressions. The exponential graph increases much faster. Obviously in a quadratic there are negative values; therefore, the x value domain must be greater than or equal to two. Using technology, a general statement and formula was found. This was found by using a quadratic regression, when listing all the points. The general formula and statement is $y = 0.5x^2$ -

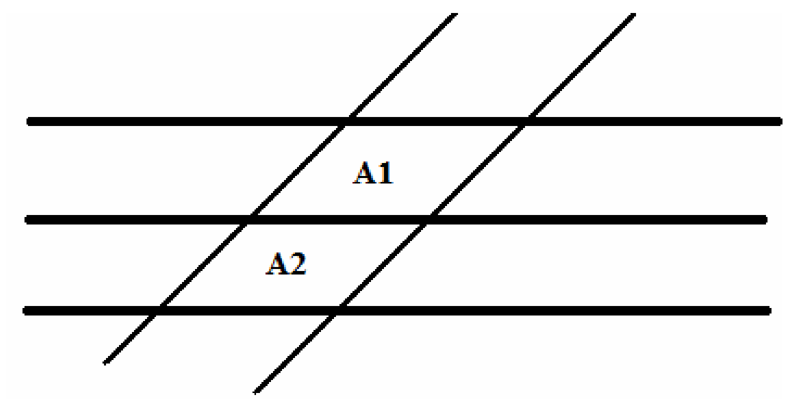
0.5x. To make sure this function is valid; numbers can be substituted into this function. Also when looking at the table and analyzing this regression, since the formula is quadratic, when squaring each of these numbers, they must be subtracted by the value of themselves and then divided by two. Therefore, another statement without technology is $\frac{N^2 - N}{2}$, N is the number of transversals. When using this formula, the numbers of parallelograms are found for each transversal.

Firstly substituting 2 into this function would be the first number, because of the domain, x has to be greater than or equal to 2 because the least number of transversals needed to form a parallelogram is 2. Therefore, $y = (0.5) \times (2)^2 - (0.5) \times (2)$. $Y = 2 - 1 = 1$ parallelogram.

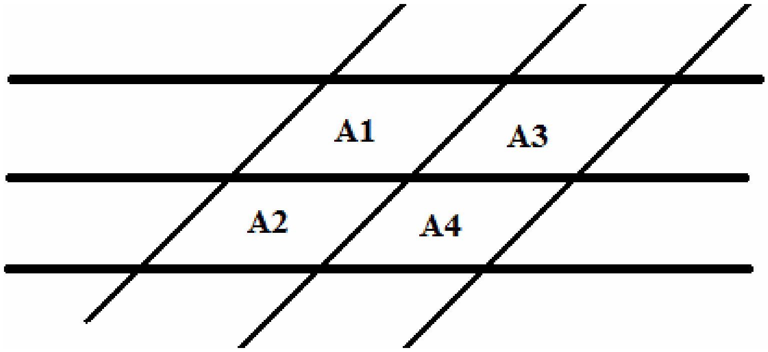
Substituting 3 into this function, $y = (0.5) \times (3)^2 - (0.5) \times (3)$. This equals, $y = 4.5 - 1.5 = 3$ parallelograms.

Substituting 4 into this function, $y = (0.5) \times (4)^2 - (0.5) \times (4)$. $Y = 8 - 2 = 6$ parallelograms. Obviously these numbers follow the table of values and these numbers are following the pattern of parallelograms.

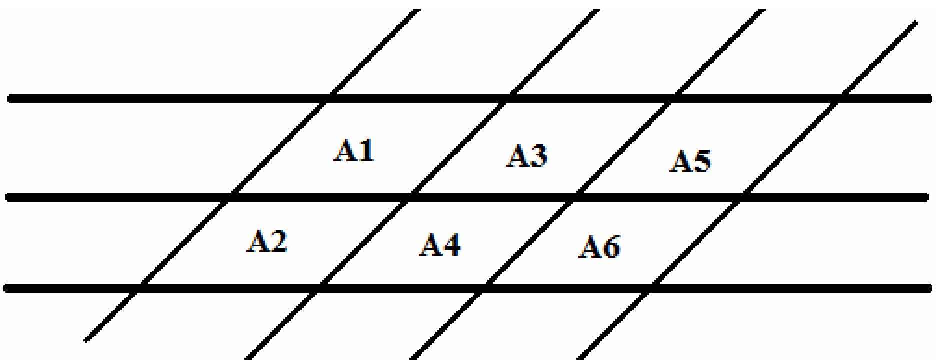
Extending this diagram, adding a horizontal parallel line to make three parallel horizontal lines will form many more parallelograms. Also a relationship between these new values and the other values can be shown, resulting in a general statement for all cases. Shown below, this is a figure with two transversals and three horizontal parallel lines.



In this diagram, there are three different parallelograms. Compared to the pair of horizontal parallel lines, instead of having one parallelogram, when another horizontal line is added this figure, this creates three parallelograms. Firstly, A_1 and A_2 are shown; however, when combining $A_1 \cup A_2$, this forms another parallelogram, known as A_3.



This figure represents another transversal added to the three horizontal parallel lines. Within this figure, there are nine parallelograms shown. Firstly there are A_1, A_2, A_3, and A_4. A_5 is shown when combining A_1 with A_2. $A_1 \cup A_3$, this makes another parallelogram, forming A_6. Then A_7 is formed when combining $A_3 \cup A_4$. A_8 is formed when combining $A_2 \cup A_4$. Lastly, A_9 is formed when combining all of these together, $A_1 \cup A_2 \cup A_3 \cup A_4$.



In this figure, there are a total of eighteen parallelograms within this figure. Firstly there are A_1, A_2, A_3, A_4, A_5, and A_6. Also A_7 is formed when combining $A_1 \cup A_2$. A_8 is formed when combining $A_1 \cup A_3 \cup A_5$. A_9 is formed when combining $A_1 \cup A_2 \cup A_3 \cup A_4$. A_{10} is formed when combining, $A_3 \cup A_5$. A_{11} is formed when combining $A_3 \cup A_4 \cup A_5 \cup A_6$. A_{12} is formed when combining $A_2 \cup A_4 \cup A_6$. A_{13} is formed when combining $A_3 \cup A_4$. A_{14} is formed when $A_5 \cup A_6$. A_{15} is formed when combining $A_2 \cup A_4$. A_{16} is formed when combining $A_4 \cup A_6$. Also when combining $A_1 \cup A_3$, this forms the seventeenth parallelogram, A_{17}. Lastly, when combining all of these together this forms A_{18}, $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$.

Clearly when analysing these values, when comparing them to the two horizontal parallel lines, all of these values are multiplied by three. The table below shows this easier.

Number of	Number of	Number of	Number of

Transversals for 2 Horizontal Parallel Lines	Parallelograms for 2 Horizontal Parallel Lines	Transversals for 3 Horizontal Parallel Lines	Parallelograms for 3 Horizontal Parallel Lines
2	1	2	3
3	3	3	9
4	6	4	18
5	10	5	30
6	15	6	45
7	21	7	63

This table shows that the number of parallelograms for 2 horizontal parallel lines are multiplied by three for them to equal the vaules for parallelograms for 3 horizontal parallel lines. Therefore, for three horizontal parallel lines the general statement and formula is the same as the first formula multiplied by three. Therefore, the formula for three horizontal lines is $3 \times \frac{N^2 - N}{2}$. To simplify this the formula can be written as, $\frac{3N^2 - 3N}{2}$. Testing this formula is shown below:

When N =2, N = $\frac{3(2)^2 - 3(2)}{2} = \frac{12 - 6}{2} =$ 3 Parallelagrams, this matches the value in the table.

When N =3, N = $\frac{3(3)^2 - 3(3)}{2} = \frac{27-9}{2} =$ 9 Parallelagrams. This also matches the actual value.

When N = 7, N = $\frac{3(7)^2 - 3(7)}{2} = \frac{147-21}{2} =$ 63 Parallelegrams, this shows that the formula is correct because this is the actual value as well.

To further extend the results, by looking at the horizontal lines added each time, it is noticable that because these are all parallel lines, they have the same formula except different variables. When flipping the diagram, it appears to be the same. Therefore, the transversals act the same as as the horizontal parallel lines. Moreover, the formula for the overall formula dealing with the variables M, horizontal lines, and N, transversals is $\frac{N^2 - N}{2} \times \frac{M^2 - M}{2}$.

The limitation of this formula and general statement are non existant. These formulas are completely correct, if the correct domain is being used. The domain for this formula is that N and M have to be greater than or equal to 2. However, for a quadratic regrission the limitation is that the x values must be always positive and never negative.