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ABSTRACT

This essay will examine theoretical and experimental probability in relation to the Korean card game called ‘Sut-Da’. First, a definition of probability and how it is used in general life will be examined. Each hand of ‘Sut-Da’ provides the theoretical probability for a player to win the game. It is clear however, that the theoretical value of winning in ‘Sut-Da’ does not always apply in real life games. Secondly, the experimental probability of winning for each hand is examined. To find out the probability of winning with each hand, I am using permutation & combination, theoretical probability and experimental probability. Experimental probability data was gained from my friend and I playing the game. Finally, within my evaluation, I looked at experimental probability using excel spread sheets and also using calculations that were compared with the experimental probability data gained from actually playing the game. The theory of probability has been covered in a number of textbooks and I used these textbooks to help me get used to the formula. I have then worked out all the possibilities of hands and their probabilities for winning, performing all the calculations myself and using my own numbers in presenting my data for the experimental probability data.

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Introduction

It all began when I started to watch a Korean drama called “Ta-JJa”. I decided to watch this drama as one of my favourite actor was the main character in the drama. This drama was about this man getting into the world of gambling in order to be successful in his life. This drama was only based on the tricks that could have been used while playing the game “Sut-Da”. Then the drama made me to wonder whether this game plays under the concept of probability and also questioned me if the game was a fair game or not. Hence came to decision to make this as my extended essay topic.

I will first introduce what is probability and the game called “Sut-Da”. Everyone has different definition for probability and it depends on which perspective the people looks from, but generally, probability is the measure of how likely for an event to occur.

Throughout this essay, these two games will be set under few conditions. These conditions are;

1. Number of players are 2
2. The deck is shuffled 10 times after the game in order to create a fair deck every round

My research question becomes:
What are the possibilities of winning in “Sut-Da” and how does the theoretical value compare with the experimental value?

I will start by giving brief idea on what is probability and how it relates to the real life world situations. I will then start examining the chances of getting each hand with “Sut-Da” then compare with the experimental value. I will be using the probability theory and combination theory to calculate each hand for the “Sut-Da” game and use trial and error for experimental value. I will then investigate similar concepts that are being used in different parts in the world.

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Understanding the Concepts

1. Probability

1. What is Probability?

There are number of different definitions of mathematical probability which have been proposed by various authors. The scientific definition of a 'fundamental' concept such as "the concept of probability is merely the refinement and logical processing of a series of very simple observations and practical methods that have proved their value by successful use over a long period of time". (GendenkoB., SecklerB.) Also, it could be written as a quantitative measure of the degree of certainty of the observer or the relative frequency of occurrence of the even in a large number of trials or it could be defined as mathematical method to describe the 'chance' or 'likelihood' of an event happening. (UrbanPaul, MartinDavid, HaeseRobert)

2. History of Probability

It is known that the development of probability theory was begun in 17th century; however, it was firstly performed by Girocardano (1501-1576) an Italian mathematician, physician and gambler. He created a manual discussing many of the basic concepts of probability. However, his work did not get revealed until 1663 and received little attention.

The development of modern probability theory began in 1653. It started when one gambler made a contact with the mathematician, 'Blaise Pascal', with a problem on how to divide the stacks when the gambling game has been interrupted. Pascal involved 'Pierre de Fermat' and together solved the problem. During this process, they created the foundation of the laws of probability.

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3. Experimental Probability

The number of trial is the total number of times the experiment is repeated.

The outcomes are the different results possible for one trial of the experiment

The frequency of a particular outcome is the number of times that this outcome is observed.

The relative frequency of an outcome is the frequency of that outcome expressed as a percentage of the total number of trials.

For example, when a wooden cone was thrown 200 times it fell on its side 123 times and on its base 77 times.

The relative frequency of side and base are;

123/200 = 61.5% and 77/200 = 38.5%

4. Theoretical Probability

Theoretical probability is based on what we theoretically predict to occur. It measures the chance of that event occurring in any trials of an experiment.

Probability = $\frac{\text{(number of chance of that event occur)}}{\text{(total number of chances that could occur)}}$

For example, when a dice was thrown chance of getting a '6' is 1/6 and when a dice was thrown chance of getting '2' and '3' is 1/2.

2. Game

"Sut-Da"

The game is called "Sut-Da", this game uses cards called "Hwa Too" which was originated from Japanese cards called "Danafuda". Cards were spread around early 19th century, when Japanese merchants visited Korea frequently. The cards are made up of 48 cards, 4 cards each represent each month of the year.

Each month has different names; January is Song-Hak, February is Mae-Jo, March is But-Ggot, April is Huk-Ssa-Ri, May is Nan-Cho, June is Mo-Ran, July is Hong-Ssa-Ri, August is Gong-San, September is Kook-Jun,

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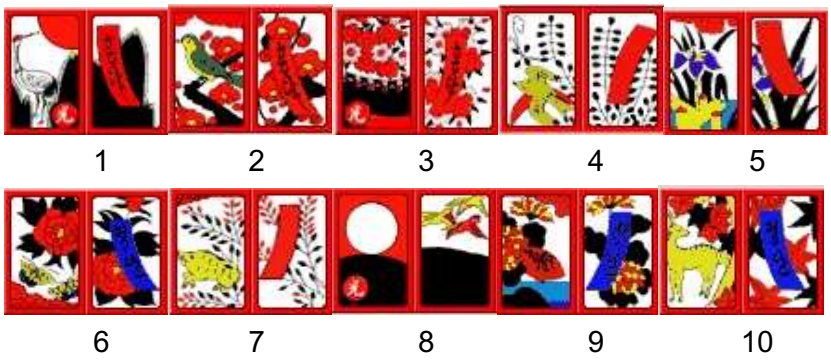
October is Dan-Feng, November is Oh-Dong and December is Bi.
This game "Sut-Da" does not use all these 48 cards but only uses 20 cards. This game can hold two to ten people in one game, however in this essay, there will be 2 people playing the game.
There are two ways of dealing; first one is each player receive two cards each and then reveal the cards to see who has the highest hand. Second one is each player receive three cards, create a highest hand possible with using only two cards from three and burn the unused card. The word 'burn' in previous sentence means to throw away and not use the card in the game. But in this essay, the first dealing type will be used.

These 20 cards are 2 cards from each month, except for November and December.

Diagram 1. Cards Used to Play Sut-Da



There are total of, 12 types of hands from highest to lowest; 3-8 Guang-Ddaeng, Guang-Ddaeng, Ddaeng, Ali, Dok-sa, Gu-Bbing, Jang-Bbing, Jang-Sa, Sae-Luk, Gab-Oh, Gget, Mang-Tong.

3-8 Guang-Ddaeng is when a player gets March and August card with 光.This Chinese character 光 locates in right bottom corner of January and left bottom corners for March and August. Different types of cards may have different Chinese characters replacing the original character 光.

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Diagram 2. Example of 3-8 Guang-Ddaeng

Guang-Ddaeng is when a player gets January and March or January and August card with 光.

Diagram 3,4. Examples of Guang-Ddaeng

Ddaeng is when a player gets any same months, October is highest and January is lowest.

Diagram 5. Examples of Each Ddaeng

Ali is when a player gets two cards formed from January and February.

Diagram 6. Examples of Ali

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Dok-sa is when a player gets two cards formed from January and April.

Diagram 7. Examples of Dok-Sa

Gu-Bbing is when a player gets two cards formed from January and September.

Diagram 8. Examples of Gu-Bbing

Jang-Bbing is when a player gets two cards formed from January and October.

Diagram 9. Examples of Jang-Bbing

Jang-Sa is when a player gets two cards formed from April and October.

Diagram 10. Examples of Jang-Sa

Sae-Luk is when a player gets two cards formed from April and June.

Diagram 11. Examples of Sae-Luk

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Gab-Oh is when a player gets two cards' numbers get added up and the 1st digit is 9.

Diagram 12. Examples of Gab-Oh

Gget is when a player gets two cards' numbers get added up and become 1 ~ 8 (8 is highest, 1 is lowest).

Diagram 13. Examples of Each Gget

Mang-Tong is when a player gets two cards' numbers get added up and become 0.

Diagram 14. Example of Mang-Tong

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Calculations

3-8 Guang-Ddaeng

3-8 Guang-Ddaeng is when a player gets March and August card with 光.

For player 1 to get this hand, they firstly need to receive one of these two cards from the deck, which is 2/20. Then player 2 needs to receive any cards that are not those two cards, which is 18/19. As player 1 already has one of those two cards, for player 1 to get that hand, player 1 must get the other card to match the hand which is 1/18. As player 1 has the highest hand in the whole game, the card that player 2 has does not matter anymore, hence the probability of him getting any card is 17/17.

Hence probability of winning with this hand is;

P(1) = (2/20) x (18/19) x (1/18) x (17/17).

=1/190

= 0.0052631579

Guang-Ddaeng

Guang-Ddaeng is when a player gets January and March or January and August card with 光.

There are two different situations of getting these hands, getting January with 光 first and getting either of March or August with 光.

P(a) = 2 x P(player 1 getting January with 光) x P(player 2 getting any cards excluding March or August with 光) x P(player 1 getting either March or August with 光) x P(player 2 getting any card)

= 2 x (1/20) x (18/19) x (2/18) x (17/17)

= 1/95

P(b) = P(player 1 getting either March or August with 光) x P(player 2 getting any cards excluding January with 光) x P(player 1 getting January with 光) x P(player 2 getting any card)

= (2/20) x (18/19) x (1/18) x (17/17)

= 1/190

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P(all) = P(a) + 2P(b)

= 1/95+ 1/190

= 0.0157894737

Ddaeng

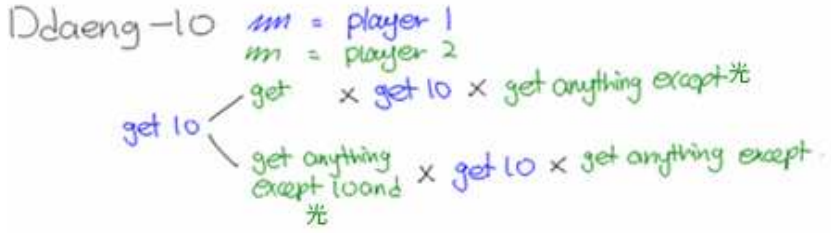
Ddaeng is when a player gets any same months, October is highest and January is lowest.

Starting with October, probability of winning with this hand is split into two different ways. First one is when player 2 has one card that has 光 and second one is when player 2 does not have card with 光.

1st and 3rd bracket is the probability of cards that player 1 receive and 2nd and 4th bracket is the probability of cards that player 2 receive.

In P(1) and P(2), 1st bracket represents two cards that is in the deck of 20, which are October. Second bracket is three cards that is 光 in the deck of 20, third bracket is one card left in the deck of 20 which is October and the last bracket is representing the amount of cards left that is not 光 for player 2 to get, which makes the hand lower than Ddaeng-10.

Graph 1. Tree Diagram of Ddang-10



P(1) = (2/20) x (3/19) x (1/18) x (15/17)

= 1/1292

P(2) = (2/20) x (15/19) x (1/18) x 17/17)

= 1/228

P(all) = (1/1292) + (1/228)

= 5/969

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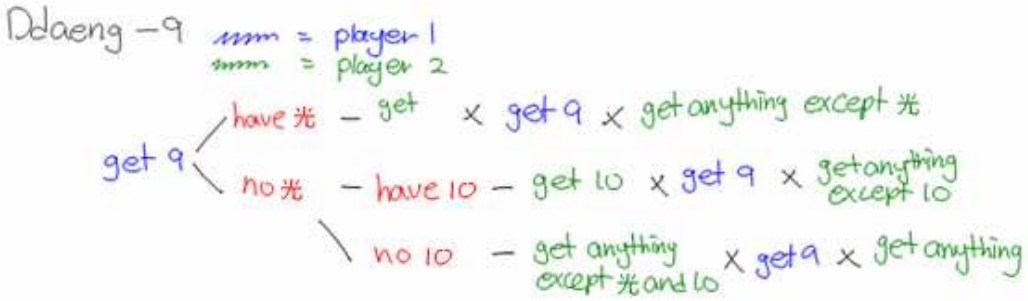
Probability of winning with September (Ddaeng-9) is split into three different possibilities.

First one is when player 2 has one card that has 光, second one is when player 2 does not have 光 and does have October and last one is when player 2 does not have both 光 and October.

Graph 2. Tree Diagram of Ddang-9



P(1) = (2/20) x (3/19) x (1/18) x (1/17)
= 1/1292

P(2) = (2/20) x (2/19) x (1/18) x (16/17)
= 16/29070

P(3) = (2/20) x (13/19) x (1/18) x (17/17)
= 13/3420

P(all) = (1/1292) + (16/29070) + (13/3420)
= 0.005125559

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Probability of winning with August (Ddaeng-8) is split into four different possibilities.

First one is when player 2 has one card that has 光, second one is when player 2 does not have 光 but has one 10, third one is when player 2 does not have 光 and 10 but has 9 and last one when player 2 does not have 光, 10 and 9.

Graph 3. Tree Diagram of Ddang-8

P(1) = (2/20) x (2/19) x (1/18) x (16/17)
= 16/29070

P(2) = (2/20) x (2/19) x (1/18) x (16/17)
= 16/29070

P(3) = (2/20) x (2/19) x (1/18) x (16/17)
= 16/29070

P(4) = (2/20) x (12/19) x (1/18) x (17/17)
= 1/285

P(all) = (16/29070) + (16/29070) + (16/29070) + (1/285)
= 0.0051599587

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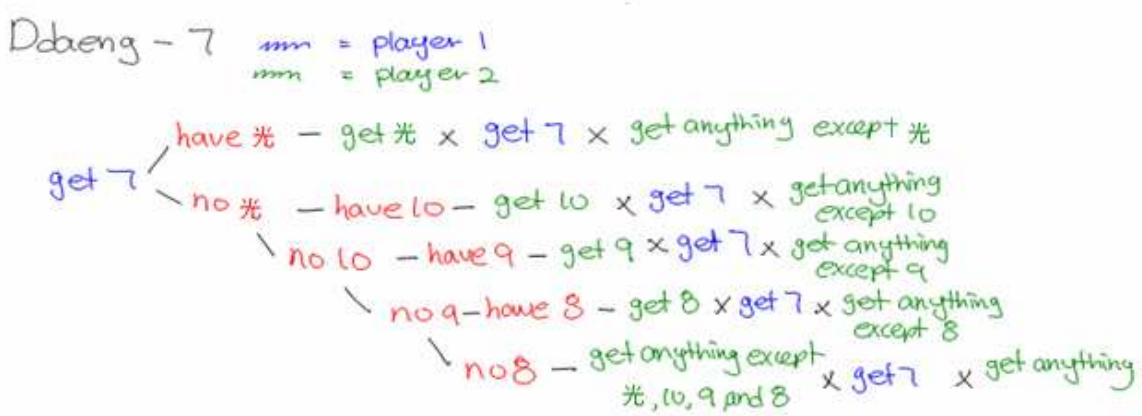
Probability of winning with July (Ddaeng-7) is split into six different possibilities. However, from this hand, since the numbers of possibilities are increasing, the mathematical equation with the 'combination' notation will be used, instead of using the expanded fraction method.

C is representing 'combination', where C_2^{20} represents $\frac{20\times19}{1\times2}$. The equation above represents, simply taking away the probability of opponent winning from 1 and multiplying by the probability for player 1 to receive a specific hand.

Graph 4. Tree Diagram of Ddang-7



$$P(\text{win}) = \frac{1}{C_2^{20}} \times \left\{1 - \left(\frac{6}{C_2^{18}}\right)\right\}$$

$$= 5.0567595 \times 10^{-3}$$

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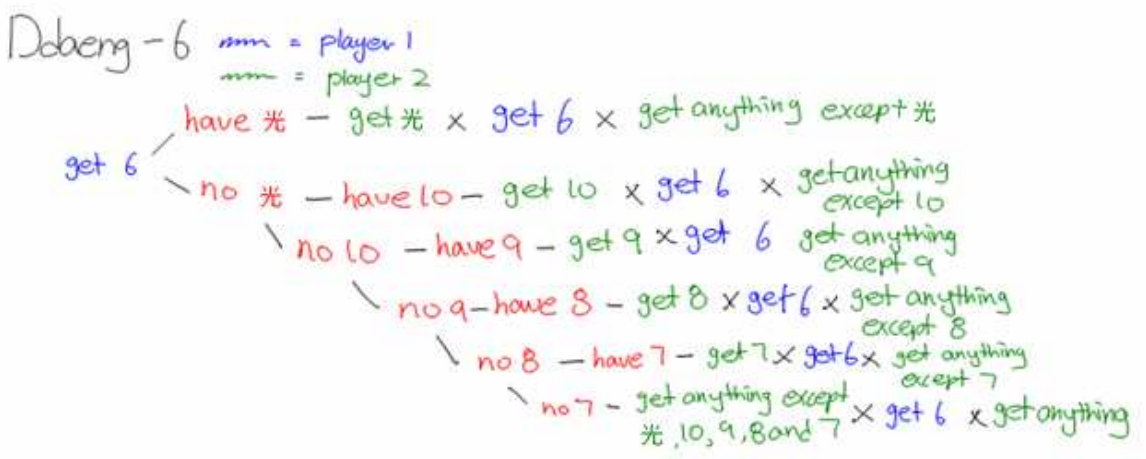
Probability of winning with June (Ddaeng-6) is split into seven different possibilities.

Graph 5. Tree Diagram of Ddang-6

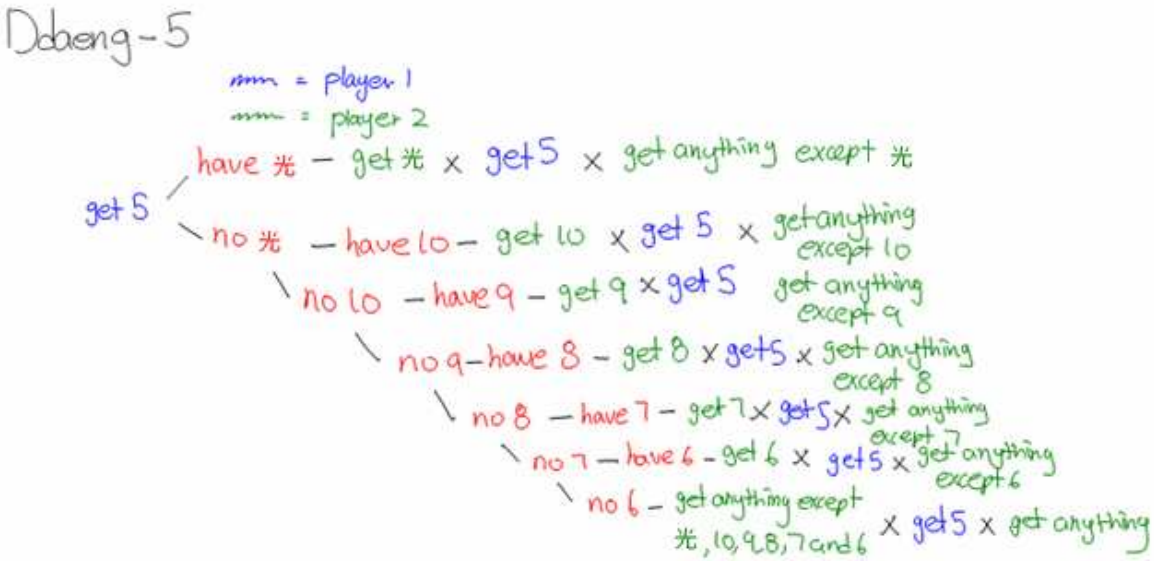


$$P(\text{win}) = \frac{1}{C_2^{20}} \times \left\{1 - \left(\frac{7}{C_2^{18}}\right)\right\}$$

$$= 5.0225698 \times 10^{-3}$$

Probability of winning with May (Ddaeng-5) is split into seven different possibilities.

Graph 6. Tree Diagram of Ddang-5



$$P(\text{win}) = \frac{1}{C_2^{20}} \times \left\{1 - \left(\frac{8}{C_2^{18}}\right)\right\}$$

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$= 4.9879601 \times 10^{-3}$

Probability of winning with April (Ddaeng-4) is split into eight different possibilities

Graph 7. Tree Diagram of Ddang-4

$$P(\text{win}) = \frac{1}{C_2^{20}} \times \left\{1 - \left(\frac{9}{C_2^{18}}\right)\right\}$$

$= 4.9545604 \times 10^{-3}$

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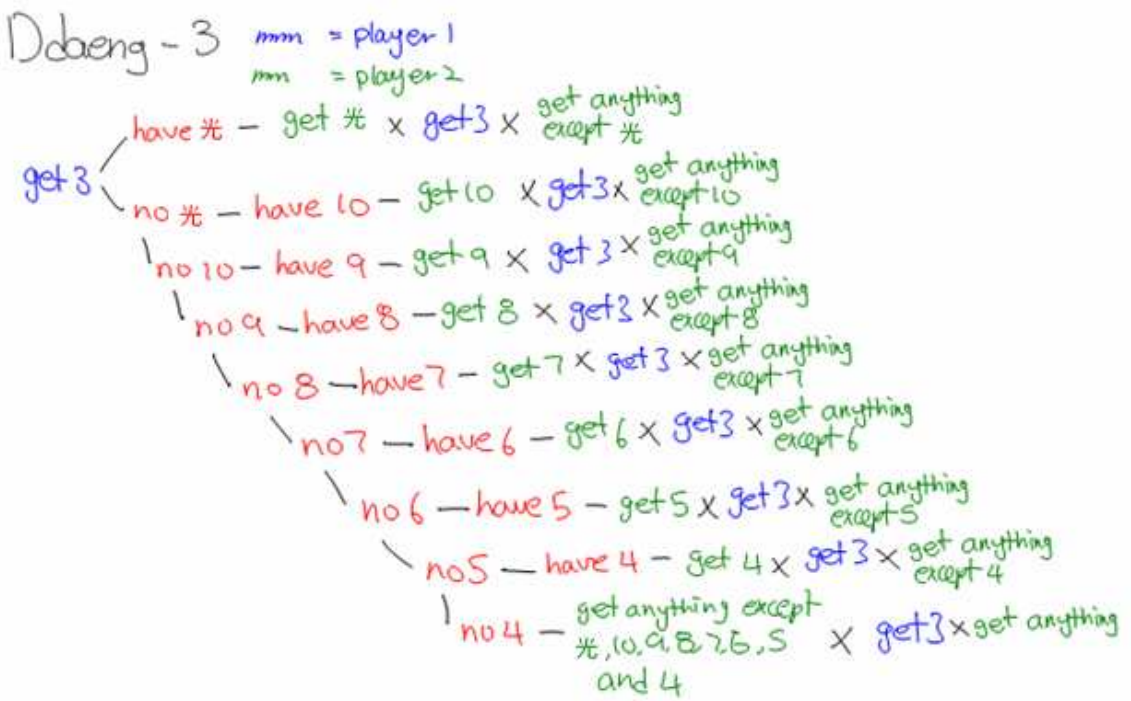
Probability of winning with March (Ddaeng-3) is split into nine different possibilities.

Graph 8. Tree Diagram of Ddang-3



For Ddaeng-3, it is slightly different than other hands. This is because Ddaeng-3 includes a card that has 光. As this hand requires 光 card to complete the hand, 3-8 Guang-Ddaeng and 1-3 Guang-Ddaeng cannot be created. Hence these two hands were eliminated from the probability of opponent winning.

$$P(\text{win}) = \frac{1}{C_2^{20}} \times \left\{1 - \left(\frac{8}{C_2^{18}}\right)\right\}$$

$= 4.9879601 \times 10^{-3}$

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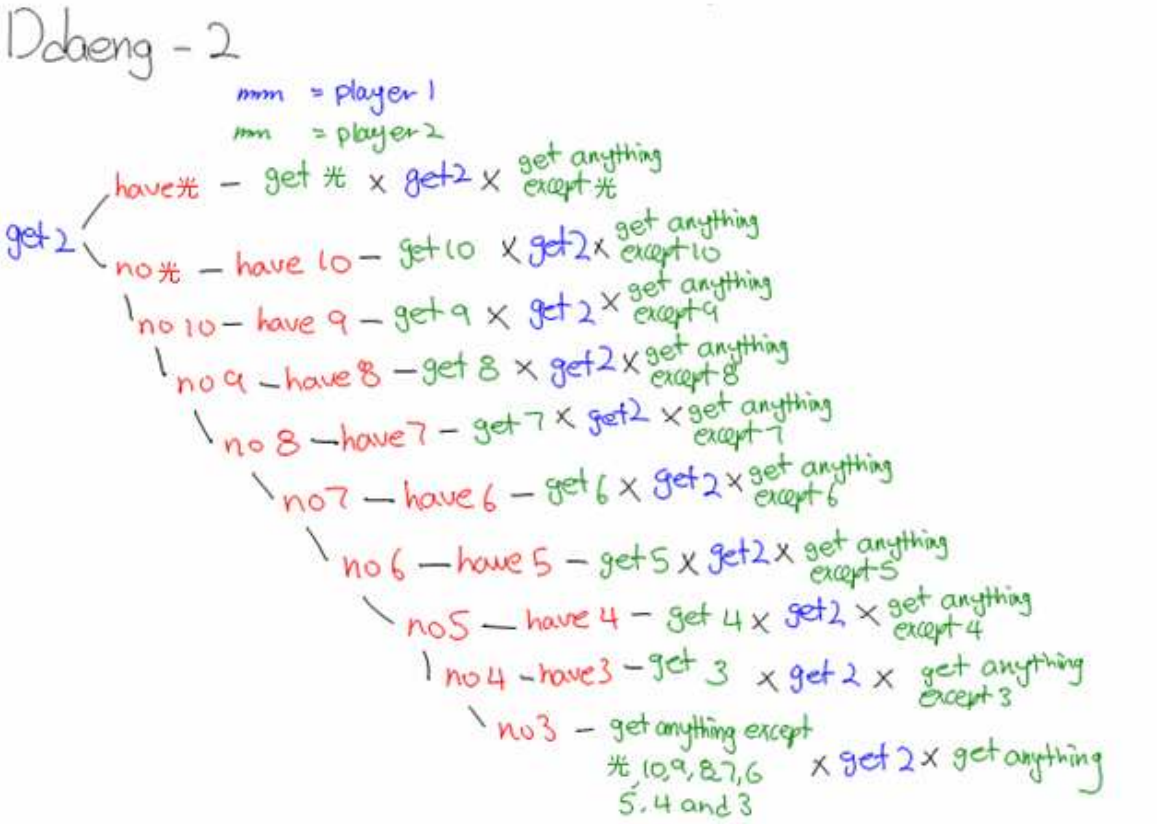
Probability of winning with February (Ddaeng-2) is split into ten different possibilities.

Graph 9. Tree Diagram of Ddang-2



$$P(\text{win}) = \frac{1}{C_2^{20}} \times \left\{1 - \left(\frac{11}{C_2^{18}}\right)\right\}$$

$$= 4.8847609 \times 10^{-3}$$

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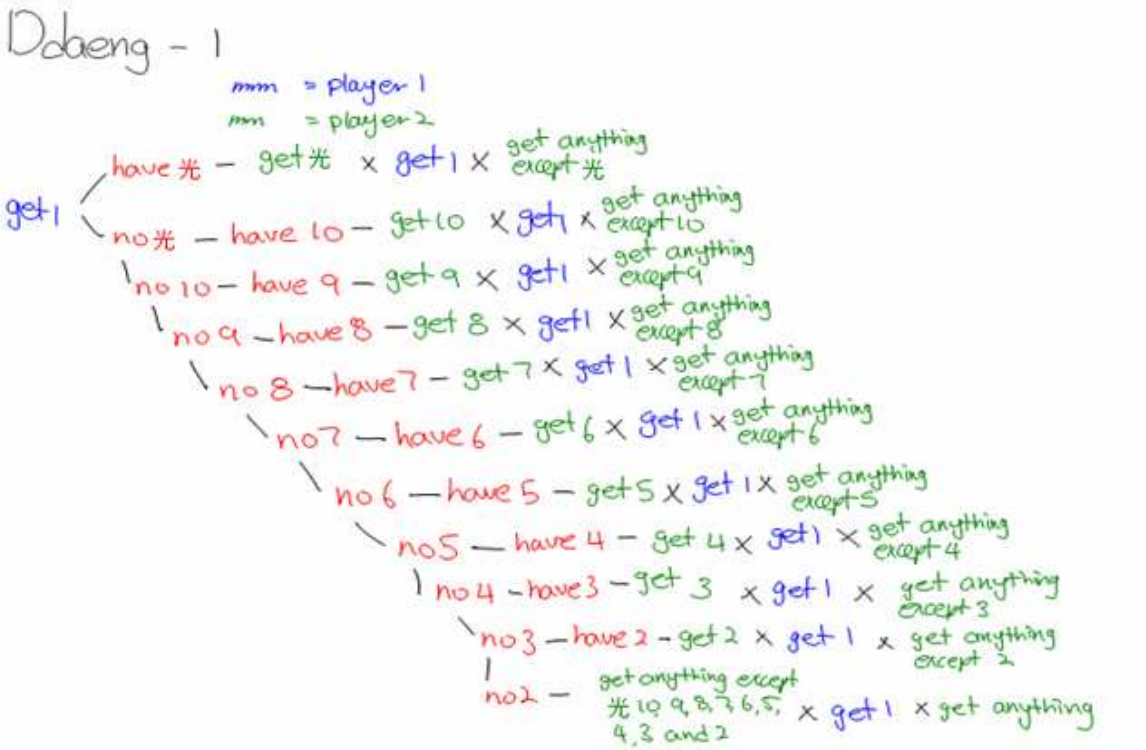
Probability of winning with January (Ddaeng-1) is split into eleven different possibilities.

Graph 10. Tree Diagram of Ddang-1



For this hand, it is similar with Ddaeng-3. Because this hand requires 光 card, 1-3 Guang-Ddaeng and 1-8 Guang-Ddaeng cannot be created. Hence these two hands were eliminated from the probability of opponent winning.

$$P(\text{win}) = \frac{1}{C_2^{20}} \times \left\{1 - \left(\frac{10}{C_2^{18}}\right)\right\}$$

$$= 4.9191606 \times 10^{-3}$$

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Ali

Probability for player 1 to get Ali is $2/^{20}C_2$. Ali could be formed using one January and one February card; hence there are four combinations and $^{20}C_2$ represents getting two cards from a deck of 20 cards. However, this needs to be split into two different ways where the January card may be 光 card or not. Hence, the probability for player 1 to win with Ali is (1/2) x $(2/^{20}C_2)$ x (1 – $(10/^{18}C_2)$ – $(12/^{18}C_2)$. $(10/^{18}C_2)$ is the sum of probabilities where player 2 will win when player 1 has Ali with January 光 card. $(12/^{18}C_2)$ is the sum of probabilities where player 2 will win then player 1 has Ali with normal January card.

Hence, by subtracting from 1, 2nd bracket represents the sum of probabilities of player 2 receiving a lower hand than player 1.

P(player 1 win with Ali) $= \frac{1}{2} \times \frac{2}{C_2^{20}} \times \left\{2 - \left(\frac{10}{C_2^{18}}\right) - \left(\frac{12}{C_2^{18}}\right)\right\}$

$= 9.76952185 \times 10^{-3}$

Dok-Sa

Probability for player 1 to get Dok-Sa is $2/^{20}C_2$. Dok-Sa could be formed using one January and one April card; hence there are two combinations and $^{20}C_2$ represents getting two cards from a deck of 20 cards. Again split into player 1 having 光 card or not.

P(player 1 win with Dok-Sa) =

$$\frac{1}{2} \times \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{10}{C_2^{18}}\right) + \left(\frac{12}{C_2^{18}}\right) + 2(1.95390437 \times 10^{-2})\right\}\right]$$

$= 9.5638477 \times 10^{-3}$

Gu-Bbing

Probability for player 1 to get Gu-Bbing is $2/^{20}C_2$. Gu-Bbing could be formed using one January and one September card; hence there are two combinations and $^{20}C_2$ represents getting two cards from a deck of 20 cards. Again split into player 1 having 光 card or not.

P(player 1 win with Gu-Bbing) =

$$\frac{1}{2} \times \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{10}{C_2^{18}}\right) + \left(\frac{12}{C_2^{18}}\right) + 2(0.0386667391)\right\}\right]$$

$= 9.36250355 \times 10^{-3}$

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Jang-Bbing

Probability for player 1 to get Jang-Bbing is $2/{}^{20}C_2$. Jang-Bbing could be formed using one January and one October card; hence there are four combinations and ${}^{20}C_2$ represents getting two cards from a deck of 20 cards. Again split into player 1 having 光 card or not.

P(player 1 win with Jang-Bbing)=

$$\frac{1}{2} \times \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{10}{C_2^{18}}\right) + \left(\frac{12}{C_2^{18}}\right) + 2(0.0573917462\right\}\right]$$

$$= 9.1653982 \times 10^{-3}$$

Jang-Sa

Probability for player 1 to get Jang-Bbing is $4/{}^{20}C_2$. Jang-Bbing could be formed using one April and one October card; hence there are four combinations and ${}^{20}C_2$ represents getting two cards from a deck of 20 cards.

The equation below represents, simply taking away the probability of opponent winning from 1 and multiplying by the probability for player 1 to receive a specific hand. Because to create Jang-Sa, no 光 cards are needed, firstly taking out all the Guang-Ddaeng and taking Ali, Dok-sa and Jang-Bbing.

P(win) $= \frac{1}{2} \times \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{12}{C_2^{18}}\right) + (0.757225426\right\}\right]$

$= 8.903643 \times 10^{-3}$

Sae-Luk

Probability for player 1 to get Jang-Bbing is $4/{}^{20}C_2$. Jang-Bbing could be formed using one April and one June card; hence there are four combinations and ${}^{20}C_2$ represents getting two cards from a deck of 20 cards.

The equation below represents, simply taking away the probability of opponent winning from 1 and multiplying by the probability for player 1 to receive a specific hand. Because to create Sae-Luk, no 光 cards are needed, firstly taking out all the Guang-Ddaeng and taking Ali, Dok-sa, Jang-Bbing and Jang-Sa.

P(win) $= \frac{1}{2} \times \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{12}{C_2^{18}}\right) + (0.0935298286)\right\}\right]$

$= 8.7161979 \times 10^{-3}$

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Gab-Oh

Gab-Oh could be created with five different combinations; 1&8, 2&7, 5&4, 10&9 and 3&6. For 1&8 and 3&6, they are slightly different from others because there is 光 in the hand, then it must be split into have 光 or no.

P(1&8) $= \frac{1}{C_2^{20}} \times \left[4 - \left\{\left(\frac{29}{C_2^{18}}\right) + \left(\frac{30}{C_2^{18}}\right) + \left(\frac{30}{C_2^{18}}\right) + \left(\frac{32}{C_2^{18}}\right)\right\}\right]$

$= 1.68902649 \times 10^{-2}$

P(2&7) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{38}{C_2^{18}}\right)\right\}\right]$

$= 1.58238734 \times 10^{-2}$

P(3&6) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{36}{C_2^{18}}\right) + \left(\frac{38}{C_2^{18}}\right)\right\}\right]$

$= 1.59614723 \times 10^{-2}$

P(4&5) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{34}{C_2^{18}}\right)\right\}\right]$

$= 1.6374269 \times 10^{-2}$

P(10&9) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{36}{C_2^{18}}\right)\right\}\right]$

$= 1.60990712 \times 10^{-2}$

P(all) $= 1.68902649 \times 10^{-2} + 1.58238734 \times 10^{-2} + 1.59614723 \times 10^{-2} + 1.6374269 \times 10^{-2} + 1.60990712 \times 10^{-2}$

$= 8.11489508 \times 10^{-2}$

Gget-8

Gget-8 could be created with four different combinations; 1&7, 2&6, 3&5 and 8&10. Again on 1&7, 3&5 and 8&10, each contains a card with 光, therefore, it must be split into have 光 or no.

P(1&7) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{45}{C_2^{18}}\right) + \left(\frac{47}{C_2^{18}}\right)\right\}\right]$

$= 1.47230822 \times 10^{-2}$

P(2&6) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{51}{C_2^{18}}\right)\right\}\right]$

$= 1.4035088 \times 10^{-2}$

P(3&5) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{53}{C_2^{18}}\right) + \left(\frac{55}{C_2^{18}}\right)\right\}\right]$

$= 1.42052758 \times 10^{-2}$

P(8&10) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{51}{C_2^{18}}\right) + \left(\frac{49}{C_2^{18}}\right)\right\}\right]$

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$= 1.4172687 \times 10^{-2}$

P(all) $= 1.47230822 \times 10^{-2} + 1.4035088 \times 10^{-2} + 1.42052758 \times 10^{-2} + 1.4172687 \times 10^{-2}$

$= 5.9219698 \times 10^{-2}$

Gget-7

Gget-7could be created with five different combinations; 1&6, 2&5, 3&4, 8&9 and 7&10. Again on 1&6, 3&4 and 8&9, each contains a card with 光, therefore, it must be split into have 光 or no.

P(1&6) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{56}{C_2^{18}}\right) + \left(\frac{54}{C_2^{18}}\right)\right\}\right]$

$= 1.3484692 \times 10^{-2}$

P(2&5) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{66}{C_2^{18}}\right)\right\}\right]$

$= 1.1971104 \times 10^{-2}$

P(3&4) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{68}{C_2^{18}}\right) + \left(\frac{66}{C_2^{18}}\right)\right\}\right]$

$= 1.1833505 \times 10^{-2}$

P(8&9) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{66}{C_2^{18}}\right) + \left(\frac{64}{C_2^{18}}\right)\right\}\right]$

$= 1.2108703 \times 10^{-2}$

P(7&10) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{64}{C_2^{18}}\right)\right\}\right]$

$= 1.2246302 \times 10^{-2}$

P(all) $= 1.3484692 \times 10^{-2} + 1.1971104 \times 10^{-2} + 1.1833505 \times 10^{-2} + 1.2108703 \times 10^{-2} + 1.2246302 \times 10^{-2}$

$= 6.1644306 \times 10^{-2}$

Gget-6

Gget-6 could be created with four different combinations; 1&5, 2&4, 6&10 and 7&9. Again on 1&5, each contains a card with 光, therefore, it must be split into have 光 or no.

P(1&5) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{75}{C_2^{18}}\right) + \left(\frac{73}{C_2^{18}}\right)\right\}\right]$

$= 1.0870313 \times 10^{-2}$

P(2&4) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{77}{C_2^{18}}\right)\right\}\right]$

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$= 1.0457516 \times 10^{-2}$

P(6&10) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{79}{C_2^{18}}\right)\right\}\right]$

$= 1.0182319 \times 10^{-2}$

P(7&9) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{84}{C_2^{18}}\right)\right\}\right]$

$= 0.9356725 \times 10^{-2}$

P(all) $= 1.0870313 \times 10^{-2} + 1.0457516 \times 10^{-2} + 1.0182319 \times 10^{-2} + 0.9356725 \times 10^{-2}$

$= 4.0866873 \times 10^{-2}$

Gget-5

Gget-5 could be created with four different combinations; 2&3, 5&10, 6&9 and 7&8. Again on 2&3 and 7&8, each contains a card with 光, therefore, it must be split into have 光 or no.

P(2&3) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{81}{C_2^{18}}\right) + \left(\frac{79}{C_2^{18}}\right)\right\}\right]$

$= 1.0044720 \times 10^{-2}$

P(5&10) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{79}{C_2^{18}}\right)\right\}\right]$

$= 1.0182319 \times 10^{-2}$

P(6&9) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{81}{C_2^{18}}\right)\right\}\right]$

$= 0.9907121 \times 10^{-2}$

P(7&8) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{83}{C_2^{18}}\right)\right\}\right]$

$= 0.9631923 \times 10^{-2}$

P(all) $= 1.0044720 \times 10^{-2} + 1.0182319 \times 10^{-2} + 0.9907121 \times 10^{-2} + 0.9631923 \times 10^{-2}$

$= 3.9766083 \times 10^{-2}$

Gget-4

Gget-4 could be created with three different combinations; 1&3, 5&9 and 6&8. Again on 1&3 and 6&8, each contains a card with 光, therefore, it must be split into have 光 or no.

P(1&3) $= \frac{1}{C_2^{20}} \times \left[4 - \left\{\left(\frac{102}{C_2^{18}}\right) + \left(\frac{100}{C_2^{18}}\right) + \left(\frac{100}{C_2^{18}}\right) + \left(\frac{99}{C_2^{18}}\right)\right\}\right]$

$= 0.7258342 \times 10^{-2}$

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P(5&9) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{106}{C_2^{18}}\right)\right\}\right]$

$= 0.6467148 \times 10^{-2}$

P(6&8) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{108}{C_2^{18}}\right) + \left(\frac{106}{C_2^{18}}\right)\right\}\right]$

$= 0.6329549 \times 10^{-2}$

P(all) $= 0.7258342 \times 10^{-2} + 0.6467148 \times 10^{-2} + 0.6329549 \times 10^{-2}$

$= 2.0355039 \times 10^{-2}$

Gget-3

Gget-3 could be created with four different combinations; 3&10, 4&9, 5&8 and 6&7. Again on 3&10 and 5&8, each contains a card with 光, therefore, it must be split into have 光 or no.

P(3&10) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{114}{C_2^{18}}\right) + \left(\frac{112}{C_2^{18}}\right)\right\}\right]$

$= 5.3663571 \times 10^{-3}$

P(4&9) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{115}{C_2^{18}}\right)\right\}\right]$

$= 5.2287582 \times 10^{-3}$

P(5&8) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{119}{C_2^{18}}\right) + \left(\frac{117}{C_2^{18}}\right)\right\}\right]$

$= 4.8159615 \times 10^{-3}$

P(6&7) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{115}{C_2^{18}}\right)\right\}\right]$

$= 5.3663571 \times 10^{-3}$

P(all) $= 5.3663571 \times 10^{-3} + 5.2287582 \times 10^{-3} + 4.8159615 \times 10^{-3} +$

5.3663571×10^{-3}

$= 2.10746478 \times 10^{-2}$

Gget-2

Gget-2 could be created with three different combinations; 3&9, 4&8 and 5&7. Again on 3&9 and 4&8, each contains a card with 光, therefore, it must be split into have 光 or no.

P(3&9) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{130}{C_2^{18}}\right) + \left(\frac{128}{C_2^{18}}\right)\right\}\right]$

$= 3.3023736 \times 10^{-3}$

P(4&8) $= \frac{2}{C_2^{20}} \times \left[2 - \left\{\left(\frac{132}{C_2^{18}}\right) + \left(\frac{130}{C_2^{18}}\right)\right\}\right]$

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$= 3.0271758 \times 10^{-3}$

P(5&7) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{128}{C_2^{18}}\right)\right\}\right]$

$= 3.4399725 \times 10^{-3}$

P(all) $= 3.3023736 \times 10^{-3} + 3.0271758 \times 10^{-3} + 3.4399725 \times 10^{-3}$

$= 9.7694769 \times 10^{-3}$

Gget-1

Gget-1 could be created with four different combinations; 2&9, 3&8, 4&7 and 5&6. Again on 3&8, each contains a card with 光, therefore, it must be split into have 光 or no.

P(2&9) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{139}{C_2^{18}}\right)\right\}\right]$

$= 1.9263846 \times 10^{-3}$

P(3&8) $= \frac{1}{C_2^{20}} \times \left[4 - \left\{\left(\frac{139}{C_2^{18}}\right) + \left(\frac{137}{C_2^{18}}\right) + \left(\frac{137}{C_2^{18}}\right) + \left(\frac{136}{C_2^{18}}\right)\right\}\right]$

$= 2.1671827 \times 10^{-3}$

P(4&7) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{137}{C_2^{18}}\right)\right\}\right]$

$= 2.2015824 \times 10^{-3}$

P(5&6) $= \frac{4}{C_2^{20}} \times \left[1 - \left\{\left(\frac{135}{C_2^{18}}\right)\right\}\right]$

$= 2.4767802 \times 10^{-3}$

P(all) $= 1.9263846 \times 10^{-3} + 2.1671827 \times 10^{-3} + 2.2015824 \times 10^{-3} + 2.4767802 \times 10^{-3}$

$= 8.7719299 \times 10^{-3}$

Mang-Tong

Mang-Tong could be created with two different combinations; 2&8 and 3&7. Again on 2&8 and 3&7, each contains a card with 光, therefore, it must be split into have 光 or no. However, probability of winning with Mang-Tong is 0. Because, since this hand is the lowest hand of all the possible hand in the game, hence when a player receives Mang-Tong, he automatically loses the game.

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Table 1. Summary Table for the Theoretical Probability of each Hand

Name of the Hand	Probability
3-8 Guang Ddaeng	0.00526315790
Guang Ddaeng	0.01578947370
Ddaeng-10	0.00515995872
Ddaeng-9	0.00512555900
Ddaeng-8	0.00515995870
Ddaeng-7	0.00505675890
Ddaeng-6	0.00502256980
Ddaeng-5	0.00498796010
Ddaeng-4	0.00495456040
Ddaeng-3	0.00498796010
Ddaeng-2	0.00488476090
Ddaeng-1	0.00491916060
Ali	0.00976952185
Dok-Sa	0.00956384770
Gu-Bbing	0.00936250355
Jang-Bbing	0.00916539820
Jang-Sa	0.00890364300
Sae-Luk	0.00871619790
Gab-Oh	0.08114895080
Gget-8	0.05921969800
Gget-7	0.06164430600
Gget-6	0.04086687300
Gget-5	0.03976608300
Gget-4	0.02355039000
Gget-3	0.02107464780

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Gget-2	0.00976947690
Gget-1	0.00877192990
Mang-Tong	0.00000000000

Practical Probability

These probability in table 2 was gained through number of experiments under the two conditioned mentioned in the beginning, where there are two players playing the game and the deck is shuffled ten times before the game starts to create a fair deck. The total number of games played is 1000 and there were 484 win and 516 lose.

Table 2. Summary Table for the Practical Probability of each Hand

Name of the Hand	Number of Hands Received	Probability
3-8 Guang Ddaeng	4	0.004
Guang Ddaeng	8	0.008
Ddaeng-10	6	0.006
Ddaeng-9	6	0.006
Ddaeng-8	8	0.008
Ddaeng-7	6	0.006
Ddaeng-6	4	0.004
Ddaeng-5	4	0.004
Ddaeng-4	6	0.006
Ddaeng-3	4	0.004
Ddaeng-2	6	0.006
Ddaeng-1	6	0.006
Ali	14	0.014
Dok-Sa	12	0.012
Gu-Bbing	28	0.028
Jang-Bbing	6	0.006
Jang-Sa	10	0.01
Sae-Luk	8	0.008

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Gab-Oh	78	0.078
Gget-8	54	0.054
Gget-7	66	0.066
Gget-6	42	0.042
Gget-5	38	0.038
Gget-4	26	0.026
Gget-3	18	0.018
Gget-2	8	0.008
Gget-1	8	0.008
Mang-Tong	0	0

Table 3. Percentage Difference between Theoretical and Practical Probability

Name of the Hand	Theoretical Probability	Practical Probability	Percentage Difference (%)
3-8 Guang Ddaeng	0.00526315790	0.004	49.33
Guang Ddaeng	0.01578947370	0.008	19.53
Ddaeng-10	0.00515995872	0.006	16.28
Ddaeng-9	0.00512555900	0.006	17.06
Ddaeng-8	0.00515995870	0.008	55.04
Ddaeng-7	0.00505675890	0.006	18.65
Ddaeng-6	0.00502256980	0.004	20.36
Ddaeng-5	0.00498796010	0.004	19.81
Ddaeng-4	0.00495456040	0.006	21.10
Ddaeng-3	0.00498796010	0.004	19.81
Ddaeng-2	0.00488476090	0.006	22.83
Ddaeng-1	0.00491916060	0.006	21.97
Ali	0.00976952185	0.014	43.30
Dok-Sa	0.00956384770	0.012	25.47
Gu-Bbing	0.00936250355	0.028	199.07
Jang-Bbing	0.00916539820	0.006	34.54
Jang-Sa	0.00890364300	0.010	12.31

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Sae-Luk	0.00871619790	0.008	8.22
Gab-Oh	0.08114895080	0.078	3.88
Gget-8	0.05921969800	0.054	8.81
Gget-7	0.06164430600	0.066	7.07
Gget-6	0.04086687300	0.042	2.77
Gget-5	0.03976608300	0.038	4.44
Gget-4	0.02355039000	0.026	10.40
Gget-3	0.02107464780	0.018	14.59
Gget-2	0.00976947690	0.008	18.11
Gget-1	0.00877192990	0.008	8.80
Mang-Tong	0.00000000000	0.000	0.00

Conclusion and Evaluation

The theoretical probabilities for each hand were calculated under the assumption that an event occurs under certain probability. Also, the experimental probabilities for each hand were recorded under the trial and error experiments.

Comparing these two probabilities, it was expected that experimental probability would not be similar to the theoretical probability when there are small frequencies of trials but it would get closer to the theoretical probability as the number of trials increased. For example, when the die with six sides was thrown six times, the data may not be spread equally. However, when it was thrown 600 times, then the data would then be evenly spread. Considering the same concept, this game, Sut-Da, was played 1000 times.

The theoretical probability of winning was around 0.473. However experimentally, player 1 won 484 times out of 1000 which ended up being 0.484. There was only 2.33 percentage difference between the theoretical and experimental probability. Also, for hands such as; Guang Ddaeng, Ddaeng-10, Ddaeng-9, Ddaeng-7, Ddaeng-5, Ddaeng-

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3, Jang-Sa, Sae-Luk, Gab-Oh, Gget-8, Gget-7, Gget-6, Gget-5, Gget-4, Gget-3, Gget-2 and Gget-1, the experimental probabilities were closer to the theoretical values. These hands had less than 20 percentage difference between the theoretical and experimental probability.

Also, there were hands that had a percentage difference which were greater than 20 percent. Hands such as, 3-8 Guang Ddaeng, Ddaeng-6 and Jang-Bbing did not come as frequently as expected. Also, hands such as, Ddaeng-8, Ddaeng-4, Ddaeng-2, Ddaeng-1, Ali, Dok-Sa and Gu-Bbing, turned out to be more frequent than expected. Ddaeng-8 and Gu-Bbing had extremely high frequencies when compared to the theoretical values.

By looking at the comparison between theoretical probability and experimental probability, two conclusions can be made. The first conclusion is that the game setup was not balanced. In the introduction, it states that the deck will be shuffled ten times to create a fair deck. However, by repeating shuffling cards, it was observed that the cards did not mix as well and same hand appeared twice in a row. With this observation, if the shuffling of cards were performed in a more random way, the deck will become fairer.

The second conclusion is that the number of trials was too low to illustrate the overall spread of data. As the lowest probability is in the $\frac{1}{1000}$ range, it was expected that all the probabilities would be spread out when 1000 trials were performed. However, as stated above, some of the hands evenly appeared but there were also hands that were extremely higher or lower than the theoretical probability. Hence, it could be concluded that if the number of trials were increased, then the data will be evenly spread.

This game's rules are constructed into two main parts; selection and distribution and win or lose. Firstly, selection and distribution is about the cards that the players receive which always have a limitation. Selection and distribution deal with expectation which makes this game a game where the player has limited information. Since there

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are only 20 cards and the cards are being dealt by the dealer, there is no choice of cards that the players can pick, and the cards are just given randomly. Also, no matter what a player receives, those two cards belong to only one hierarchy of hands. This becomes a limiting factor and the players are only allowed to participate in the game with the received cards; therefore, there is no choice for the players in what hands they receive.

The second part of the game deals with winning or loses. For this game 'Sut-Da', winning or loses is split into three parts; scarcity, rule and addition of the cards. Scarcity means that the hand with lowest probability of occurring will have a higher chance of winning. The rule describes situations where the probability is same, but one will win because it is in a higher place in the hierarchy. This applies to hands such as Ali, Dok-Sa, Gu-Bbing, Jang-Bbing, Jang-Sa and Sae-Luk. For example, to receive 1 and 4, has same probability as receiving 2 and 3. However, 1 and 4 is Ali and 2 and 3 is Gget-5. Since Ali is a higher hand in the hierarchy, Ali will win over Gget-5. The last part, the addition of cards acts as a balance throughout the game. Hands from Gab-Oh to Mang-Tong represent single digits after the cards are added. These hands balance the validity of the game as they redistrict the probability of winning to be just below 50 percent.

However, considering that the game is limited, a game where the player has limited information, there are not many situations in different parts of the world that undergoes similar concepts. However, this could be seen similarly in combats between two groups. For example, when there is a debate between two parties, then both sides are going through the same concept compared to 'Sut-Da'. Since they lack in information, they are handicapped in preparing for the discussion, however, they do have choice in what they will do in order to win the discussion.

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To conclude, it was obtained by the calculations that the percentage difference between the theoretical probability and the experimental probability is similar for some hands but also have huge difference in other hands. To fix this problem, when the number of trials were increased, then the probability would be more spread out than by just having 1000 trials. Also, this concept of 'Sut-Da' could be seen in the combat between two groups such as debating.

Appendices

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Appendix 1. Photos of My Friend and I Playing 'Sut-Da'

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