

Introduction

This particular mathematics graph interpretation will discuss how graph of $y = f(x)$ differs from $y = |f(x)|$ and $y = f(|x|)$. The purpose of this project is to study the relationship among those functions. The guiding question is: How do they differ from one another and what patterns do they have?

Hypothesis

The function $y = |f(x)|$ will differ from the other function $y = f(x)$ by having all the parts of the function $y = f(x)$ below the x-axis, where negative units lie for y – axis, as a horizontal reflection. Because absolute value can only have positive value as a result, the function will also have positive value only. Therefore, when having an absolutely value graph, because only positive value exists, the negative value part of the graph will reflect upon x –axis. In addition, the original graph's y – intercept and range will have positive sign of the original value and be equal to or greater than zero respectively for absolute value function.

X - Intercept = x value when $y = 0$

Y – Intercept = y value when $x = 0$

Domain – from where to where x value reaches

Range – from where to where y value reaches

Asymptote – a virtual line that the graph never meets or intersects

Ex. $y = x^2 - 3x - 4$

(Done this way to find x- intercepts and y – intercepts for every functions below)

Y- Intercept

$$y = 0^2 - 3(0) - 4$$

$$y = -4$$

∴ Y-Intercept is -4 when $x = 0$

X- Intercept

$$0 = x^2 - 3x - 4$$

$$0 = (x + 1)(x - 4)$$

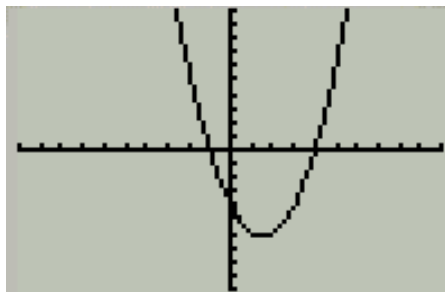
$$∴ x = -1 \text{ or } x = 4$$

∴ X- Intercept is -1 and 4
when $y = 0$

Comparing $y = f(x)$ and $y = |f(x)|$

Absolute value graph of $y = f(x)$ and $y = |f(x)|$

$$y = x^2 - 3x - 4$$



x - Intercept $x = -1$ and $x = 4$

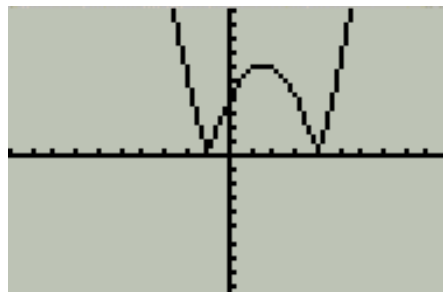
y - Intercept $y = -4$

Domain $x \in \mathbb{R}$

Range $y \geq -4.25$

Asymptote No asymptote

$$y = |x^2 - 3x - 4|$$



x - Intercept $x = -1$ and $x = 4$

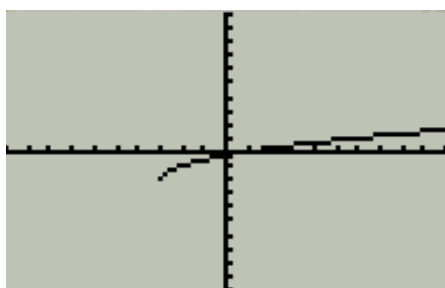
y - Intercept $y = 1.5$

Domain $x \in \mathbb{R}$

Range $y \geq 0$

Asymptote No asymptote

$$y = \sqrt{x+3} - 2$$



x - Intercept $x = 1$

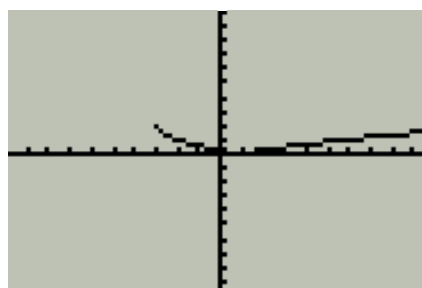
y - Intercept $y = -2$

Domain $x \geq -3$

Range $y \geq -2$

Asymptote No Asymptote

$$y = |\sqrt{x+3} - 2|$$



x - Intercept $x = 1$

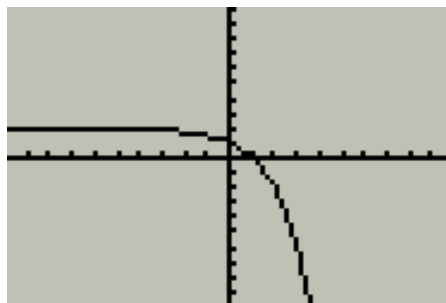
y - Intercept $y = 0$

Domain $x \geq -3$

Range $y \geq 0$

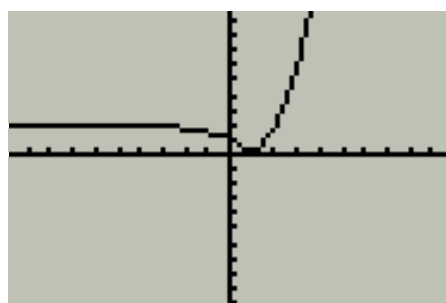
Asymptote No Asymptote

$$y = -2^x + 2$$



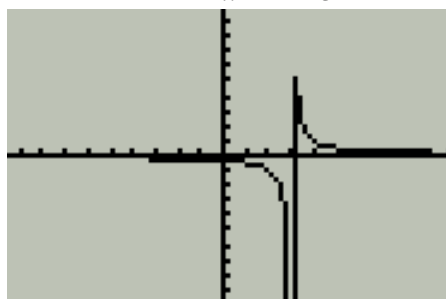
<u>x - Intercept</u>	$x = 1$
<u>y - Intercept</u>	$y = 1$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \leq 2$
<u>Asymptote</u>	$x = 2$

$$y = |-2^x + 2|$$



<u>x - Intercept</u>	$x = 1$
<u>y - Intercept</u>	$y = 1$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \geq 0$
<u>Asymptote</u>	$x = 2$ (For the third quadrant)

$$y = \frac{1}{x - 3}$$



<u>x - Intercept</u>	Does not exist
<u>y - Intercept</u>	$y = -\frac{1}{3}$
<u>Domain</u>	$x \neq 3$
<u>Range</u>	$y \neq 0$
<u>Asymptote</u>	$x = 3$ and $y = 0$

$$y = \left| \frac{1}{x - 3} \right|$$



<u>x - Intercept</u>	Does not exist
<u>y - Intercept</u>	$y = \frac{1}{3}$
<u>Domain</u>	$x \neq 3$
<u>Range</u>	$y > 0$
<u>Asymptote</u>	$x = 3$ and $y = 0$

General Statement

When changing the equation from $y = f(x)$ to $y = |f(x)|$, the bottom part (the part where y has a negative value) of the graph reflects upon x -axis and in the graph, the change in y -intercept and range occurs and everything else such as x -intercept, domain, and asymptote remains the same.

Changing the equation from $y = f(x)$ to $y = |f(x)|$

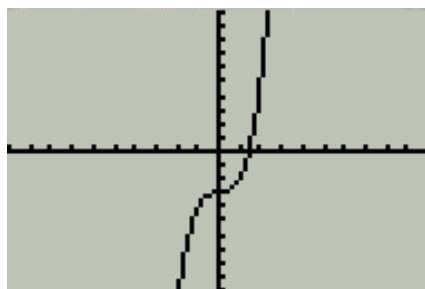
When we have a graph of $y = |f(x)|$, we take the absolute value of y . If it is positive, it leaves the number unchanged; absolute value of four is just four. Therefore, wherever $f(x)$ is positive, the graph of $|f(x)|$ appears the same. However, the absolute value of a negative number is the number itself without the sign; absolute value of -4 is 4 , which you can think as $-(-4)$. So wherever $f(x)$ is negative, you can regard $|f(x)|$ as $-f(x)$. For a given value of " x ", if " y " is negative, " y " can be replaced with $-y$, making it positive. Therefore, absolute value over the whole function will have the part below the x -axis in the graph to fold up to be above the x -axis.

Even though some graphs that do not include negative y -values will not look as if they obey this pattern, every function is going to obey these patterns. Therefore, when a function has an absolute value, the graph will be reflected vertically upon x -axis.

Conjecture

$$y = x^3 - 3 \quad \text{and} \quad y = |x^3 - 3|$$

$$y = x^3 - 3$$



<u>x - Intercept</u>	$x = \sqrt[3]{3}$
<u>y - Intercept</u>	$y = -3$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \in \mathbb{R}$
<u>Asymptote</u>	No asymptote

$$y = |x^3 - 3|$$

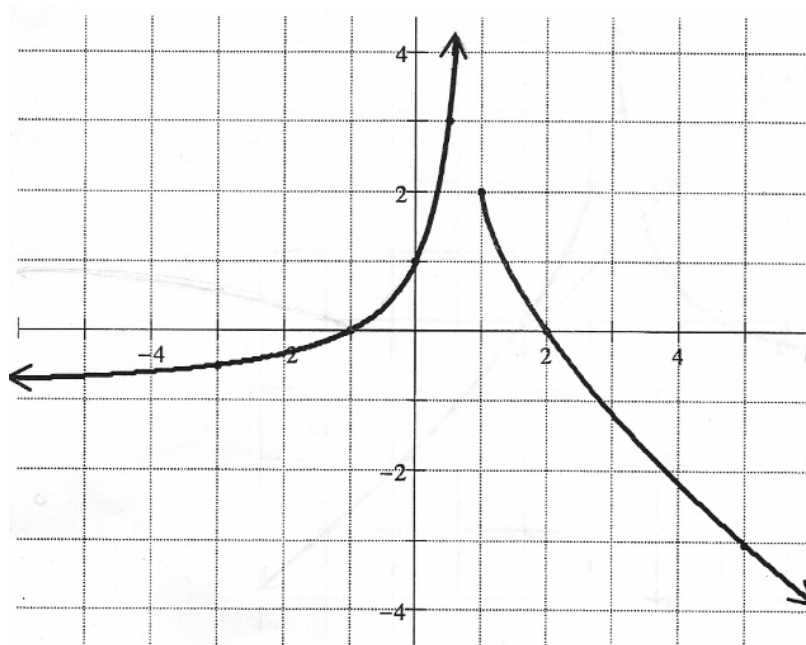


<u>x - Intercept</u>	$x = \sqrt[3]{3}$
<u>y - Intercept</u>	$y = 3$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \geq 0$
<u>Asymptote</u>	No asymptote



∴ The graph above is when the graph of $y = |x^3 - 3|$ and $y = x^3 - 3$ which is drawn in bold line are drawn together. By looking at the graph, it is clear that the function of $y = x^3 - 3$'s negative y-value part reflects upon x-axis to form the graph of absolute value. It makes x-intercepts, domain and asymptote for both of the function to remain the same while the y-intercept changes its sign because of the absolute value. However, if the graph had asymptotes, the function of absolute value and the original function will have the same asymptote except that the asymptote for function of absolute value will only work in 2nd and 3rd quadrants. Also, because it has done horizontal reflection, the range becomes $y \geq 0$. Therefore, the conjecture is always true and any function will obey this pattern.

Drawing $y = |f(x)|$ graph on given graph of $y = f(x)$



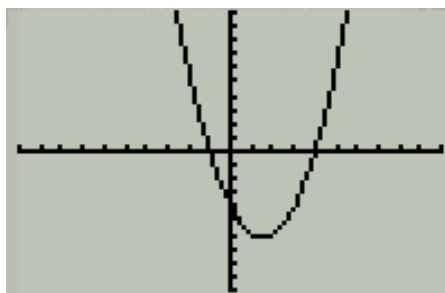
Comparing $y = f(x)$ and $y = f(|x|)$

Hypothesis

Unlike the relationship between $y = f(x)$ and $y = |f(x)|$ graphs, the function $y = f(|x|)$ will differ from the other function $y = f(x)$ by having all the left part of the function $y = f(x)$ about y-axis, where negative units lie for x – axis, as vertical reflection. Because absolute value of “x” can only have positive value of “x”, as a result, the function will also have positive value of “x” only. Therefore, when having an absolute value of “x” graph, because only positive value of “x” exists, the positive value part of the graph will reflect upon y –axis so that the graph is symmetrical to y – axis. Furthermore, original graph’s x – intercept will have the same value but different sign such as ± 4 and range will remain the same as the original graph’s range.

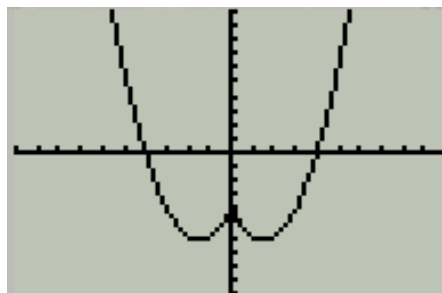
Absolute Value Graphs - $y = f(x)$ and $y = f(|x|)$

$$y = x^2 - 3x - 4$$



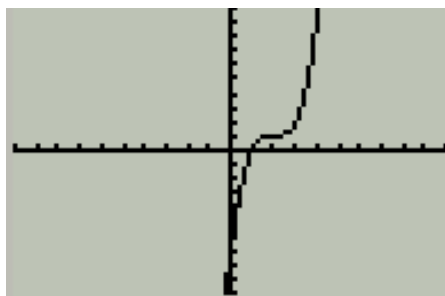
<u>x - Intercept</u>	$x = -1$ and $x = 4$
<u>y - Intercept</u>	$y = -4$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \geq -4.25$
<u>Asymptote</u>	No asymptote

$$y = |x|^2 - 3|x| - 4$$



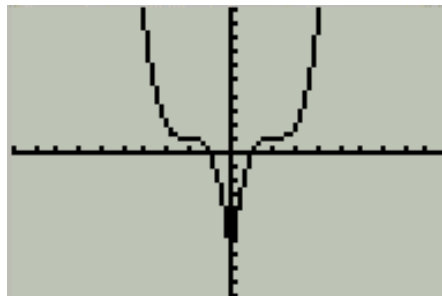
<u>x - Intercept</u>	$x = 4$ and $x = -4$
<u>y - Intercept</u>	$y = -4$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \geq -4.25$
<u>Asymptote</u>	No asymptote

$$y = (x - 2)^3 + 1$$



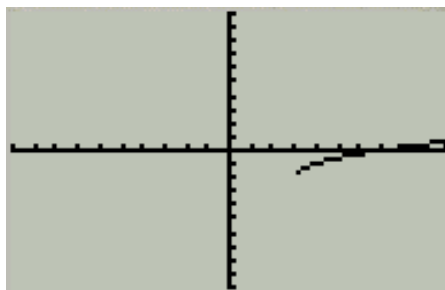
<u>x - Intercept</u>	$x = 3$
<u>y - Intercept</u>	$y = 1$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \in \mathbb{R}$
<u>Asymptote</u>	No asymptote

$$y = (|x| - 2)^3 + 1$$



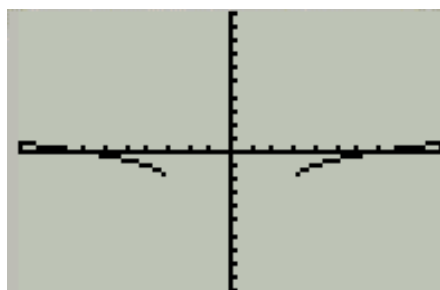
<u>x - Intercept</u>	$x = 1$ and $x = -1$
<u>y - Intercept</u>	$y = 1$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \in \mathbb{R}$
<u>Asymptote</u>	No asymptote

$$y = \sqrt{x-3} - 2$$



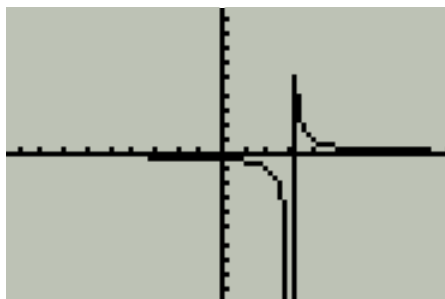
<u>x - Intercept</u>	$x = 7$
<u>y - Intercept</u>	Does not exist
<u>Domain</u>	$x \leq 7$
<u>Range</u>	$y \geq -2$
<u>Asymptote</u>	No asymptote

$$y = \sqrt{|x|-3} - 2$$



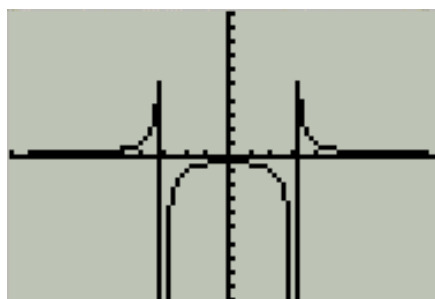
<u>x - Intercept</u>	$x = 7$ and $x = -7$
<u>y - Intercept</u>	Does not exist
<u>Domain</u>	$x \leq 7$ and $x \geq -7$
<u>Range</u>	$y \geq -2$
<u>Asymptote</u>	No asymptote

$$y = \frac{1}{x-3}$$



<u>x - Intercept</u>	Does not exist
<u>y - Intercept</u>	$y = -\frac{1}{3}$
<u>Domain</u>	$x \neq 3$
<u>Range</u>	$y \neq 0$
<u>Asymptote</u>	$x = 3$ and $y = 0$

$$y = \frac{1}{|x|-3}$$



<u>x - Intercept</u>	Does not exist
<u>y - Intercept</u>	$y = -\frac{1}{3}$
<u>Domain</u>	$x \neq 3$ and $x \neq -3$
<u>Range</u>	$y \neq 0$
<u>Asymptote</u>	$x = \pm 3$ and $y = 0$

General Statement

When changing the equation from $y = f(x)$ to $y = f(|x|)$, the graph on the right side reflects upon y-axis. Also, in the graph, x-intercept, domain and asymptote change except for certain circumstances such as having no x-intercept, and y – intercept remains the same. Furthermore, the absolute value of “x” value function has the range from the minimum value of the positive “x” value side. (1st and 2nd quadrants’)

Changing the equation from $y = f(x)$ to $y = f(|x|)$

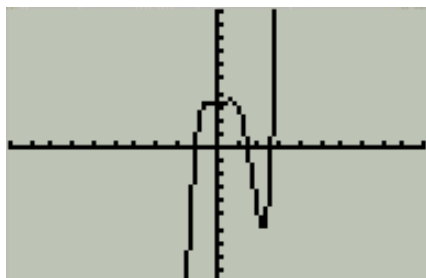
Because the absolute value is on “x” and it makes every x-value positive, it creates both positive and negative “x” values to have the same y-value. For an example, if $1 = f(|4|)$. Because $|4|$ is an absolute value, x can be either 4 or -4. Whatever the case is, the y-value is not going to change from 1. Therefore, whichever value “x” has, “y” value is going to remain the same for both positive case and negative case.

Every function is going to obey this pattern however the x-intercepts, y-intercepts, domain, range, and asymptote may vary depending on the graph. When the graph does not have x-intercept or asymptote but has absolute value of x in the function, x-intercept and asymptote of both graphs will not change from “does not exist” or “unidentified”.

Conjecture

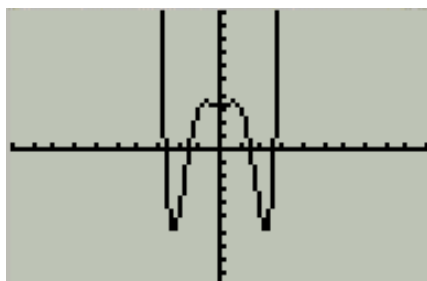
$$y = x^5 - 3x^4 + 2x^2 + 3 \quad \text{and} \quad y = |x|^5 - 3|x|^4 + 2|x|^2 + 3$$

$$y = x^5 - 3x^4 + 2x^2 + 3$$

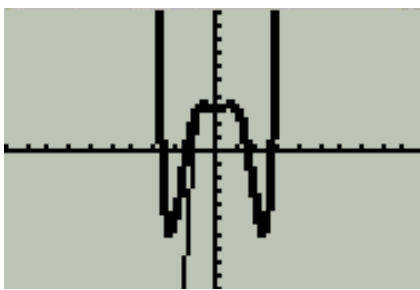


<u>x – Intercept</u>	$x = -1, 1.5, 2.6$
<u>y – Intercept</u>	$y = 3$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \in \mathbb{R}$
<u>Asymptote</u>	No asymptote

$$y = |x|^5 - 3|x|^4 + 2|x|^2 + 3$$

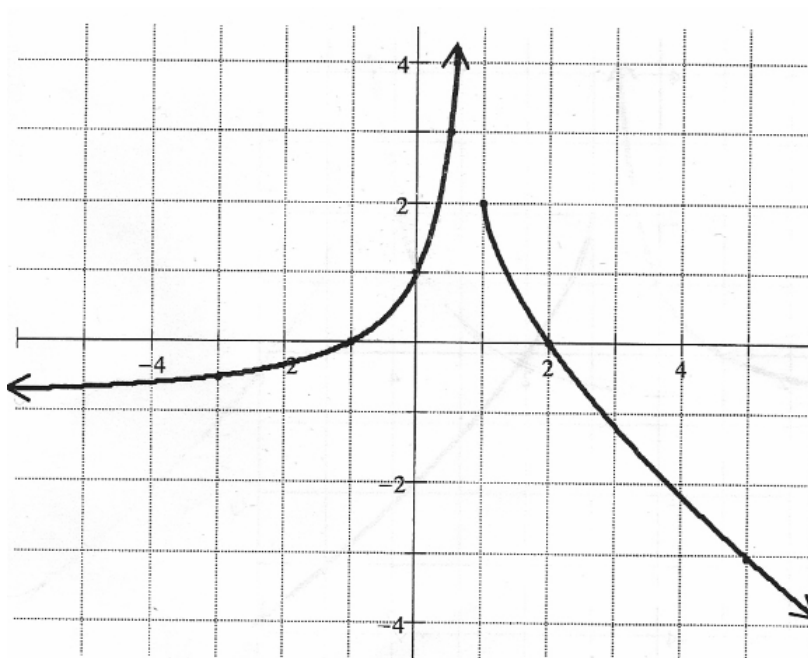


<u>x - Intercept</u>	$x = \pm 1.5, \pm 2.6$
<u>y - Intercept</u>	$y = 3$
<u>Domain</u>	$x \in \mathbb{R}$
<u>Range</u>	$y \geq -5.8$
<u>Asymptote</u>	No asymptote



∴ The graph above is when the graph of $y = x^5 - 3x^4 + 2x^2 + 3$ and $y = |x|^5 - 3|x|^4 + 2|x|^2 + 3$ which is drawn in bold line are drawn together. The graph gives clear interpretation of how the function of $y = x^5 - 3x^4 + 2x^2 + 3$ transform the function of $y = |x|^5 - 3|x|^4 + 2|x|^2 + 3$: the part of the graph of 1st and 4th quadrant of $y = x^5 - 3x^4 + 2x^2 + 3$ reflects upon y-axis. It makes x-intercept, domain and asymptote to change and y - intercept to remain the same. Furthermore, because the function of absolute value has done vertical reflection, the range will start from the minimum or maximum point of the 1st and 4th quadrants of the graph. Therefore, the conjecture is always true and any function will obey this pattern.

Drawing $y = f(|x|)$ graph on given graph of $y = f(x)$



Conclusion

After comparing these functions, it was noticeable that an absolute value makes significant differences in the graphs. When comparing $y = f(x)$ and $y = |f(x)|$, it was found that the negative y-value part of the graph reflects upon the x-axis because an absolute value creates only positive values, and negative values can never exist. When comparing $y = f(x)$ and $y = f(|x|)$, it was found that the positive x-value part of the graph reflects upon the y-axis, because the absolute value of “x” always creates the positive value of “y” regardless of x-value’s sign. Therefore, these functions differ from each other, and the function of the absolute value of either the whole function or the x-value only, each has its own specific patterns.