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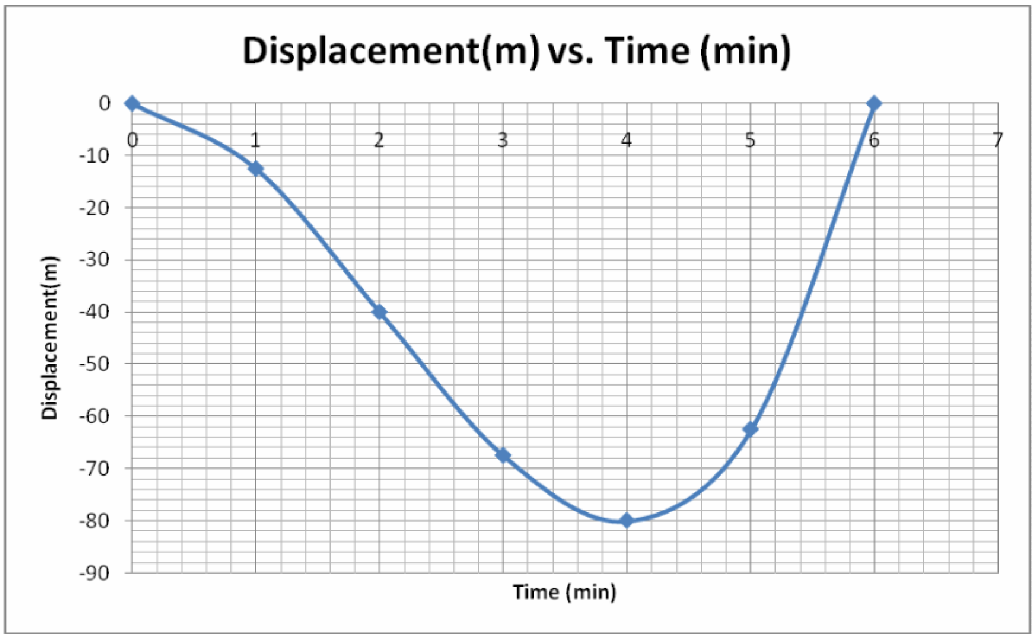
Introduction

This paper attempts to analyze the formula modeling the motion of a freight elevator:
$y(t) = 2.5t^3 - 15t^2$

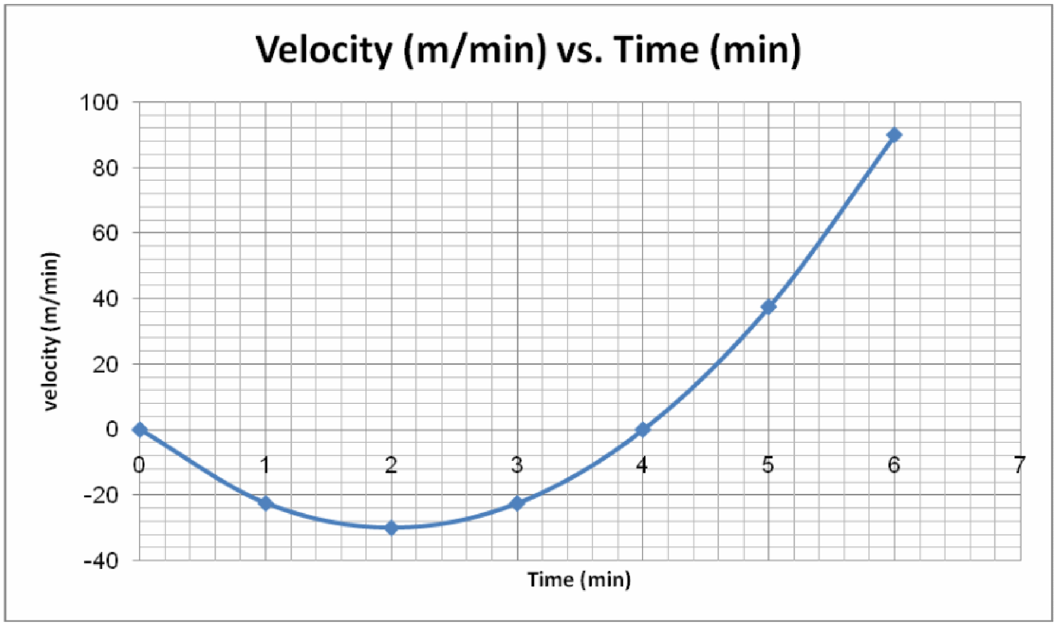
The formula above is then differentiated to obtain the velocity and acceleration equations. Based on the strengths and weaknesses of the original model, a new model is founded to fix all the problems of the original model while maintaining its strengths and usefulness. Later, the founded model is modified to fit into other applications such as residential elevators, pulleys and cranes.

1. Use these functions to:

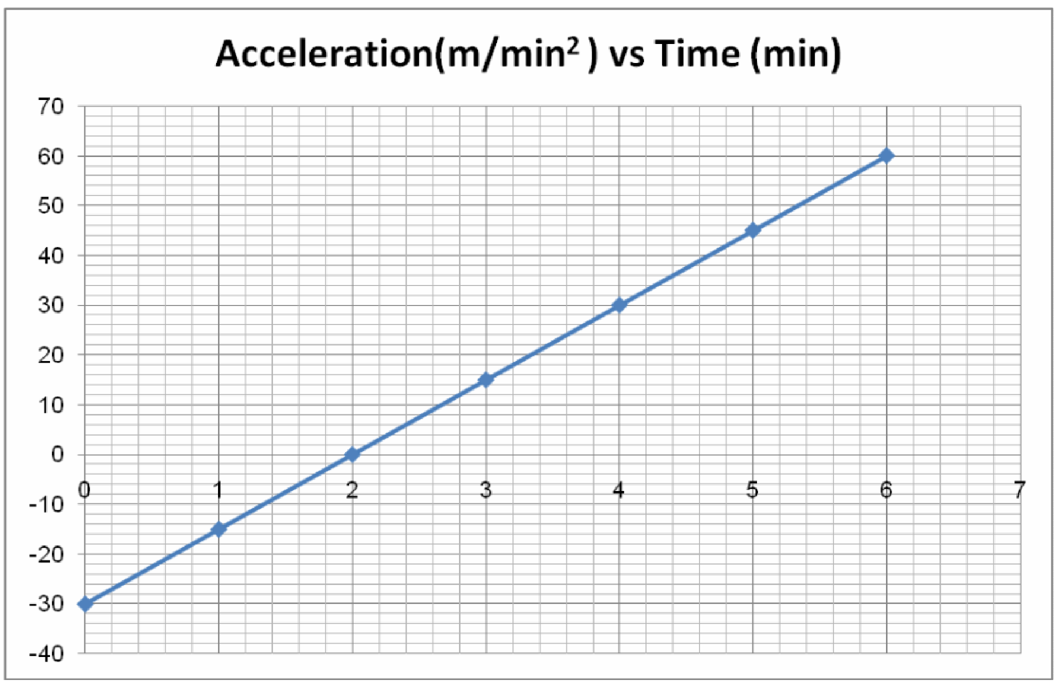
a) Interpret the original vertical line motion simulation



Graph 1 $y(t) = 2.5t^3 - 15t^2$



Graph 2: $v(t) = 7.5t\text{^}2 - 30t\ (m/min)$



Graph 3: $a(t) = 15t - 30$

From the three graphs above, the motion of the freight elevator can be described as below.

From the graph of displacement versus time, the elevator starts moving from t=0 to t=1. It then speeds up from t = 1 to t = 4. It stopped at -80m below the ground at t = 4. From t = 4 to t = 6, the elevator moves back up to the ground.

From the graph of velocity versus time, the elevator speeds up at first from t = 0 to t = 2. From t = 2 and t = 4, the velocity slows down, comparing with the displacement graph, the elevator has slowed down when it comes near the shaft. From t = 4 to t = 6, the velocity increases, which is parallel to the displacement graph where the elevator moves towards the ground from t = 4 and t = 6.

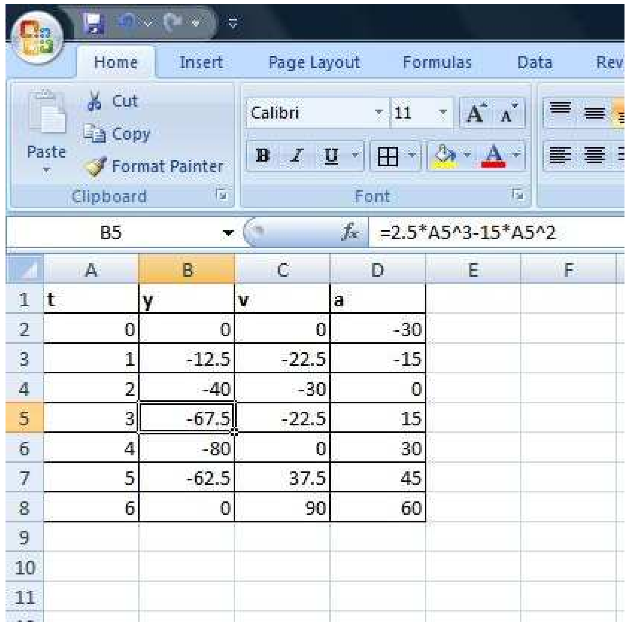
From the graph of acceleration versus time, at first, the elevator speeds up at first from t = 0 and t =2. Comparing to the velocity graph, it can be seen that both acceleration and velocity are negative from t = 0 to t = 2. From t = 2, a(2) = 0, this is when the velocity decreases, so it reflects the changes in the concavity of the velocity curve. From t = 2 to t = 4, the acceleration increases to positive values while the velocity values are still negative, this shows a decrease in speed of the elevator. From t = 4 to t = 6, the elevator speeds up again when both acceleration and velocity are positive.

b) Explain the meaning of the negative, positive and zero values of the velocity graph

The velocity function is derived by differentiating the displacement function, in other words, the velocity is the first derivative function of the displacement function.

$y(t) = 2.5t^3 - 15t^2$

$v(t) = y'(t) = 7.5t^2 - 30t$



	A	B	C	D
1	t	y	v	a
2	0	0	0	-30
3	1	-12.5	-22.5	-15
4	2	-40	-30	0
5	3	-67.5	-22.5	15
6	4	-80	0	30
7	5	-62.5	37.5	45
8	6	0	90	60

Table 1 Values of displacement (y), velocity (v) and acceleration(a) based on the original model formula

At t = 0 and t = 4, v(0) = v(4) = 0. This indicates that the elevator is instantaneously at rest. These values fit with the displacement graph since at t = 0, the elevator is at the origin while at t = 4, the elevator has reached the shaft.

At t = 1, t = 2, t = 3, v(t) < 0. This indicates that the elevator is moving away from the ground and towards the shaft, which on the motion simulation can be seen as moving to the left of the origin.

At t = 5 and t = 6, v(t) > 0. This shows that the elevator is moving towards the ground and away from the shaft, which on the motion simulation can be seen as moving to the right of the origin.

c) Explain the relationship between velocity and acceleration in the intervals when the elevator speeds up, slows down, and is at rest.

Acceleration function is the second derivative of the displacement function.

$$y(t) = 2.5t^3 - 15t^2$$

$$v(t) = y'(t) = 7.5t^2 - 30t$$

$$a(t) = y''(t) = 15t - 30$$

When the elevator slows down, a(t) is negative (a(t) < 0), at the same time, the velocity is decreasing (from -22.5m/min to -30m/min from t = 0 to t = 1). When the elevator speeds up, a(t) is positive (a(t) > 0), the velocity is increasing from t = 3 to t = 6). When t = 2, a(2) = 0, this means the instantaneous velocity at t = 2 is at the minimum point.

When the elevator is at rest, $v(t) = 0$. This happens at $t = 0$ and $t = 4$, which are the times when the elevator reaches either the ground floor or the shaft. At $t = 0$ and $t = 4$, $a(0) = -30$ m/min^2 while $a(4) = 30m/min^2$. Therefore, at these t times, the values of acceleration have the same absolute value but different signs.

d) Evaluate the usefulness and identify the problems of the model in the given situati on.

The usefulness of the model is that it travels the distance of 80m within 6 minutes. This characteristic helps to transport a great deal of goods (minerals) from a deep mine to the ground.

However, there are some problems with the model in the given situation. As can be seen in graph 1, the elevator takes 4 minutes to reach the shaft but only 2 minutes to return to the ground. This is not ideal for the elevator since it would travel faster than expected and may do some damage to other structures. Moreover, when the graph is extended for more than 6 minutes, the elevator keeps going up, not down, this is not applicable in the situation when it needs to come down to transport goods.

Another problem can be seen in graph 2 with velocity. Even though when the elevator comes down, at first it speeds up and then slows down, the velocity when it reaches the ground again is too high: 90m/s. This indicates that the elevator is not going to stop and hence may go off the ground. The same problem also occurs to veloci ty when it is not likely for the velocity to repeat the same cycle, instead, it keeps increasing forever. So does the acceleration.

2. List specifications for a redesign of the freight elevator model

The redesign of the freight elevator model should ha ve the following characteristics:

When the elevator is at the initial conditions, the velocity should be equal to 0 or really close to close (the difference between the velocity and 0 at this time should be negligible). When the elevator starts going down to the shaft, the velocity should at first speed ups and then slow down to nearly zero when it is coming near the shaft. When the elevator returns to the ground, the velocity should speed up at first and slows down to nearly zero when it comes near the ground.

3 and 4. To ensure the periodic phenomenon in the movement of the elevator, a trigonometry equation is employed. Explain how your model addresses the problems of the given model and satisfies the specifications of a well-functioning elevator for a mining company.

The model has the general formula $y(t) = AsinB(t - C) + D$

The period is 6 minutes, so

$$6 = \frac{2\pi}{B}$$
$$\therefore B = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$The\ amplitude = \frac{\max - min}{2} = \frac{0 - (-80)}{2} = 40$$
$$\therefore A = 40$$

The principal axis is the middle between the maximum and minimum values:

$$\therefore D = \frac{\max + min}{2} = \frac{0 + (-80)}{2} = -40$$

$$\therefore y(t) = 40sin\left[\frac{\pi}{3}(t - C)\right] - 40$$

At t = 0, y(0) = 0 $\quad \therefore 40sin\left[\frac{\pi}{3}(-C)\right] - 40 = 0$

$$sin\left[\frac{\pi}{3}(-C)\right] = 1$$
$$\frac{\pi}{3}(-C) = \frac{\pi}{2}$$
$$C = -\frac{3}{2}$$

$$\therefore y(t) = 40sin\left[\frac{\pi}{3}\left(t+\frac{3}{2}\right)\right] - 40\,(m)$$

$$\therefore v(t) = 40cos\left[\frac{\pi}{3}\left(t+\frac{3}{2}\right)\right] \times \frac{\pi}{3}\,(m/min)$$

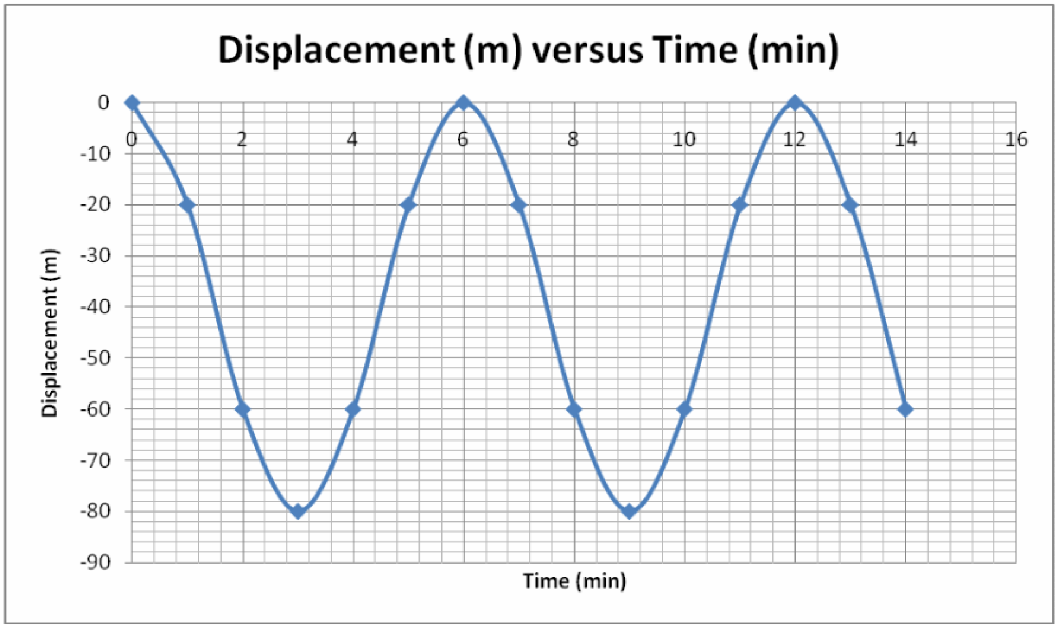
$$\therefore \mathrm{a(t)} = -40\sin\left[\frac{\pi}{3}\left(t+\frac{3}{2}\right)\right] \times \frac{\pi}{9}\,(\mathrm{m/min^2})$$

C6 =40*COS(PI()/3*(A6+1.5))*PI()/3

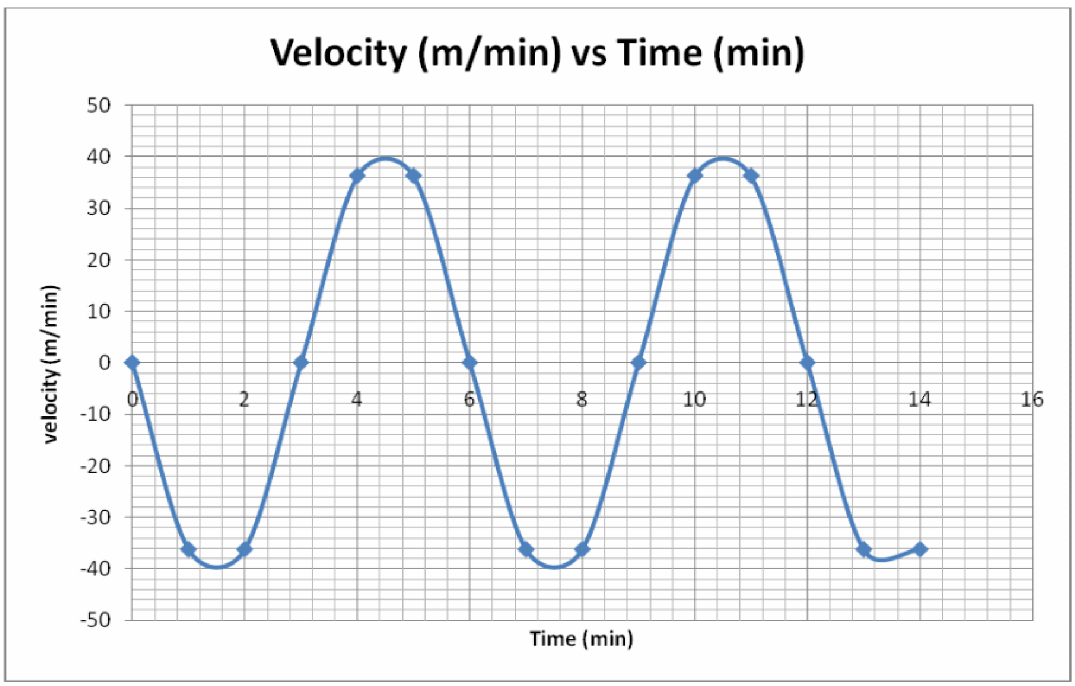
	A	B	C	D
1	t	y	v	a
2	0	0	2.56594E-15	-43.8649
3	1	-20	-36.2759873	-21.9325
4	2	-60	-36.2759873	21.93245
5	3	-80	-7.6978E-15	43.86491
6	4	-60	36.27598728	21.93245
7	5	-20	36.27598728	-21.9325
8	6	0	5.00337E-14	-43.8649
9	7	-20	-36.2759873	-21.9325
10	8	-60	-36.2759873	21.93245
11	9	-80	-1.7962E-14	43.86491
12	10	-60	36.27598728	21.93245
13	11	-20	36.27598728	-21.9325
14	12	0	9.75014E-14	-43.8649
15	13	-20	-36.2759873	-21.9325
16	14	-60	-36.2759873	21.93245

Table 2 Values of displacement (y), velocity (v) and acceleration (a) based on the new model formula

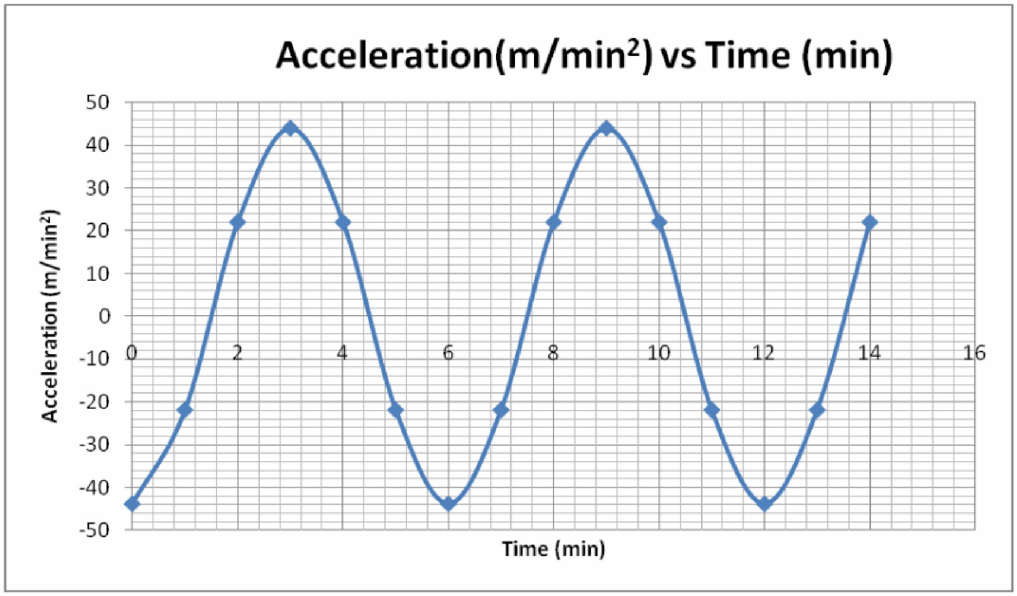
In the table values, using the same displacement distance (which is -80m), it can be seen the time for the elevator to come down to the mineshaft is equal to the time it takes for the elevator to come up to the ground. This is an improvement for the problems of the original model. When the elevator comes near the shaft or the ground, the velocities are really small, nearly zero, which are -7.7 x 10^{-15} and 5.0 x 10^{-15}. Fulfilling the specifications listed above, the elevator first speeds up and then slows down again when it comes to either the ground or the mine shaft. These problems can be seen on the graphs of displacement, velocity and acceleration.



Graph 4 $y(t) = 40sin\left[\frac{\pi}{3}\left(t + \frac{3}{2}\right)\right] - 40$



Graph 5 $v(t) = 40cos\left[\frac{\pi}{3}\left(t + \frac{3}{2}\right)\right] \times \frac{\pi}{3}(m/min)$



Graph 6 $a(t) = -40\sin\left[\frac{\pi}{3}\left(t+\frac{3}{2}\right)\right] \times \frac{\pi}{9}(m/min^2)$

However, the velocity is quite hazardous and unrealistic, because the goods transported may experience inertia, so they may move around when being inside the elevator, with such a large velocity they may do damage onto the elevator as well as they may not the same when they collide with each other.

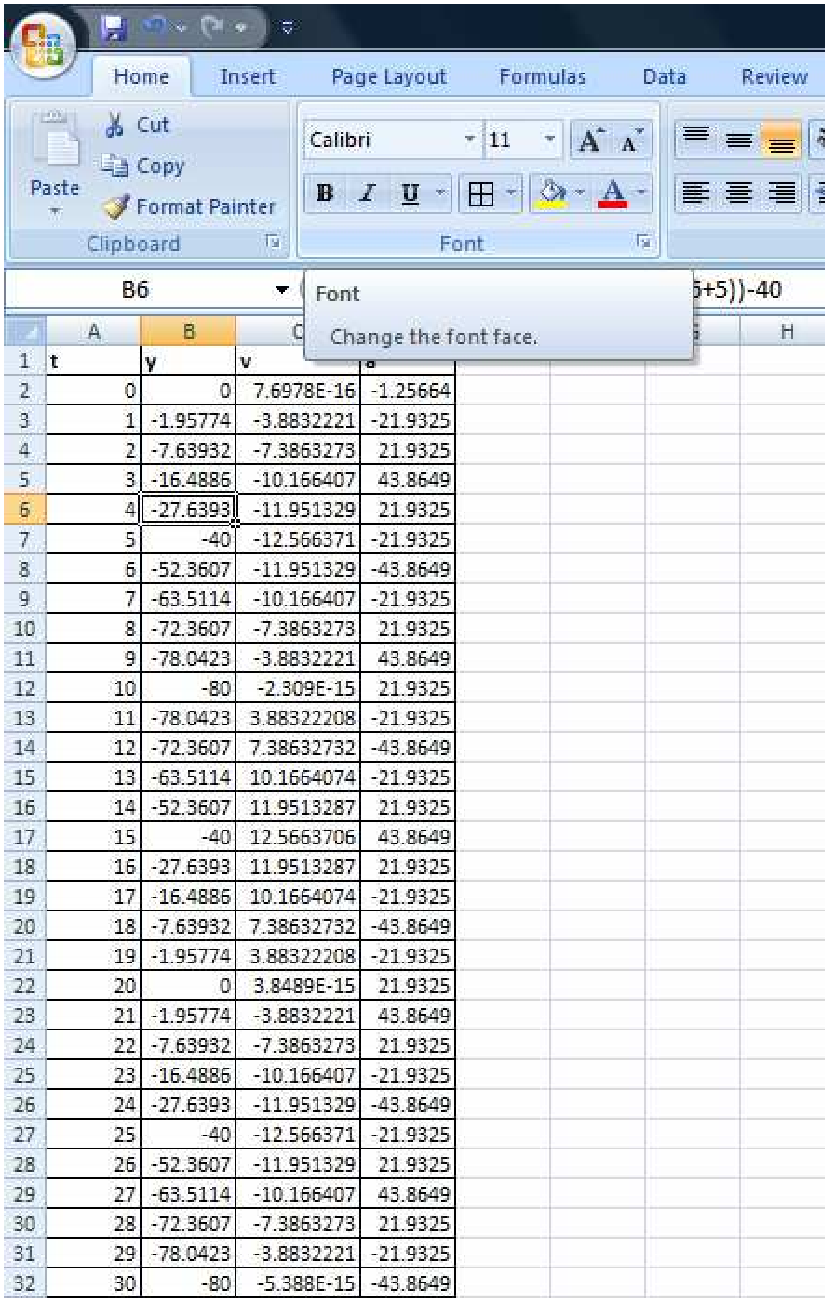
Hence, it is quite impossible to keep the time the same while making sure that the elevator will not suffer from any damage because of inertia. A refined version of this model should last for a longer period of time, but it ensures that the goods are not damaging the interior parts of the elevator.

The velocity should not exceed 15m/s, to ensure the goods are maintained. Therefore, the B value should be changed to extend the time period to lessen the velocity. Increasing the time from 6 minutes to 20 minutes while keeping the distance -80m the same, the following equations are evoked which should help to solve the problem of inertia and high velocity.

$$\therefore y(t) = 40sin\left[\frac{\pi}{10}(t+5)\right] - 40$$

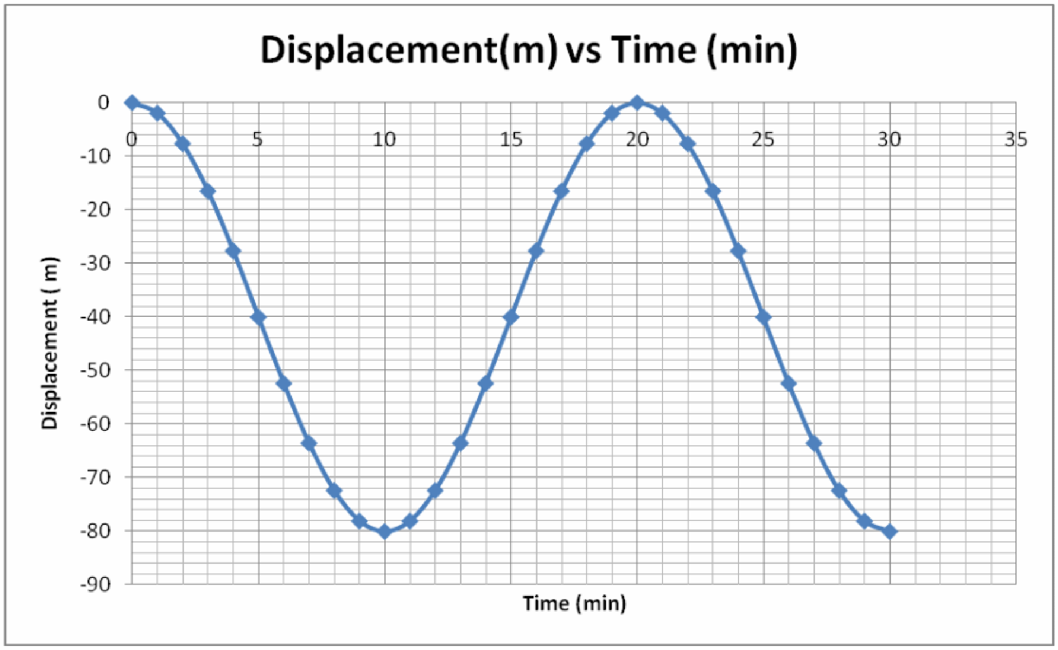
$$\therefore v(t) = 40cos\left[\frac{\pi}{10}(t+5)\right] \times \frac{\pi}{10}$$

$$\therefore a(t) = -40\sin\left[\frac{\pi}{10}(t+5)\right] \times \frac{\pi}{100}$$

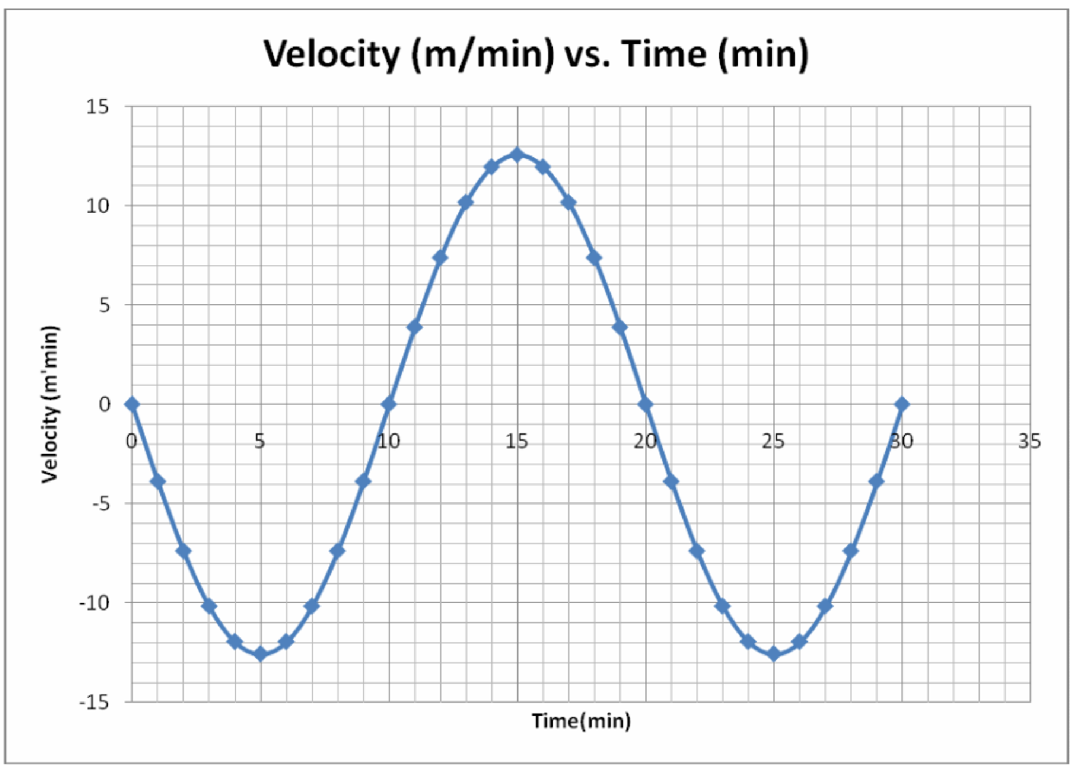


t	y	v	a
0	0	7.6978E-16	-1.25664
1	-1.95774	-3.8832221	-21.9325
2	-7.63932	-7.3863273	21.9325
3	-16.4886	-10.166407	43.8649
4	-27.6393	-11.951329	21.9325
5	-40	-12.566371	-21.9325
6	-52.3607	-11.951329	-43.8649
7	-63.5114	-10.166407	-21.9325
8	-72.3607	-7.3863273	21.9325
9	-78.0423	-3.8832221	43.8649
10	-80	-2.309E-15	21.9325
11	-78.0423	3.88322208	-21.9325
12	-72.3607	7.38632732	-43.8649
13	-63.5114	10.1664074	-21.9325
14	-52.3607	11.9513287	21.9325
15	-40	12.5663706	43.8649
16	-27.6393	11.9513287	21.9325
17	-16.4886	10.1664074	-21.9325
18	-7.63932	7.38632732	-43.8649
19	-1.95774	3.88322208	-21.9325
20	0	3.8489E-15	21.9325
21	-1.95774	-3.8832221	43.8649
22	-7.63932	-7.3863273	21.9325
23	-16.4886	-10.166407	-21.9325
24	-27.6393	-11.951329	-43.8649
25	-40	-12.566371	-21.9325
26	-52.3607	-11.951329	21.9325
27	-63.5114	-10.166407	43.8649
28	-72.3607	-7.3863273	21.9325
29	-78.0423	-3.8832221	-21.9325
30	-80	-5.388E-15	-43.8649

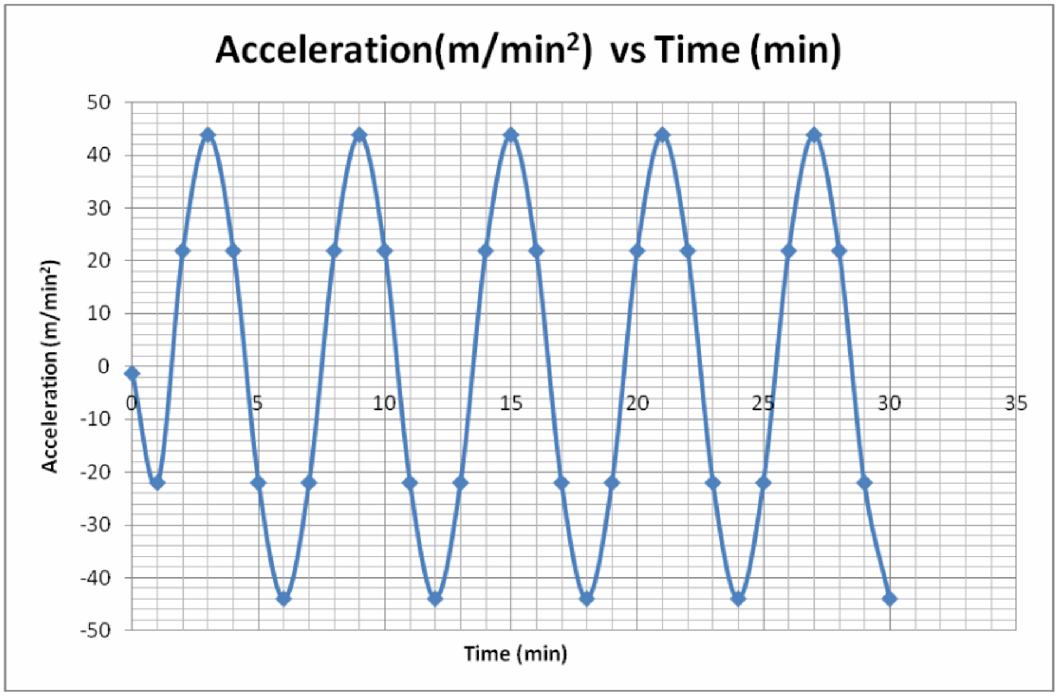
Table 3 Values of displacement (y), velocity (v) and acceleration of the refined version of the new model formula



Graph 7 $y(t) = 40sin\left[\frac{\pi}{10}(t+5)\right] - 40$



Graph 8 $v(t) = 40cos\left[\frac{\pi}{10}(t+5)\right] \times \frac{\pi}{10}$



Graph 9 $a(t) = -40\sin\left[\frac{\pi}{10}(t+5)\right] \times \frac{\pi}{100}$

4. Explain how your model may be modified to be useful in other situations

The model founded is to be applied upon the elevator for a mining company. This elevator moves into the underground, in other words, below the ground surface. Other situations may involve an object moving upwards such as an elevator in a hotel, a pulley, or a crane. Not taking the time spending at the maximum distance, the initial position or anything in the range, the formula can be applied to calculate the height, velocity and acceleration of the object.

The model has the general formula $y(t) = A\sin B(t - C) + D$

The period is:

$$t = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{t}$$

$$The\ amplitude\ A = \frac{\max - min}{2}$$

The principal axis is the middle between the maximum and minimum values:

$$\therefore D = \frac{\max + min}{2}$$

$$\therefore y(t) = \left(\frac{\max - min}{2}\right) sin\left[\frac{2\pi}{t}(t - C)\right] + \frac{\max + min}{2}$$
$$\therefore y(t) = \left(\frac{\max - min}{2}\right) sin\left[2\pi - \frac{2\pi C}{t}\right] + \frac{\max + min}{2}$$

At t = 0, y(0) = 0

$$\left(\frac{\max - min}{2}\right) sin\left[2\pi - \frac{2\pi C}{t}\right] + \frac{\max + min}{2} = 0$$

$$sin\left[2\pi - \frac{2\pi C}{t}\right] = \frac{max + min}{min - max}$$
$$C = \left[-arcsin\left(\frac{max + min}{min - max}\right) + 2\pi\right]\frac{t}{2\pi}$$

Conclusion:

Some conclusions can be drawn upon the results obtained in this investigation. First, based on the displacement, velocity and acceleration graphs, the reliability and the practicality of a formula on how a model works.

The original model has the displacement equation, velocity equation and acceleration equation as shown below:

$$y(t) = 2.5t^3 - 15t^2$$

$$v(t) = y'(t) = 7.5t^2 - 30t$$

$$a(t) = y''(t) = 15t - 30$$

To improve the weaknesses of the original model which include unrealistic and hazardous velocity as well as acceleration, a refined model has been made:

$$y(t) = 40sin\left[\frac{\pi}{10}(t + 5)\right] - 40$$
$$v(t) = 40cos\left[\frac{\pi}{10}(t + 5)\right] \times \frac{\pi}{10}$$
$$\mathrm{a(t)} = -40\sin\left[\frac{\pi}{10}(t + 5)\right] \times \frac{\pi}{100}$$

The general formula for an object to have similar motions to the elevator is:

$$y(t) = AsinB(t - C) + D$$

Where
$$A = \frac{\max - min}{2}$$

$$B = \frac{2\pi}{t}$$

$$C = \left[-arcsin\left(\frac{max + min}{min - max}\right) + 2\pi\right]\frac{t}{2\pi}$$

$$D = \frac{\max + min}{2}$$