Math I.B Internal Assessment: SL Type 1

Stellar Numbers

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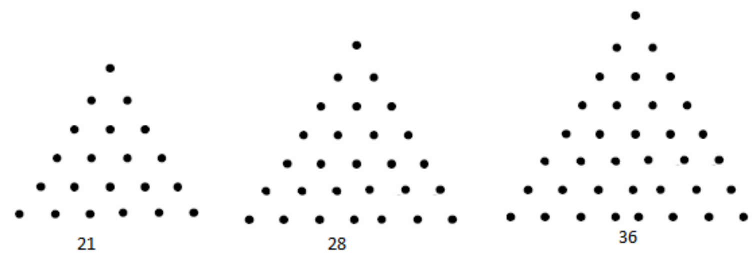
TRIANGULAR NUMBERS

TRIANGULAR NUMBERS WITH THREE MORE TERMS



GENERAL STATEMENT: N^{TH} TRIANGULAR NUMBERS IN TERMS OF N.

The differences between the sequences of terms:

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1 ↘ ↙ 3 ↘ ↙ 6 ↘ ↙ 10 ↘ ↙ 15 ↘ ↙ 21 ↘ ↙ 28 ↘ ↙ 36

2 ↘ ↙ 3 ↘ ↙ 4 ↘ ↙ 5 ↘ ↙ 6 ↘ ↙ 7 ↘ ↙ 8

1 1 1 1 1 1

According to Finite Differences if the 3 rd difference of a pattern is 1, then the general term is a quadratic equation: $y = ax^2 + bx + c$

X	Y	Y= number of dots on triangle X= number of dots on 1 side of triangle
0	0	
1	1	
2	3	
3	5	

Thus to find the general term we must first find the value of 'c':

$$0 = a(0)^2 + b(0) + c$$

$$0 = 0 + 0 + c$$

$$c = 0$$

$$1 = a(1)^2 + b(1) + 0$$

$$3 = a(2)^2 + b(2) + 0$$

$$\therefore$$

In order to find the values of 'a' and 'b' we must solve a quadratic using simultaneous equations, thus:

$$2 = a + b$$

$$3 = 4a + 2b$$

$$\therefore$$

$$4 = 2a + 2b$$

$$3 = 4a + 2b$$

$$\therefore$$

$$1 = -2a$$

$$a = \frac{1}{2}$$

Substitute the value of 'a' of one equation:

$$4\left(\frac{1}{2}\right) + 2b = 3$$

$$\therefore$$

$$2 + 2b = 3$$

$$\therefore$$

$$2b = 1$$

$$\therefore$$

$$b = \frac{1}{2}$$

Therefore $a = \frac{1}{2}$, $b = \frac{1}{2}$ and 'c'=0

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To find the General statement that represents the n th triangular number in terms of 'n', we substitute the value of 'y' by U_n and the value of 'x' by n, thus:

$$U_n = \frac{1}{2}n^2 + \frac{1}{2}$$

$$\therefore$$

$$U_n = \frac{n^2 + n}{2}$$

$$\therefore$$

$$U_n = \frac{n(n+1)}{2}$$

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STELLAR NUMBERS

NUMBER OF DOTS TO S_6 STAGE

S_1	S_2	S_3	S_4	S_5	S_6
1	13	37	73	121	181

Thus, using finite difference:

$$\begin{array}{ccccccccccc} 1 & & 13 & & 37 & & 73 & & 121 & & 181 \\ & 12 & & 24 & & 36 & & 48 & & 60 & \\ & & 12 & & 12 & & 12 & & 12 & & \end{array}$$

The most obvious pattern is that the 1 st row all numbers and odd and the second row all are even.
Also all these numbers are some multiples of 12 + 1, for example: 12 also turns out to be the half of 6.

$$12(0) + 1 = 1$$

$$12(1) + 1 = 13$$

$$12(3) + 1 = 37$$

$$12(6) + 1 = 73$$

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6 STELLAR NUMBER AT STAGE S_7

1 + 1 (12) + 2 (12) + 3 (12) + 4(12) + 5(12) + 6(12) = 253

GENERAL STATEMENT FOR 6 STELLAR NUMBER AT STAGE S_N IN TERMS OF N

If you notice the multiples are triangular numbers thus the general statement will, in some way contain the general s tatement of the triangle numbers. Because the final difference of the stellar numbers is 12 the multiple is Thus:

$$12\left(\frac{n-1}{2}(n)\right)+1$$

$$6n(n-1)+1$$

$$U_n = 6n^2 - 6n + 1$$

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4 STELLAR NUMBER

NUMBER OF DOTS AT 4 STELLAR

S_1	S_2	S_3	S_4	S_5	S_6
1	9	25	49	81	121

4 STELLAR NUMBER AT STAGE S_7

1 + 1 (8) + 2 (8) + 3 (8) + 4(8) + 5(8) + 6(8) = 169 (8 being multiple of 2 and 4)

GENERAL STATEMENT FOR 4 STELLAR NUMBER AT STAGE S_N IN TERMS OF N

Thus the pattern for 4 stellar, some multiples of 8 with triangular numbers:

$$8(1) + 1 = 1$$

$$8(3) + 1 = 9$$

$$8(6) + 1 = 25$$

Thus:

$$8\left(\frac{n-1}{2}(n)\right) + 1$$

$$4n(n-1) + 1$$

$$U_n = 4n^2 - 4n + 1$$

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8 STELLAR NUMBER

NUMBER OF DOTS AT 8 STELLAR

S_1	S_2	S_3	S_4	S_5	S_6
1	17	49	97	161	241

8 STELLAR NUMBER AT STAGE S_7

1 + 1 (16) + 2 (16) + 3 (16) + 4(16) + 5(16) + 6(16) = 337 (Taking the double of 8)

GENERAL STATEMENT FOR 8 STELLAR NUMBER AT STAGE S_N IN TERMS OF N

Thus the pattern for 8 stellar, some multiples of 16 with triangular numbers:

$$16(1) + 1 = 1$$

$$16(3) + 1 = 9$$

$$16(6) + 1 = 25$$

Thus:

$$16\left(\frac{n-1}{2}(n)\right) + 1$$

$$8n(n-1) + 1$$

$$U_n = 8n^2 - 8n + 1$$

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GENERAL STATEMENT IN TERMS OF *P* AND *N*

The general statement is:

$$p\left(\frac{n-1}{2}(n)\right)+1$$

$$pn(n-1)+1$$

$$\mathrm{Sn} = p(n)^2 - p(n) + 1$$

VALIDITY

1. Find the number of dots, in a star with 10 vertices when n = 5

$$S(5) = 10(5)^2 - 10(5) + 1$$

$$S(5) = 250 - 50 + 1$$

$$S(5) = 201$$

2. Find the number of dots, in a star with 1 ver tices when n = 5

$$S(1) = 10(1)^2 - 10(1) + 1$$

$$S(1) = 10 - 10 + 1$$

$$S(1) = 1$$

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3. Find the number of dots, in a star with 3 vertices when n = 3.5

$$S(3.5) = 3(3.5)^2 - 3(3.5) + 1$$

$$S(3.5) = 36.75 - 10.5 + 1$$

$$S(3.5) = 26.25$$

PROFF

1. 1 + 1 (20) + 2 (20) + 3 (20) + 4(20) = 201
2. 1 = 1
3. –

LIMITATIONS OF THE GENERAL STATEMENT AND ARRIVAL AT GENERAL STATEMENT

Due to the fact that each vertex on the star has a triangular pattern on it, it is easy to determine the formula: you multiply the general statement of the triangular pattern by the number of vertices that the star has and then add 1. The reason for this addition is so that you can add the dot in the middle of the star.

S_3

The next difficulty faced is: if I multiply by the number of vertices the points on the closest will also be multiplied twice, and this could not be.

But then how do we add the inside star to the final result. In the example below the star on the inside has 12 dots (excluding the point in middle), and the outside star if you count the dots on the inside twice you will count a total of 12 dots, thus covering the dots of the inside star –

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This is the same for all other stellar shapes.