IB Standard Portfolio Assignment

Type I – Mathematical Investigation

Logarithm Bases

This investigation will determine the relation between different sets of sequences. The sequences include logarithms. This investigation will be tested using technology.

The sets of sequences are as follows:

$Log_2 8$, $Log_4 8$, $Log_8 8$, $Log_{16} 8$, $Log_{32} 8$, …

$Log_3 81$, $Log_9 81$, $Log_{27} 81$, $Log_{81} 81$, …

$Log_5 25$, $Log_{25} 25$, $Log_{125} 25$, $Log_{625} 25$, …

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$\text{Log}_m m^k, \log_{m^2} m^k, \log_{m^3} m^k, \log_{m^4} m^k, \ldots$

By following these sequences a pattern can be shown. The base of each term in the sequences changes but the exponents are constant. The following two terms of each sequence were determined:

$Log_2 8$, $Log_4 8$, $Log_8 8$, $Log_{16} 8$, $Log_{32} 8$, $\text{Log}_{64} 8$, $\text{Log}_{128} 8$

$Log_3 81$, $Log_9 81$, $Log_{27} 81$, $Log_{81} 81$, $\text{Log}_{243} 81$, $\text{Log}_{729} 81$, $\text{Log}_{2187} 81$

$Log_5 25$, $Log_{25} 25$, $Log_{125} 25$, $Log_{625} 25$, $\text{Log}_{3125} 25$, $\text{Log}_{15625} 25$, $\text{Log}_{78125} 25$

$\text{Log}_m m^k, \log_{m^2} m^k, \log_{m^3} m^k, \log_{m^4} m^k, \log_{m^5} m^k, \log_{m^6} m^k$

Let's start with the first sequence ($Log_2 8$, $Log_4 8$, $Log_8 8$, $Log_{16} 8$, $Log_{32} 8$, $\text{Log}_{64} 8$, $\text{Log}_{128} 8$) and determine an expression for the nth term:

1	2	3	4	5	6	7
2	2	2	2	2	2	2
2^1	2^2	2^3	2^4	2^5	2^6	2^7
2	4	8	16	32	64	128

The value 2 was used to determine the nth term. $\log_{2n} 8$ is worked out from the table above in the form of $\frac{p}{q}$ by applying the change of base rule which states that:

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}.$$

An application of this rule is as follows:

$$\frac{log8}{log2n}$$

The equations shown below will be used to determine and solve the sequences shown in the portfolio:

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}.$$

$$k\log a = \log_a k$$

$$a_n = \log_{m^n} m^k$$

Terms in the form of $\frac{p}{q}$:

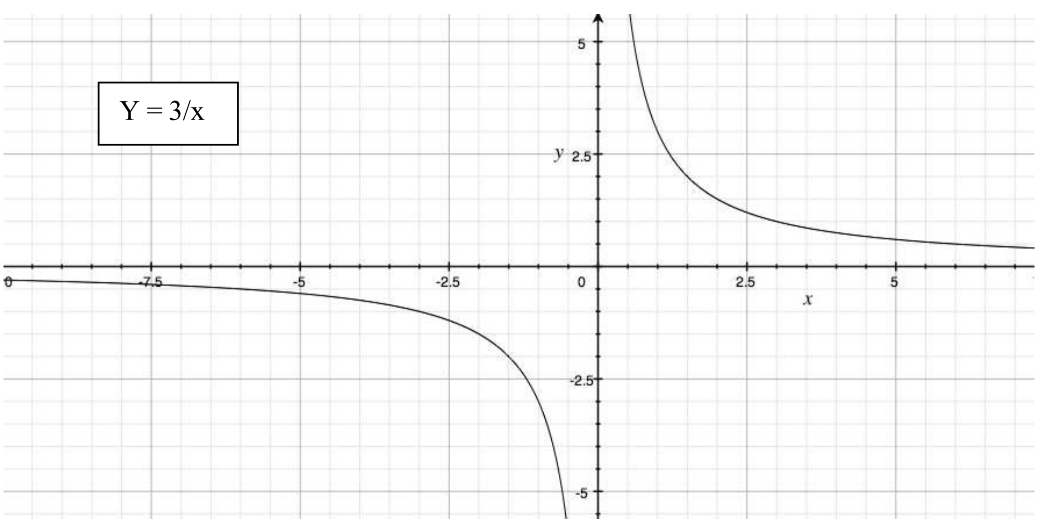
$$\frac{log8}{log2n}$$

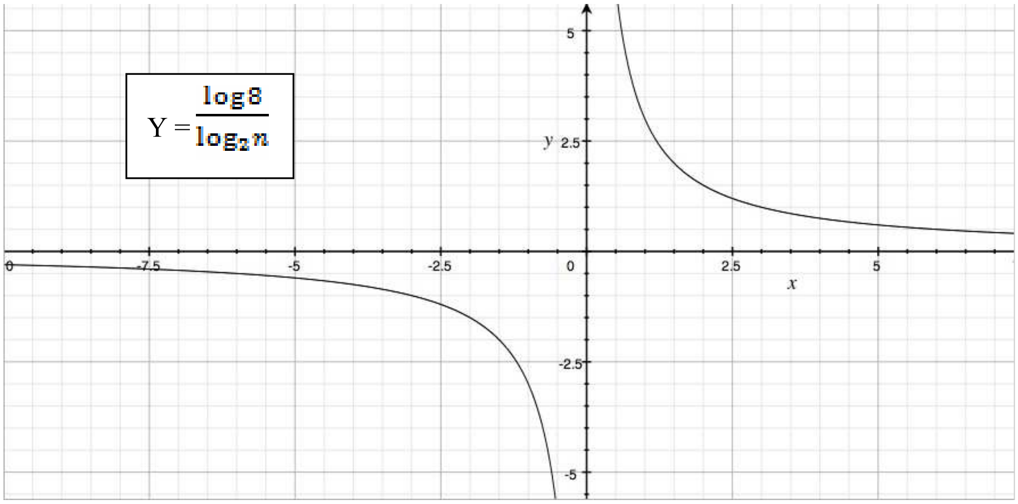
$$\therefore \frac{log2^3}{log2^n}$$

$$\therefore \frac{3log2}{nlog2}$$

The numerator and denominator both have the value log2 in them which means they can be canceled out thus leaving us with $\frac{3}{n}$

Here are two graphs to test its validity:





Both graphs are identical towards each other which indicate that both functions of the graphs are the same.

Let's take the second sequence ($\text{Log}_3 81$, $\text{Log}_9 81$, $\text{Log}_{27} 81$, $\text{Log}_{81} 81$, $\text{Log}_{243} 81$, $\text{Log}_{729} 81$, $\text{Log}_{2187} 81$) into consideration:

1	2	3	4	5	6	7
3	3	3	3	3	3	3
3^1	3^2	3^3	3^4	3^5	3^6	3^7
3	9	27	81	243	729	2187

I used the value 3 to determine the nth term since the value of 2 did not work. $\log_{3n} 81$ is worked out from the table above in the form of $\frac{p}{q}$ by applying the change of base rule therefore:

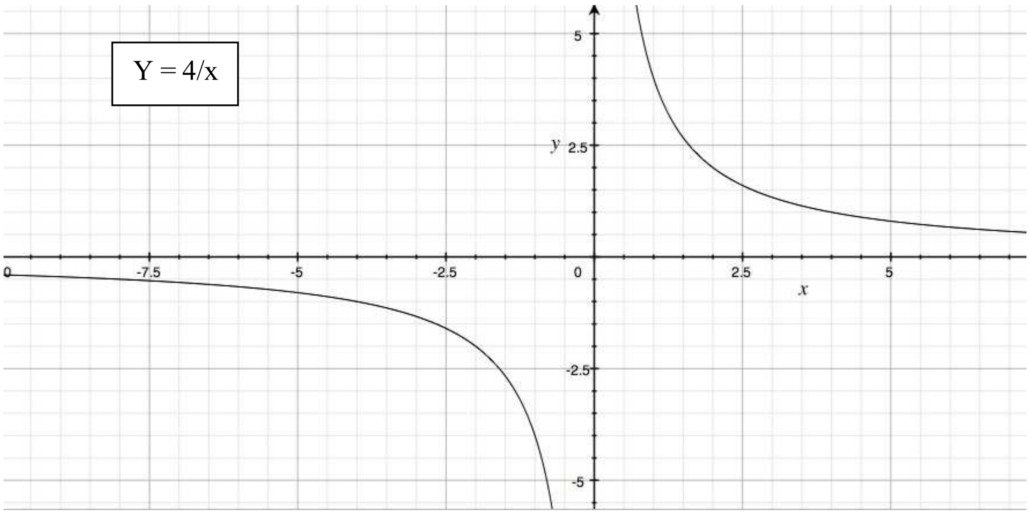
$$\therefore \frac{\log 81}{\log_3 n}$$

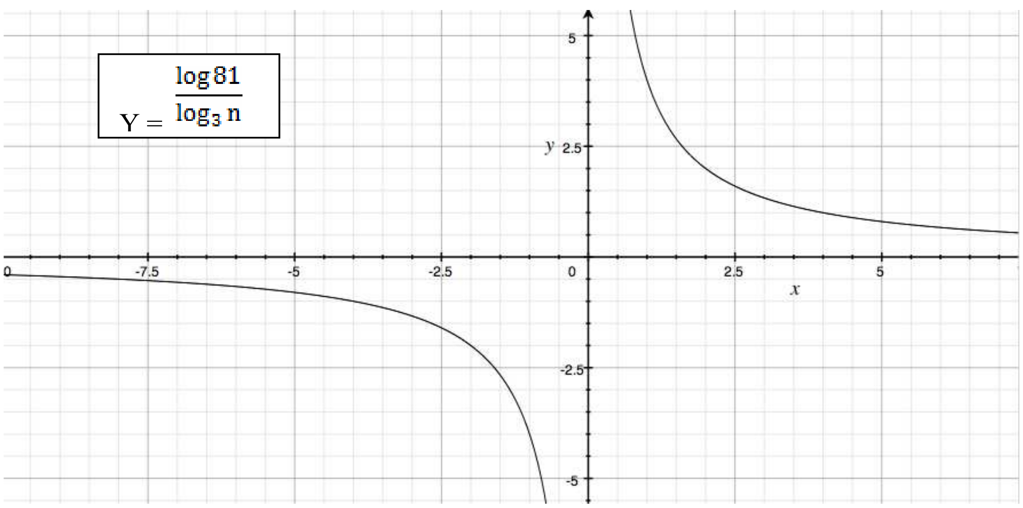
$$\therefore \frac{log 3^4}{log 3^n}$$

$$\frac{4 log 3}{n log 3}$$

The numerator and denominator both have the value log3 in them which means they can be canceled out thus leaving us with $\frac{4}{n}$

Here are two graphs to test its validity and accuracy:





Both graphs are identical towards each other which indicate that both functions of the graphs are the same.

Let's take the third sequence ($Log_5 25$, $Log_{25} 25$, $Log_{125} 25$, $Log_{625} 25$, $Log_{3125} 25$, $Log_{15625} 25$, $Log_{78125} 25$) into consideration:

1	2	3	4	5	6	7
5	5	5	5	5	5	5
5^1	5^2	5^3	5^4	5^5	5^6	5^7
5	25	125	625	3125	15625	78125

I used the value 5 to determine the nth term. $\log_{5n} 25$ is worked out from the table above in the form of $\frac{p}{q}$ by applying the change of base rule therefore:

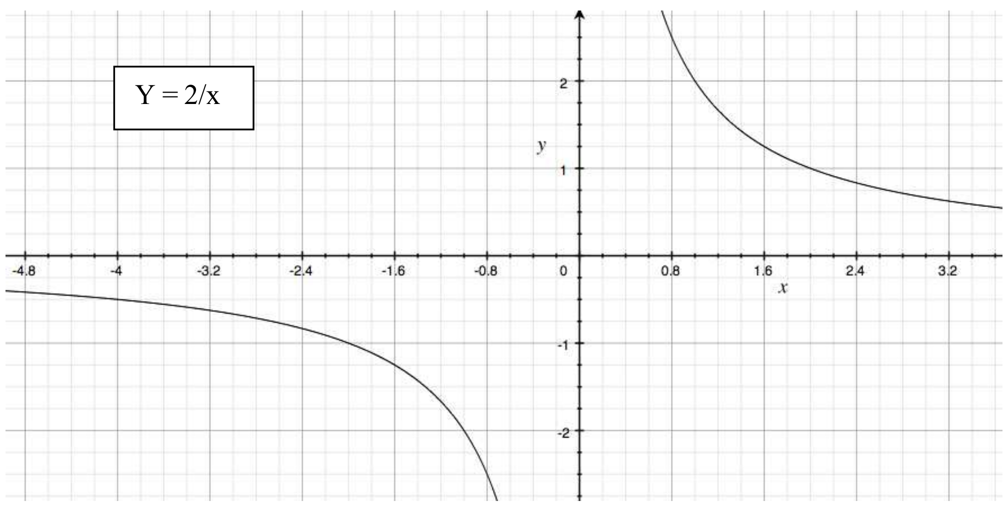
$$\therefore \frac{log25}{log5n}$$

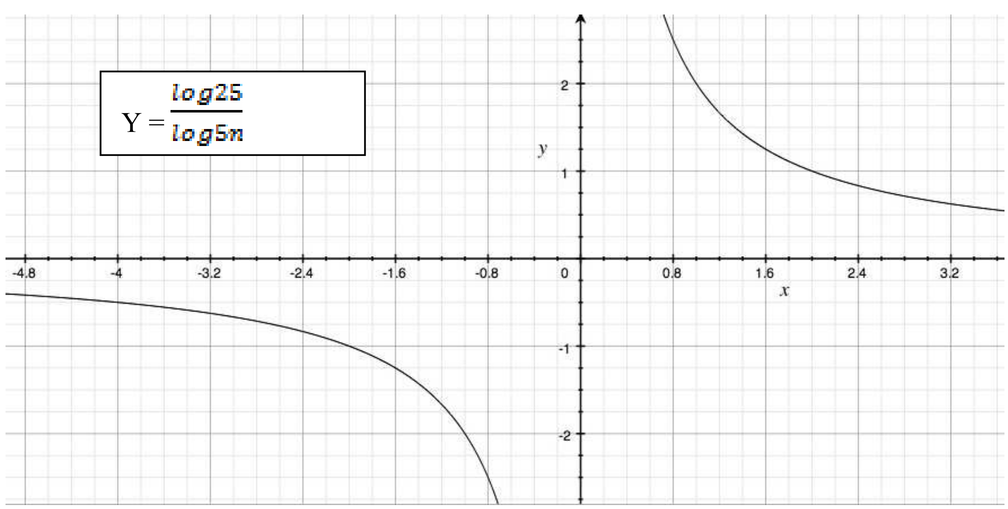
$$\therefore \frac{log5^2}{log5^n}$$

$$\frac{2log5}{nlog5}$$

The numerator and denominator both have the value log5 in them which means they can be canceled out thus leaving us with $\frac{2}{n}$

Here are two graphs to test its validity and accurateness:





Both graphs are identical towards each other which indicate that both functions of the graphs are the same.

Let's take the fourth and final sequence ($\text{Log}_m m^k$,$\log_{m^2} m^k$, $\log_{m^3} m^k$, $\log_{m^4} m^k$, $\log_{m^5} m^k$, $\log_{m^6} m^k$) into concern:

1	2	3	4	5	6
m	m	m	m	m	m
m^1	m^2	m^3	m^4	m^5	m^6

I used the value m to determine the nth term. $\log_{m^n} m^k$ is worked out from the table above in the form of $\frac{p}{q}$ by applying the change of base rule therefore:

$$\frac{\log_m k}{\log_m n}$$

$$\therefore \frac{klogm}{nlogm}$$

The numerator and denominator both have the value log m in them which means they can be canceled out thus leaving us with $\frac{k}{n}$

Now I will calculate the following giving the answers in the form of $\frac{p}{q}$ by using the sequences shown above:

$\log_4 64, \log_8 64, \log_{32} 64=$

$$\frac{\log 64}{\log 4} = 3,$$

$$\therefore \frac{\log 64}{\log 8} = 2,$$

$$\therefore \frac{\log 64}{\log 32} = 1.2$$

$$\therefore \log_{32} 64 = 1.2 = \frac{6}{5} = \frac{3x2}{3+2} = \frac{\log_4 64 \, x \, \log_8 64}{\log_4 64 + \log_8 64}$$

$\log_7 49, \log_{49} 49, \log_{343} 49=$

$$\frac{\log 49}{\log 7} = 2,$$

$$\therefore \frac{\log 49}{\log 49} = 1,$$

$$\therefore \frac{\log 49}{\log 343} = 0.666667$$

$$\therefore \log_{343} 49 = 0.66667 = \frac{2}{3} = \frac{2x1}{2+1} = \frac{\log_7 49 \, x \, \log_{49} 49}{\log_7 49 + \log_{49} 49}$$

$$\log_{\frac{1}{5}} 125 \ , \ \log_{\frac{1}{125}} 125 \ , \ \log_{\frac{1}{625}} 125$$

$$\therefore \frac{\log 125}{\log \frac{1}{5}} = -3$$

$$\therefore \frac{\log 125}{\log \frac{1}{125}} = -1$$

$$\frac{\log 125}{\log \frac{1}{625}} = -0.75, \ -\frac{3}{4}$$

$$\therefore \log_{\frac{1}{625}} 125 = -0.75 = -\frac{3}{4} = \frac{-3x-1}{-3-1} = \frac{\frac{\log 125}{\log \frac{1}{5}} \, x \, \frac{\log 125}{\log \frac{1}{125}}}{\frac{\log 125}{\log \frac{1}{5}} + \frac{\log 125}{\log \frac{1}{125}}}$$

$$\log_8 512, \ \log_2 512, \ \log_{1632} 512 =$$

$$\frac{\log 512}{\log 8} = 3,$$

$$\therefore \frac{\log 512}{\log 2} = 9,$$

$$\therefore \frac{\log 512}{\log 16} = 2.25$$

$$\therefore \log_{16} 512 = 2.25 = \frac{9}{4} = \frac{3x9}{3+9} = \frac{\log_8 512 \, x \, \log_2 512}{\log_8 512 + \log_2 512}$$

The General Statement:

$$\log_a x = c \text{ and } \log_b x = d$$

$$\log_{ab} x = \frac{1}{\log_x ab}$$

$$= \frac{1}{\log_x a + \log_x b} = \frac{1}{\frac{1}{\log_a x} + \frac{1}{\log_b x}} = \frac{1}{\frac{1}{c} + \frac{1}{d}} = \frac{1}{\frac{d+c}{cd}} = \frac{cd}{d+c}$$

Now I'm going to test the validity of my general statement using other values of a, b, and x.

$$\log_{ab} x = \frac{\log_a x \times \log_b x}{\log_a x + \log_b x}$$

Limitations:

There are some limitations that were taken into concern. The variables a, b and x cannot be any number below zero. They all have to be positive integers. Only values and variables greater than zero are taken into thought. Negative values in graphs are ignored. The values in each sequence should all be positive integers since the denominator cannot be negative.

Clarification of How I Arrived at My General Statement:

The sequences given were firstly observed. I determined and have assured that each sequence had a constant exponent but the bases of each one were not. After observing the sequences I then switched the exponents and bases together with the help of the change of base rule. Basically I inversed them:

$$\log_{ab} x = \frac{1}{\log_x ab}$$

After observing the sequences and determining the nth term we had to test the validity of our general statement using other values of a, b, and x. The validity was proved by finding this formula:

$$\log_a(xy) = \log_a x + \log_a y$$