Type 1 Portfolio: Matrix Binomials

Hudson Liao
12/7/2008

I was given the expression X = $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ and Y = $\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$, where I calculate X^2,X^3,X^4 [illegible]. Below I calculated X^2 and made my way up to X^4, where I also did the same with Y^2 to Y^4.

X^2= $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} x \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}(1\times1)+(1\times1) & (1\times1)+(1\times1)\\(1\times1)+(1\times1) & (1\times1)+(1\times1)\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$

X^3 or X^2 * X^1 = $\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} x \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}(2\times1)+(2\times1) & (2\times1)+(2\times1)\\(2\times1)+(2\times1) & (2\times1)+(2\times1)\end{bmatrix} = \begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}$

X^4 or X^3 * X^1 = $\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix} x \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}(4\times1)+(4\times1) & (4\times1)+(4\times1)\\(4\times1)+(4\times1) & (4\times1)+(4\times1)\end{bmatrix} = \begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}$

Y^2 = $\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} x \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} = \begin{bmatrix}(1\times1)+(-1\times-1) & (1\times-1)+(-1\times-1)\\(-1\times1)+(1\times-1) & (-1\times-1)+(1\times1)\end{bmatrix} = \begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$

Y^3 or Y^2*Y^1 =
$\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix} x \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} = \begin{bmatrix}(2\times1)+(-2\times-1) & (2\times-1)+(-2\times1)\\(-2\times1)+(2\times-1) & (-2\times-1)+(2\times1)\end{bmatrix} = \begin{bmatrix}4 & -4\\-4 & 4\end{bmatrix}$

Y^4 or Y^{3+} * Y^1 =
$\begin{bmatrix}4 & -4\\-4 & 4\end{bmatrix} x \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} = \begin{bmatrix}(4\times1)+(-4\times-1) & (4\times-1)+(-4\times1)\\(-4\times1)+(4\times-1) & (-4\times-1)+(4\times1)\end{bmatrix} = \begin{bmatrix}8 & -8\\-8 & 8\end{bmatrix}$

Now I am going to find and expression for: [X^n, Y^n, $(X+Y)^n$], by inputting different 'n' values. By doing this I can find a correlation between each variable.

X^n
$X^1 = 1X = 2^0X$
$X^2 = 2X = 2^1X$
$X^3 = 4X = 2^2X$
$X^4 = 8X = 2^3X$

Expression: $X^n = 2^{(n-1)} X$

This general statement was found by finding a relationship through values from X^1 to X^4. In the X^n table, a pattern begins to form from 1X, 2X, 4X and 8X. If we simplify these numbers by using a constant value such as $1X = 2^0X$ we can find a general statement for this expression.

Y^n
$Y^1= 1Y = 2^0Y$
$Y^2= 2Y = 2^1Y$
$Y^3= 4Y = 2^2Y$
$Y^4= 8Y = 2^3Y$

Expression: $Y^n = 2^{(n-1)} Y$

The same method to determine the general statement for the expression $X^n = 2^{(n-1)} X$ was also used for $Y^n = 2^{(n-1)} Y$.

$(X+Y)^n$
2I
4I
8I
16I

I am going to determine the expression for $\mathbf{(X+Y)^n}$ by letting X = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and Y = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Therefore the expression would look like:

$$(X+Y)^n = \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right)^2 = \left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The resultant matrix is $2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

We know that the Identity Matrix is $\mathrm{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore (X+Y)^n = (2\mathrm{I})^n$

However I^n is an identity matrix $\therefore \mathrm{I}^n = \mathrm{I}$

$\therefore (X+Y)^n = 2^n\mathrm{I}$

I am going to prove that this expression works with **$(X+Y)^n$** :

Calculation:

$(X+Y)^n$ n=2

My expression:
$(X+Y)^2 = 2^2\mathrm{I} = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \therefore 4\mathrm{I}$

With $(X+Y)^n$:

$$\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right)^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^2$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} (2\mathrm{x}2) + (0\mathrm{x}0) & (-2\mathrm{x}0) + (0\mathrm{x}2) \\ (0\mathrm{x}2) + (0\mathrm{x}2) & (0\mathrm{x}0) + (2\mathrm{x}2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \therefore 4\mathrm{I}$$

As well, since 'X' and 'Y' are singular matrices, they cannot be raised to a negative exponent since the determinant is zero.

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$X^{-1} = \frac{1}{(1x1)-(1x1)}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\det(X) = 1 - 1 = 0$	$Y^{-1} = \frac{1}{(1x-1)-(-1x1)}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $\det(Y) = -1 + 1 = 0$

In this section I am going use another two expression: $A^n = \sigma X$ and $B^n = \tau Y$, where 'σ' and 'τ' are constants. I am going to use different values for both of the constants 'σ' and 'τ' to calculate A^2, A^3, A^4 ; B^2, B^3 , B^4.

First, I am going to find A^2 for the expression $A^n = \alpha X$ and use 4 different constants for 'α', where I am going to let $X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Calculation:

n=2 α=1 (without calculator) n=2 a=2 (with calculator) n=3 a=2 n=4 a=2

$A^2 = \alpha^2 X^2$

$= (1)^2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2$

$= 1 \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$= 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \therefore A^2 = 2X$

$A^2 = (2)^2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2$

$A^2 = 8X$

$A^2 = (3)^2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2$

$A^2 = 18X$

$A^2 = (4)^2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2$

$A^2 = 32X$

Now I am going to create a table for [$A = \alpha X$] with all the different 'n' values and 'α' values:

	n=2	n=3	n=4
α=1	2X	4X	8X
α=2	8X	32X	128X
α=3	18X	108X	648X
α=4	32X	256X	2048X

Now I am going to find all the B^n by using the expression $B^n = \beta Y$ and use 4 different constants for β, where I am going to let $Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

	n=2	n=3	n=4
β=1	2Y	4Y	8Y
β=2	8Y	32Y	128Y
β=3	18Y	108Y	648Y
β=4	32Y	256Y	2048Y

By considering integer powers of A and B, find expression for A^n , B^n and $(A+B)^n$

For the statement $A=\alpha X$, I am going to determine a general formula by inputting different numbers for the constant 'α' and as well for the terms 'n'.

$A^n=(\alpha X)^n = \left(a\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^n$

If I input $\alpha=1$ and $n=2$ into $A^n=(\alpha X)^n$, the resulting value would be 2X:

$$A^2 = \left(1\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^2$$

$$= \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

$$= 2X$$

However if I continue to input $\alpha=1$ and change the terms 'n' to 3 and 4 a pattern begins to form:

$$A^3 = \left(1^3\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^3$$

$$= 1\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}$$

$$= 4X$$

$$A^4 = \left(1^4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^4$$

$$= 1\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}$$

$$= 8X$$

By changing the terms $n=2$ up to $n=4$ 'X' increases each time from, 2X, 4X to 8X. The number of X's that is being increased is resulted from this expression, "2^{n-1}". Therefore, we can convert the formula $A^n=(\alpha X)^n$ to:

$$A^n = a^n X^n$$

$$A^n = a^n 2^{n-1} X$$

The same expression can be also used for the statement $B=bY$ because both of the statements, $A=\alpha X$ and $B=bY$ have the same pattern. The only difference between the two statements is that 'X' and 'Y' have different matrices. Therefore we just change, A^n to B^n by:

$$A^n = a^n 2^{n-1} X$$

$$B^n = b^n 2^{n-1} Y$$

If the same values that were inputted for $A^n=aX$ to $B^n=bY$, the resultant values will be exactly the same with 'X' being 'Y':

$$B^2 = \left(1^2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^2 \qquad B^3 = \left(1^3\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^3 \qquad B^4 = \left(1^4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^4$$

$$= 1\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix} \qquad = 1\begin{bmatrix}4 & -4\\-4 & 4\end{bmatrix} \qquad = 1\begin{bmatrix}8 & -8\\-8 & 8\end{bmatrix}$$

$$= 2\mathrm{Y} \qquad = 4\mathrm{Y} \qquad = 8\mathrm{Y}$$

The expression for $(A+B)^n$ can be found by inputting different values in the expression for $A^n=aX$ and $B^n=bY$. Therefore by inputting different values or same values for the constants 'a' and 'b' raised to the power of 'n'

If we input the constants with the same values like a=1 and b=1 raised to the power of n=2

$$(\mathrm{A}+\mathrm{B})^n = [(a^n 2^{n-1}\mathrm{X}) + (b^n 2^{n-1}\mathrm{Y})]^n$$

$$= [(1^2 2^{2-1}\mathrm{X}) + (1^2 2^{2-1}\mathrm{Y})]^2$$

$$= \left(1\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} + 1\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2$$

$$= \begin{bmatrix}4 & 0\\0 & 4\end{bmatrix} \qquad \therefore\ 4\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 4\mathrm{I}$$

The resultant value for this expression contains an identity matrix when both of the values constants 'a' and 'b' are the same; however, there are some limitations. If the constants 'a' and 'b' contained different values such as (a=3 b=-12) the resultant matrix would differ, therefore the Identity Matrix cannot be used.

Let a=-3, b=5 and n=3

$$(\mathrm{A}+\mathrm{B})^n = [(a\mathrm{X}) + (b\mathrm{Y})]^n$$

$$= [(-3\mathrm{X}) + (5\mathrm{Y})]^3$$

$$= \begin{bmatrix}392 & -608\\-608 & 392\end{bmatrix} \qquad \therefore 8\begin{bmatrix}49 & -76\\-76 & 49\end{bmatrix}$$

Therefore the expression for $(A+B)^n$ would be:

$$2^n = \textit{the individual numbers in the matrix raised to the power of 'n', not counting the zeros.}$$

$2^n = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

If I let a=5, b=-7 and n =3 with the expression $(\mathrm{A}+\mathrm{B})^n = [(a\mathrm{X}) + (b\mathrm{Y})]^n$ than the expression where $2^n = \textit{the individual numbers in the matrix raised to the power of 'n', not counting the zeros}$, will work as well.

I used a calculator to calculate this expression:

$(\mathrm{A}+\mathrm{B})^n = [(5\mathrm{X}) + (-7\mathrm{Y})]^3$

$= \begin{bmatrix} -872 & 1972 \\ 1872 & -872 \end{bmatrix}$ $\therefore 8\begin{bmatrix} -109 & 234 \\ 234 & -109 \end{bmatrix}$

The general statement:

$2^n = \textit{the individual numbers in the matrix raised to the power of 'n', not counting the zeros}$

$2^n = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

n =3

$2^3\begin{bmatrix} -109 & 234 \\ 234 & -109 \end{bmatrix} = \begin{bmatrix} -872 & 1972 \\ 1872 & -872 \end{bmatrix}$

In this expression there are some limitations where ‘n’ cannot equal zero nor a negative value.

The expression $M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$ can also be derived into $M = A + B$ and $M^2 = A^2 + B^2$. By proving the expression $M = A + B$, A and B needs to be substituted for ɑX and ɑY as well as keeping the constants as a variable. This will prove the expression $M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$ through $M = A + B$.

$M = aX + bY$

$$M = a\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} + b\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$$
$$M = \begin{bmatrix}a & a\\a & a\end{bmatrix} + \begin{bmatrix}b & -b\\-b & b\end{bmatrix}$$
$$M = \begin{bmatrix}a+b & a-b\\a-b & a+b\end{bmatrix}$$

Now I am going to substitute A and B for [illegible] and [illegible] for the expression $M^2 = A^2 + B^2$.

$$M^2 = (aX)^2 + (bY)^2$$
$$\begin{bmatrix}a+b & a-b\\a-b & a+b\end{bmatrix}\begin{bmatrix}a+b & a-b\\a-b & a+b\end{bmatrix} = \left(a\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^2\left(b\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^2$$

$$\begin{bmatrix}(a+b)(a+b)+(a-b)(a-b) & (a+b)(a-b)+(a-b)(a+b)\\(a-b)(a+b)+(a+b)(a-b) & (a-b)(a-b)+(a+b)(a+b)\end{bmatrix} = a^2\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}b^2\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$$

$$\begin{bmatrix}(a^2+2ab+b^2)+(a^2+2ab+b^2) & (a^2-b^2)+(a^2-b^2)\\(a^2-b^2)+(a^2-b^2) & (a^2+2ab+b^2)+(a^2+2ab+b^2)\end{bmatrix} = \begin{bmatrix}2a^2 & 2a^2\\2a^2 & 2a^2\end{bmatrix} + \begin{bmatrix}2b^2 & -2b^2\\-2b^2 & 2b^2\end{bmatrix}$$

$$\begin{bmatrix}2a^2+2b^2 & 2a^2-2b^2\\2a^2-2b^2 & 2a^2+2b^2\end{bmatrix} = \begin{bmatrix}2a^2+2b^2 & 2a^2-2b^2\\2a^2-2b^2 & 2a^2+2b^2\end{bmatrix}$$

Therefore the general statement that expresses M^n in terms of [illegible] and [illegible] could be expressed as:

$$M^n = (aX)^n + (bY)^n$$

Test the validity of your general statement by using different values of [illegible] and [illegible]

First I am going to use the general statement $M^n = (aX)^n + (bY)^n$ and then prove this general statement by using the expression $M = \begin{pmatrix}a+b & a-b\\a-b & a+b\end{pmatrix}$.

Let [illegible]=3, [illegible]=4, [illegible]=4 $M^n = (aX)^n + (bY)^n$ $M^4 = (3X)^4 + (4Y)^4$ $= \left(3^4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^4\right) + \left(4^4\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}^4\right)$ $= \left(81\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}\right) + \left(256\begin{bmatrix}8 & -8\\-8 & 8\end{bmatrix}\right)$ $= \begin{bmatrix}2696 & -1400\\-1400 & 2696\end{bmatrix}$	$M^n = \begin{pmatrix}a+b & a-b\\a-b & a+b\end{pmatrix}^n$ $M^4 = \begin{pmatrix}3+4 & 3-4\\3-4 & 3+4\end{pmatrix}^4$ $= \begin{pmatrix}7 & -1\\-1 & 7\end{pmatrix}^4$ $= \begin{pmatrix}2696 & -1400\\-1400 & 2696\end{pmatrix}$ $= 8\begin{pmatrix}337 & 175\\175 & 337\end{pmatrix}$

$= 8\begin{bmatrix}337 & 175\\175 & 337\end{bmatrix}$	
Let a=-1, b= 0.5, n=2 $M^n = (aX)^n + (bY)^n$ $M^2 = (-1X)^2 + (0.5Y)^2$ $= \left(-1^2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}^2\right) + \left(0.5^2\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}^2\right)$ $= \left(1\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}\right) + \left(0.25\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}\right)$ $= \begin{bmatrix}2.5 & 1.5\\1.5 & 2.5\end{bmatrix}$ $= 5\begin{bmatrix}0.5 & 0.3\\0.3 & 0.5\end{bmatrix}$	$M^n = \begin{pmatrix}a+b & a-b\\a-b & a+b\end{pmatrix}.^n$ $M^2 = \begin{pmatrix}-1+0.5 & -1-0.5\\-1-0.5 & -1+0.5\end{pmatrix}^2$ $= \begin{pmatrix}-0.5 & -1.5\\-1.5 & -0.5\end{pmatrix}^2$ $= \begin{pmatrix}2.5 & 1.5\\1.5 & 2.5\end{pmatrix}$ $= 5\begin{pmatrix}0.5 & 0.3\\0.3 & 0.5\end{pmatrix}$

By finding if there are any limitations within this expression: $M^n = (aX)^n + (bY)^n$, I am going to change the constants 'a' and 'b' as well as power 'n' into , decimals, fractions and negative exponents.

The first example I am going to prove that this general statement has some limitations:

Let a=0.3 b=0.32 and n=-3

$$M^{-3} = (0.3X)^{-3} + (0.32Y)^{-3}$$
$$M^{-3} = \left(0.3\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^{-3} + \left(0.32\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^{-3}$$

As I plug this equation into my calculator it comes up with: (error domain). As a result this equation cannot be solved because the matrix cannot be raised to a negative power. Therefore the limitation in this expression is that 'n' cannot be a negative number.

In this second example I am going let 'n' equal to a positive integer and the constants a and b equal zero:

Let $a = 0 \;\; b = 0 \;\; n = 3$

$$M^3 = (0X)^3 + (0Y)^3$$
$$M^3 = \left(0\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)^3 + \left(0\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\right)^3$$
$$= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

The limitation for the expression $M^n = (aX)^n + (bY)^n$ is that 'n' cannot contain a negative exponent nor a decimal value or a fraction because if we multiply an exponent raised to a negative number it would make the value flip. However, both of the constants ‘a’ and ‘b’ can equal to any set of real numbers. Therefore the limitations and scope are:

$n \in Z^+$
$a \,\&\, b \in \mathbb{R}$.

Use an algebraic method to explain how you arrived at your general statement.

The general statement that came from $M = A + B$ and $M^2 = A^2 + B^2$ is $M^n = (aX)^n + (bY)^n$. This general statement should equal to $M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$.
To prove that this general statement equals to $M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$, I am going to expand the equation $M^n = (aX)^n + (bY)^n$ by using only variables:

$M^2 = (aX)^2 + (bY)^2$ $= \left(a\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right)^2 \left(b\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right)^2$ $= a^2\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} b^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ $= \begin{bmatrix} 2a^2 & 2a^2 \\ 2a^2 & 2a^2 \end{bmatrix} + \begin{bmatrix} 2b^2 & -2b^2 \\ -2b^2 & 2b^2 \end{bmatrix}$ $= \begin{bmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{bmatrix}$	$M^2 = \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}^2.$ $= \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix} \begin{bmatrix} a+b & a-b \\ a-b & a+b \end{bmatrix}$ $= \left[\begin{bmatrix} (a+b)(a+b)+(a-b)(a-b) & (a+b)(a-b)+(a-b)(a+b) \\ (a-b)(a+b)+(a+b)(a-b) & (a-b)(a-b)+(a+b)(a+b) \end{bmatrix}\right]$ $= \begin{bmatrix} (a^2+2ab+b^2)+(a^2-2ab+b^2) & (a^2-b^2)+(a^2-b^2) \\ (a^2-b^2)+(a^2-b^2) & (a^2+2ab+b^2)+(a^2-2ab+b^2) \end{bmatrix}$ $= \begin{bmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{bmatrix}$

$$\begin{bmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{bmatrix} = \begin{bmatrix} 2a^2+2b^2 & 2a^2-2b^2 \\ 2a^2-2b^2 & 2a^2+2b^2 \end{bmatrix}$$

Therefore the equation $M^n = (aX)^n + (bY)^n$ equals with $M = \begin{pmatrix} a+b & a-b \\ a-b & a+b \end{pmatrix}$.

However, the equation $M^n = (aX)^n + (bY)^n$ would not work unless it is proven by the binomial theorem.

$$\sum_{k=0}^{n} \frac{n}{k} x^{n-k} y^k$$

$$M^n = (A+B)^n$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$=\begin{bmatrix}2a^2 & 2a^2\\2a^2 & 2a^2\end{bmatrix}+2\begin{bmatrix}ab-ab & -ab+ab\\ab-ab & -ab+ab\end{bmatrix}+\begin{bmatrix}2b^2 & -2b^2\\-2b^2 & 2b^2\end{bmatrix}$$

$$=\begin{bmatrix}2a^2 & 2a^2\\2a^2 & 2a^2\end{bmatrix}+2\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}+\begin{bmatrix}2b^2 & -2b^2\\-2b^2 & 2b^2\end{bmatrix}$$

$$=\begin{bmatrix}2a^2 & 2a^2\\2a^2 & 2a^2\end{bmatrix}+\begin{bmatrix}2b^2 & -2b^2\\-2b^2 & 2b^2\end{bmatrix} \qquad \therefore A^2+B^2$$

This calculations tells us that AB must equal to zero for this equation to work $AB=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$. As said before the only way the equation $M^n=(aX)^n+(bY)^n$ works is because $2(aX)(bY)$ equals to a zero matrix: $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$.

In the end, the expression $M^n=(aX)^n+(bY)^n$ can be substituted into a different equation where $(aX)^n+(bY)^n$ can be replaced as $[(a^n2^{n-1}X)+(b^n2^{n-1}Y)]^n$.

$$M^n=(aX)^n+(bY)^n$$

$$M^n=(a^n2^{n-1}X)^n+(b^n2^{n-1}Y)^n$$