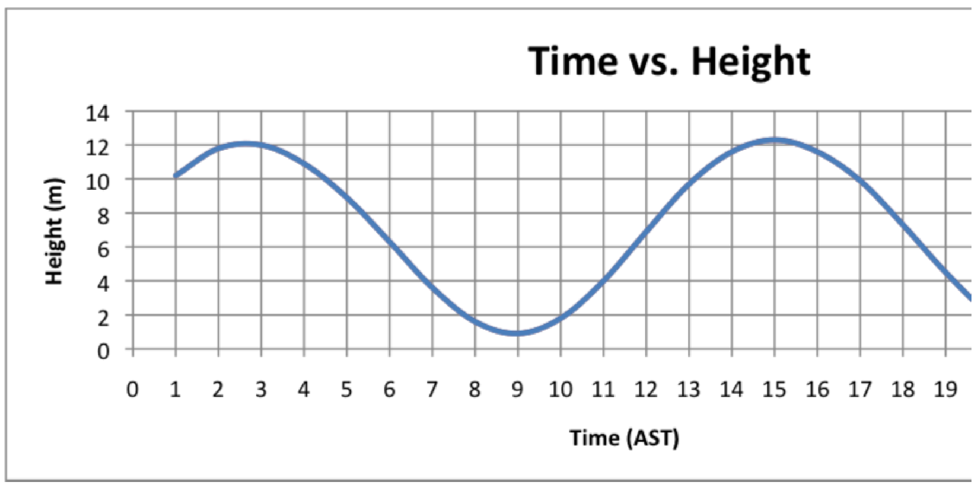
felpTide Modeling

1. Using Microsoft Excel, plot the graph of time versus height. Describe the result using the terminology you have acquired in the study of circular functions.

In order to come up with a plot graph for the bay of Fundy in Noca Scotia, Cana it was necessary to use Microsoft Excel and the data from www.lau.chs-shc.dfo-mpo.gc.ca.com. In order to plot the graph in excel all of the data had to be entered in the spreadsheet.

0:00	7.5
1:00	10.2
2:00	11.8
3:00	12
4:00	10.9
5:00	8.9
6:00	6.3
7:00	3.6
8:00	1.6
9:00	0.9
10:00	1.8
11:00	4
12:00	6.9
13:00	9.7
14:00	11.6
15:00	12.3
16:00	11.6
17:00	9.9
18:00	7.3
19:00	4.5
20:00	2.1
21:00	0.7
22:00	0.8
23:00	2.4

Once that was done a scatter graph was created. Once it was done it was possible to analyze the graph. The first thing that is possible to notice is that this is a periodical graph. However, the graph is not completely periodical since the lines don't always have the same height. Sometimes it is greater or lower. This can be attributed to the fact that it is a real life situation. Therefore, it would not be expected for it to be perfect. However, the graph is still periodical since it follows the same shape throughout.



2. Use your understanding of circular functions and their transformations to develop a mathematical model for the behavior noted in the graph. Explain your method and reasoning in detail.

In order to come up with the equation I had to come up with a series of averages. This was necessary since the graph is not completely periodical.

In order for me to find the vertical translation of the graph, meaning how much the graph was moved up I had to find the graph's sinusoidal axis. In order to find it I had to find the graphs height for both crests and do an average of it. The highest point in the first crest was 12 meters. For the second crest it was 12.3 meters. Therefore, my average was 12.15 meters. My next step was to find the lowest points, the trough. For the first one was 0.90 meters and for second one it was 0.70. Therefore, my average was 0.80 meters. Finding the average between the through and the crest gave me the sinusoidal axis. The sinusoidal axis ended up being 6.475 meters.

Calculations: Vertical Translation d= 6.475

12+12.3=24.3

,24.3-2.=12.15

0.90+0.70=1.6

,1.6-2.=1.6

0.8+12.15=12.95

,12.95-2.=6.475

My next step was to find the graphs vertical stretch also known as the graphs amplitude; or the distance between the sinusoidal axis and the crest. In order to determine, it was necessary to subtract the sinusoidal axis from the average crest. My answer for the vertical stretch was than 5.675.

Calculations: Vertical Stretch a= 5.675

12.15−6.475=5.675

The next step was to find the graphs horizontal dilation also known as the graphs period. In order to determine it is necessary to find two points exactly on the same y coordinate and different x axis. It is also important to notice that they need to be in the same position of the period. For reference it was measure the top two points of the crests, and the two low points of the trough. Once the two points are measure it is necessary to find the difference between the two points. In this case the difference was 12. Once it is found it is needed to plug it in the formula*(□□□□□□□□□□□□□□□ □□□□□□□□□□□□□□□□□□=,2*□□-□.), and solve it for b. That value will be the graphs period. In this case after using the formula the answer that was acquired was*,*□*-6.*.

Horizontal Dilation b=*,*□*-6.*.

15−3=12 Crest

21−9=12 Through

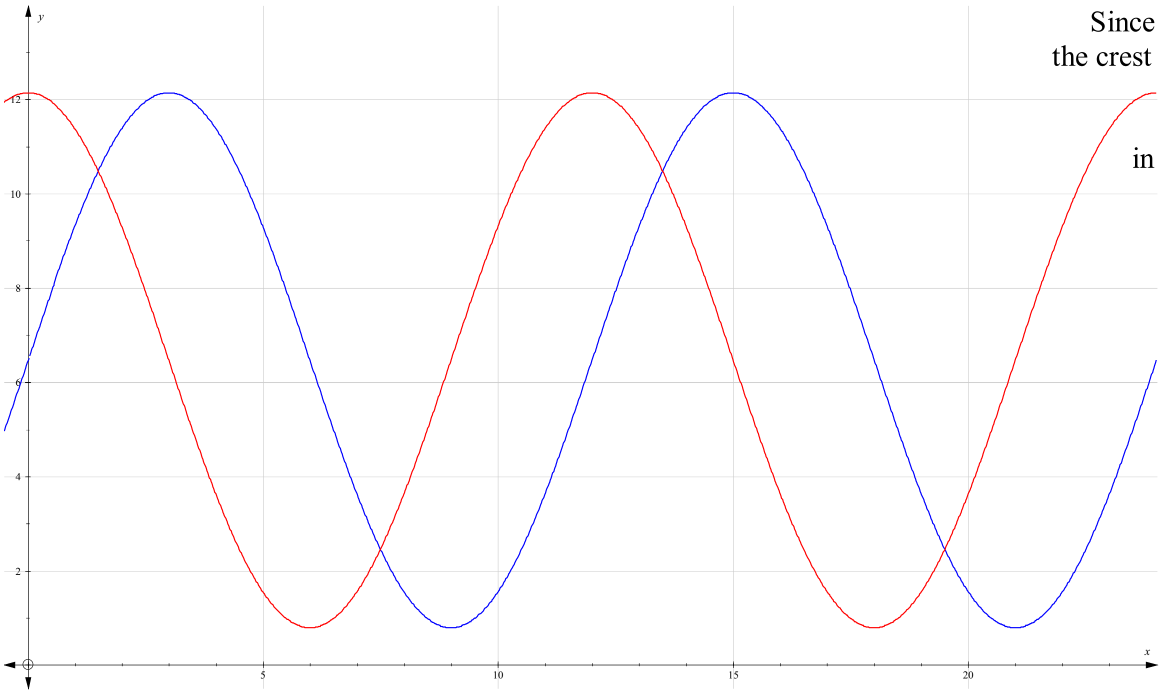
12=,2□□-□.

□*=12*

Finally, the last step in order to determine a model for the graph is its horizontal translation. In order to do the last step, it is necessary to determine if the graph is cosine or sine.

First it will be assumed it is cosine. Once that is done it is necessary to calculate how much the graph was moved from its original position. In order to determine it, the difference between the standard cosine graph and the translated one needs to be found. In order to do so two of the same points need to be analyzed for example two crests. Once the comparison is made it is possible to see that the graph was moved 3 units to the right.

Horizontal Translation c=3

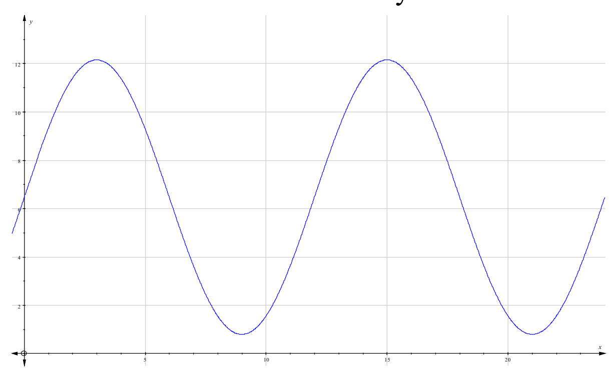


Comparisson between original cosine and sine. Standard Cosine is blue, and translated is blue.

Since the crest in the standard graph was at 0 and in the excel graph it is at 3 it is possible to assume it was moved 3 units to the right.

All of the vales above can also be used for sine since cosine and sine are only a horizontal translation of each other. No transformation would be required because as the graph is half way up it crosses the y- axis and it also happens in the transformed.

Horizontal Translation c=0



Sine graph: It shows that no horizontal translation is needed.

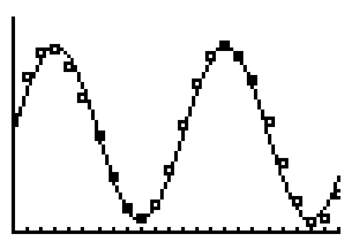
Therefore the following models are possible to come up with after the analysis:

Cosine:

□=5.675,cos-,,□-6.,□−3...+ 6.475

Sine:

□=5.675,sin-,,□-6.,□...+6.475



The graph to the side further proves that this is an accurate model because it shows that the model goes through most of the points of the original excel graph.

The line is a the model. The squares are the points of the excel data.

3. Create a graph of your model, and comment on how well the model fits the data making note of what you consider is a reasonable degree of accuracy in this project.

It is possible to conclude that both of the models are good for the data for a couple of reasons. The first one is the fact that both models are the same when graphed. Therefore, it shows that they are the same with the only difference that cosine has the horizontal translation; therefore for future problems the model for cosine will be used.

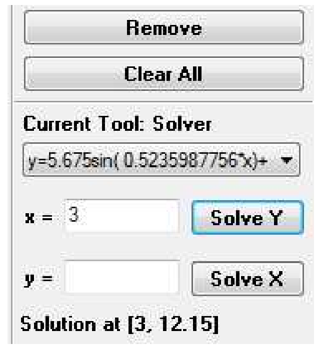


Figure 1: Crest Proof

The second reason is because when the graph from excel is compared with one of the models that was created using Graphing Package it is possible to see that they have almost the same trough (Figure 2). The graph from excel when using the average has a trough of 0.80 meters. When we look at the trough of the model graphs from Graphing Package it shows 0.80 therefore proving that for that particular point the graph is correct. Furthermore, it shows that it is a good model. Another proof is the graphs crest (Figure 1). Following the excel version and the average the crest was supposed to be 12.15 and in the graph it was 12.15 once again showing the models accuracy. In conclusion, that shows that the graph was both precise and accurate for those particular points.

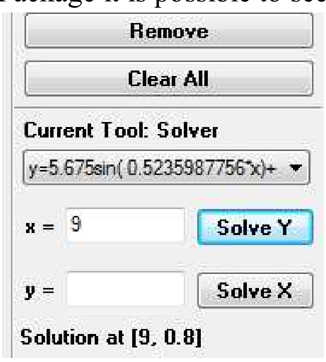


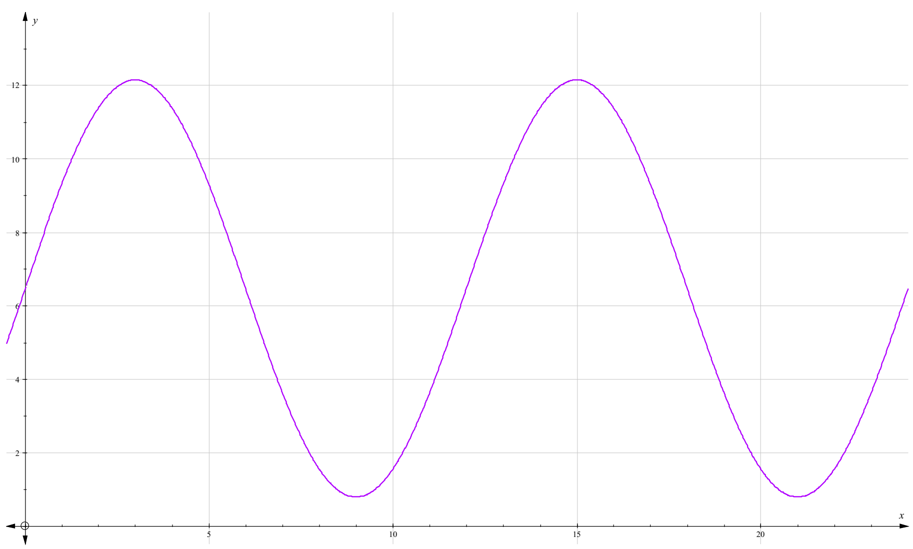
Figure 2: Through Proof

Unfortunately the models are not able to show the oscillation of heights that the excel graph presents. It rather shows a perfectly periodical graph that probably would not exist in nature under normal circumstances. After a more extensive research it was possible to notice that the model is only accurate for the average of the data exactly at the crest and through. At the other points there is a difference between the points. The difference oscillates from a difference of 0.087 to a high 0.725. Therefore in order to have a precise measurement the best degree of accuracy should be to the nearest whole number. Unluckily even then the points will not be perfect considering it would not work for the points (0,12) and (7,19). A possible solution for that would be to round up the model value to the nearest whole number. Further an unorthodox method in order to get the best degree of accuracy would be to round up the model value and to round down the average value to the nearest whole number. That would work for all numbers but the through and crest. That is probably due the fact that when the model was come up with those two points were the ones that were specifically looked at. And it was assumed that the other points in the graph would consequently be as accurate as those two specific problems.

All that work can be seen when analyzing the chart below. The points were grouped by two because those are the frequencies in which the period repeats itself, further in the model the answers were the same for both points. The values written for the model are the y coordinate values that are obtained when a one of the two points is plugged in for□. For the average they are the averages of two points from the original data that were used to create the excel graph. An average was used rather than a specific points because in order to come up with the model, averages of the through and crest were used, therefore in order to better compare and be consistent with the method the averages were used. Finally the difference is |Average – Graph| and it serves to show in numbers the difference between the model and the real one. However, when the percent error is found as whole for all of the points in the model against the average using the formula ,|□□□□□□□□ □□□□□□□□ □□□□□−□□□□□□□□□□□□□□□□□ □□□□□□□□□□|-□□□□□□□□ □□□□□.×*100* it is only %3.64 showing that the model is accurate.

Points	0,12	1,13	2,14	3,15	4,16	5,17	6,18	7,19	8,20	9,21	10,22	11,23
Model	6.475	9.313	11.39	12.15	11.39	9.313	6.475	3.638	1.56	0.8	1.16	3.638
Average	7.2	9.95	11.7	12.15	11.25	9.4	6.8	4.05	1.85	0.8	1.3	3.2
Difference	0.725	0.637	0.31	0	0.14	0.087	0.325	0.412	0.29	0	0.26	0.438

□=*5.675*,sin-,,□-*6*.,□…+*6.475* and □=*5.675*,cos-,,□-*6*.,□−*3*…+ *6.475*

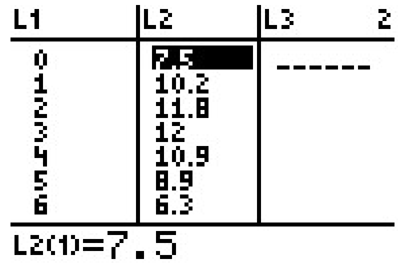


Only one color is shown because graphs overlap each other since they are the same.

4. Use the regression feature of your graphing calculator to develop a best-fit function for this data. Compare this model with the one you developed analytically.

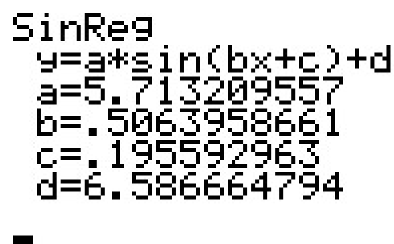
In order to come up with a best- fit line for the data, so that it is possible to see what the best model for the data would be it was used the regression feature of the TI 84. In order to use it all the data values for the data had to be inserted in the table (Figure 3). Once that was done a regression had to be found and once again the TI 84 was used. In order to find it a feature called SinReg was used.

Finally Graphing Package was used in order to graph the data and so that it could be compare to the sine model



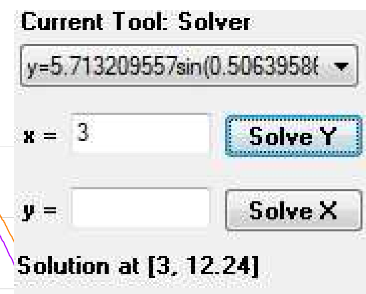
Data plugged in the TI 84

Once that was done in order to better compare the model with the best- fit graph it was decided to plug in values for □ so that it the models could better be compared. When 3 was plug in for the sine model it was obtained 12.15 as the y values, and that would be expected since it was the average obtained in step 2.



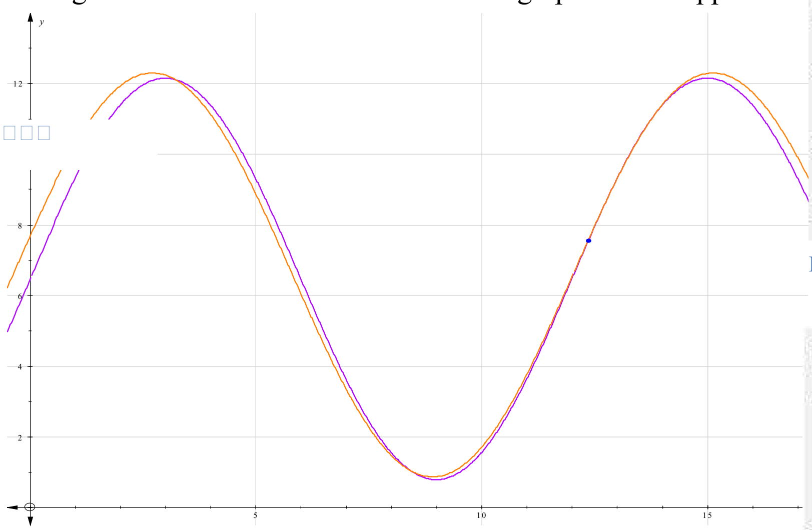
Sine Regression

However, when 3 was plug in the best- fit equation 12.24 was obtained. The difference between both was 0.091. In this case the sine model graph seems to have been better since according to the table used to create the excel graph it was supposed to be

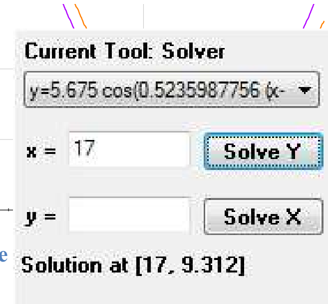


Best fit model at □=3

Best fit model at □=□□□□

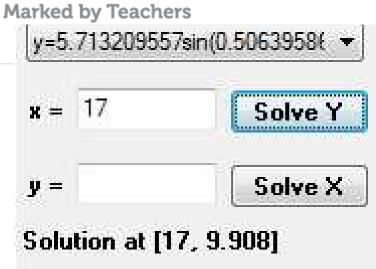


Comparison between best fit model and model. Orange is the best fit model and the



Sine model at □=□□□□

12. In order to further prove, 17 was plug in for□. However, this changed things. When it was plugged in for the sine model the value for y was 9.312. Finally, when it was plug in for the best- fit graph the value for y was 9.908. According to the data it was supposed to be 9.9. Therefore, this time the best- fit graph was better.



After further Finally a conclusion can be arrived at. The best-fit graph is better than the sine model for all points but the through and the crest. This can be explained by the fact that the model was created mainly trying to make the through and the crest as close as possible, in the other hand the best fit model was created by the calculator in a way so that all the points would be as close as possible to the real one while the sine model sacrificed the other points in order to make sure the through and the crest were as accurate as possible. Therefore, that makes the best fit line graph better in the overall than the sine model.

It is also possible to be sure that the best fit model is better than the sine model when looking at their percent error in relation to the original excel data. In the table below the points were not grouped together since for the best fit graph the values for 0 and 12 are not the same; therefore a point by point comparison was necessary. Using the same method as in number 3 with the percent error it is possible to determine that the sine models error point by point is %1.83 and for the best fit graph it was %0.25. Once again this shows that the model that the calculator came up with is better than the one that was developed analytically.

Points	0	1	2	3	4	5	6	7	8	9	10	11
Model	6.475	9.313	11.39	12.15	11.39	9.313	6.475	3.638	1.56	0.8	1.16	3.638
Best fit	7.697	10.28	11.929	12.241	11.134	8.885	6.06	3.367	1.482	0.8782	1.708	3.762
Data	7.5	10.2	11.8	12	10.9	8.9	6.3	3.6	1.6	0.9	1.8	4.0
Points	12	13	14	15	16	17	18	19	20	21	22	23
Model	6.475	9.313	11.39	12.15	11.39	9.313	6.475	3.638	1.56	0.8	1.16	3.638
Best fit	6.525	9.303	11.4	12.29	11.75	9.908	7.237	4.402	2.116	0.9517	1.202	2.804
Data	6.9	9.7	11.6	12.3	11.6	9.9	7.3	4.5	2.1	0.7	0.8	2.4

5. Apply the model you developed on the calculator to the following two situations:

a) A sailor launched his boat at 7:45 on December 27, 2003. Determine the height of the tide at that time using both analytical and graphical methods. Explain your methods and reasoning in detail.

In order to solve the problem of what was the height of the tide when the sailor left the harbor at 7:45 both a analytical and a graphical method was used.

In order to solve it graphically the software Graphing Package was used. First the equation for the best fit line was plugged in,

For further problems the equation below will be expressed in the b(x+c) form rather than the bx+c form.

□=*5.713209557*sin(*0.5063958661* θ+*0.195592963*)+6.586664794

Like this:

□=*5.713209557*sin(*0.5063958661* ,θ+0.3862451811.)+6.586664794

Once it was graphed it was then used the solver tool of the software. Since the time was 7:45 the proportion that *,45-60.=,75-100.* in order to change the hour to 7.75. That value was than plugged in for x and the solver said the answer would 1.846. Considering the fact that the hour given only had 3 significant figures the best answer would 1.85 meters. **(Place picture).**

The next step would be to find the solution using an analytical method. In this case since the θ value was given the equation would simply need to be solved by y. Therefore 7.75 was simply plug in the equation. **(Show solving for period in the best fit line?)**

□=*5.713209557,*sin-*,0.5063958661,7.75+0.3862451811..+6.586664794.*

□=*5.173209557,*sin-*,0.5063958661,8.136245181..+6.586664794.*

□=*5.173209557,*sin-*,4.120160925.+6.586664794.*

□=*5.173209557·−0.08296990146+6.586664794*

□=*−4.74024434+6.586664794*

□=*1.846420454*

The answer is the same achieved in the graphical method, therefore, confirming the answer. Furthermore, as a way of showing that no procedural errors were made the equation was plugged in the Ti 84, and the answer given was the same as the graphical method. **(Calculator Picture)** Once again it is just important to note that the answer probably should simply have three significant figures since the information that is given only has that, further, the analytical answer gives more answers than the graphical one using the software, and with 3 significant figures both answers would be the same.

b) The sailor docked his boat in the afternoon of the same day when the height of the tide was 5.2m. Determine the time of day the sailor returned, using both analytical and graphical methods. Explain your methods and reasoning in detail.

This time in order to figure out the time that the sailor came back considering the fact that the height of the tide was 5.2 meters it is given a y value, and it is needed to solve for θ.

In order to solve it graphically this time the calculator TI 84 was used. The best fit line equation was plugged in and a second equation was plugged in y=5.2. The second one was plugged in because that was the value of y that is needed in order to calculate x. Once it was graphed the intersect tool was used in order to find the solution. However, there are many solutions. But since the problem says that the sailor docked in the afternoon only one of the solutions happens after noon. The calculator said the intersection was at x= 18.709366. Since the problem asks the time of the day it is necessary to turn the x value into an hour. Therefore, the proportion *,0.709366-100.=,*□*-60.*needs to be used, and it gives the solution 0 .4256196. Since the problem talks about hours to the nearest second the solution will be rounded too only to the nearest second. Therefore the solution is 18:43.

In order to solve analytically several steps need to be taken and also the unit circle needs to be used. First it is necessary to set up the equation.

*5.2=5.713209557,*sin-*,0.5063958661,*□*+0.3862457811..+6.586664794.*

Than it is necessary to solve it algebraically

*−1.386664794=5.713209557,*sin-*,0.5063958661,*□*+0.3862457811...*

,−0.2427120483=sin-*,0.5063958661,*□*+0.3862457811...*

Once this step was achieved it was necessary to take the inverse sine in order to get rid of sine. In this step the calculator was set to degrees in order to get an answer. When the inverse cosine of -0.2427120483 was taken -14.04666311° was gotten. That point than was tried to be found in the excel graph (in this case was the best fit graph just to make it better "see friendly") the red dot in the picture. Like any other unit circle problem there was another solution. Using the unit circle it is possible to determine it is -165.9533369, therefore it was possible to see that it was out of the domain (*0≤0≤23)*, as it shown by the blue dot in the picture. Further considering the fact that the answer that is wanted to be achieved is in the afternoon it is possible to see how many times around the circle the answer is away. Also since the answer is in the down of the bump as it is shown by the green dot in the picture it is possible to see that in order to solve the problem it is better to use the point that is the blue one rather than the red. It is then possible to than see that the answer is 2 full rotations away from -165.9533369. Therefore meaning that if 720 is added to the answer the solution will be achieved. Once that is done the result achieved is 554.0466631. Now it is necessary to convert the answer back to radians by multiplying it by, □*-180.*. The answer achieved is 9.669938481. Now it is necessary to keep solving the problem algebraically in order to find the answer. **(Unit circle picture)**

9.669938481=,0.5063958661,□+0.3862457811..

19.09562202=□□+0.3862457811

□≈18.7096524

Like it was described in the graphical method, the conversion factor is needed to be used in order to figure out what □ is as an hour. After the conversion factor is used, the solution is .425619144. Also, for the same reasons of the graphical method the answer will be than 18:43. Therefore, when the tide was 5.2 meters considering he docked in the afternoon the time was 18:43.

6. Now, apply the model you developed on the calculator to the following two situations:

a) Ambitious sailors will launch their boats in the morning on an outgoing tide, specifically sometime during the interval for which the tide is decreasing from 8m to 4m. Determine the times between which a good sailor would have launched a boat on December 27, 2003using both analytical and graphical methods. Explain your methods and reasoning in detail.

In order to determine the time of the day that the sailors left the harbor considering that the tide was decreasing from 8m to 4m. In order to find the solutions graphically the same method that was used for 5b was used to find the solution graphically. (**Place solution).** The only difference is that this time it would be an interval of times rather than just one. The answers were 5.3239458 and 6.7454118. Therefore, when converting the answers to proper hours the answers were that the sailor lunched his boat from 5:19 – 6:45.

Analytically the same from 5b was used. The only difference is that it will not be needed to go around the circle in order to come up with the value. When the opposite sine of 0.2473802496 is found it is just needed to find the other solution for the same sine value, meaning it is needed to find the point in the opposite y axis, and that will be the value that will needed to be used.

*8=5.713209557,*sin-*,0.5063958661,□+0.3862451881..+ 6.586664794.*

*1.413335206=5.713209557,*sin-*,0.5063958661,□+0.3862451881...*

*0.2473802496=,*sin-*,0.5063958661,□+0.3862451881...*

□−0.2499755297=0.5063958661,□+0.3862451881.

The last operation on the left side was used in order to be able to find the other sine value in the unit circle.

2.891617124=0.5063958661,□+0.3862451881.
5.710191014= □□+0.3862451881
□≈5.323945825
Using the conversion factor it is possible to get 5:19 and that is the same as the graph.

*4=5.713209557,*sin-*,0.5063958661,□+0.3862451881..+ 6.586664794.*
*−2.586664794=5.713209557,*sin-*(0.5063958661.,□+0.3862451881.)*
*−0.4527516045=,*sin-*(0.5063958661.,□+0.3862451881.)*
*−0.4698489452=*0.5063958661(□*+0.3862451881*)
□*−0.4698489452=*0.5063958661(□*+0.3862451881*)

For the same reason as when 8 was plugged in for □ the last operation was necessary was needed in order to find the other value of sine.

-2.640413708=0.5063958661(□*+0.3862451881*)
2 □□ + -2.640413708=0.5063958661(□*+0.3862451881*)

In this case 2 □□ was added to the solution because the answer obtained was a negative value and just like in 5b it was necessary to add one rotation around the circle in order to get to the answer that was intended.

*3.611441599=*0.5063958661(□*+0.3862451881*)
7.131656952= □□+0.3862451881
□=6.745411771
Using the conversion factor presented in problem 5a it is possible to get 6:45 the same answer as the graph.

b) Lazy sailors will launch their boats in the afternoon after they've had lunch and a nap, specifically sometime from 14:00 to 16:00. Determine the tides between which a lazy sailor would have launched a boat on December 27, 2003 using both analytical and graphical methods. Explain your methods and reasoning in detail.

In order to come up with the graphical solution this it is necessary to graph the original best fit model shown in 5a. Once that is done it is needed to open the table of the graph. (**Table**). Once that is done it is needed to find the place that x is equal to 16 and write its y value. In this case it is 11.746 meters. And for 14 it is 11.4 meters.

To solve this problem analytically it is like problem 5a.

□*=5.713209557,*sin-*,0.5063958661,14+0.3862451811..+6.586664794.*
□*=5.713209557,*sin-*,0.5063958661,14.38624518..+6.586664794.*
□*=5.713209557,*sin-*(7.285135088)+6.586664794.*
□*=5.713209557·0.8425228559+6.586664794*
□*=4.813509633+6.586664794*
□*=11.40017443*

□*=5.713209557,*sin-*,0.5063958661,16+0.3862451811..+6.586664794.*
□*=5.713209557,*sin-*,0.5063958661,16.38624518..+6.586664794.*
□*=5.713209557,*sin-*,8.297926821.+6.586664794.*
□*=5.713209557·0.9030642159+6.586664794*
□*=5.159395109+6.586664794*
□*=11.7460599*

Finally, it is possible that the same answers were obtained both analytically and graphically. And so the height of the tides in which lazy sailors launched their boats was between 11.75 meters and 11.40 meters. In this problem only four significant figures were used because the problem gave the time with four significant figures.

7. **The table below lists the tide heights for December 28, 2003. Does your previous model fit the data? Why or why not? What modifications are needed? Confirm that your modified model fits the data.**

The first model does not fit the new data as can be seen by the picture (**Place picture).** Mainly it does not fit because apparently the period of the graph is different. It is possible to conclude that because the first time the graph goes up it almost follows the same points. However, than the points are more to the right than the original model. That indicates that there is no or very little horizontal translation and rather a change in period. Further the graphs crest seems to be too high compared with the new data. However, the through seems to be good.

When actual points are analyzed it is better possible to come up with the problems of the first model and the modifications necessary. The first is the fact that the first data the graph starts at 7.5 and the second at 5. Further in the first data the highest point was 12.3 and in the second data it is 11.6 explaining why the model goes over the data. However, the lowest point in the graph is .7 and for the second data it is one. These were the main differences, however, modifications still need to be made to make the model fit the new data.

In order to come up with the sinusoidal axis of the same method described in two was used. (**Put the equations).**

For the amplitdude the method is once again the same again as 2.

For the period the method is also the same.