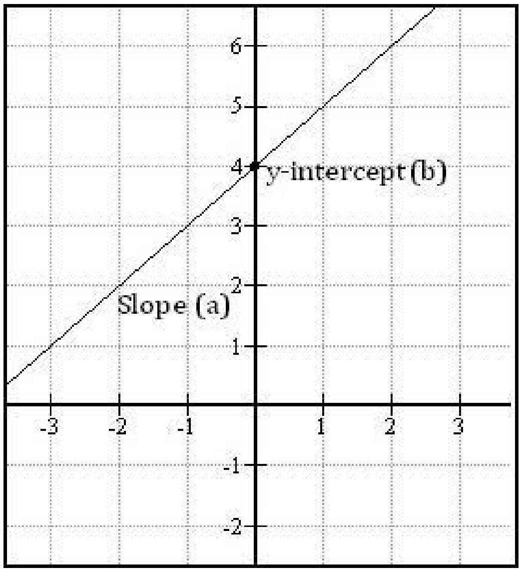
06/04 2011 William Frisch Møller and Marta Maillet Tapias,
MYP 4, NGG ID

The Straight Line

Slope-Intercept Form



The slope intercept form is probably the most frequently used way to express the equation of a line. The equation can be written in many different ways[1], but taken we are in Denmark and are part of a Danish school the equation would be:

$$\boldsymbol{y = ax + b}$$

Where:

$y = the\ y\ coordinate\ (how\ far\ up)$

$x = the\ x\ coordinate\ (how\ far\ along)$

$a = the\ slope/gradient\ (how\ steep\ the\ line\ is)$

$b = the\ y\ intercept\ (where\ the\ line\ crosses\ the\ y\ axis)$

The slope-intercept form is a type of linear equation. A linear equation is simply an algebraic equation in which each term is either a constant (fixed number) or the product of a constant and (the first power of) a single variable.

Y-intercept

The Y intercept of a straight line is simply where the line crosses the Y axis, thus it requires no calculation to find.

Examples

1. *Find the y-intercept for the following equation.*
   - $y = 4x + 2$

   Finding the y-intercept given an equation like this is extremely easy if you know your slope-intercept form, as all one has to do is find the value substituting the letter ‘b’ in the original equation:

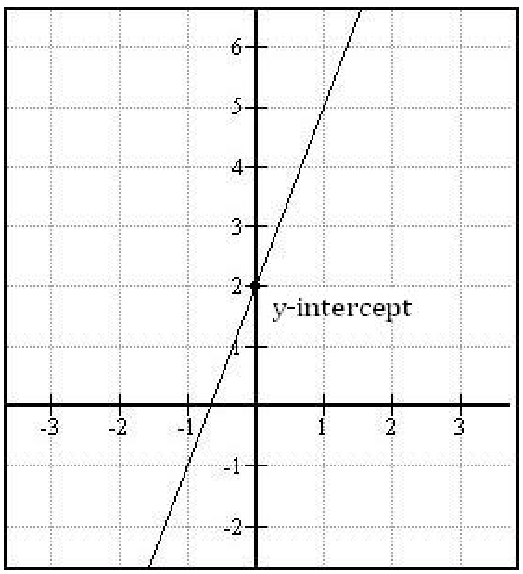
[1] See ’Notation’ in the Appendix

$$y = ax + b$$

$$y = 4x + 2$$

Thus the y-intercept for the above equation is '2'.

2. Find the y-intercept for the following straight line.



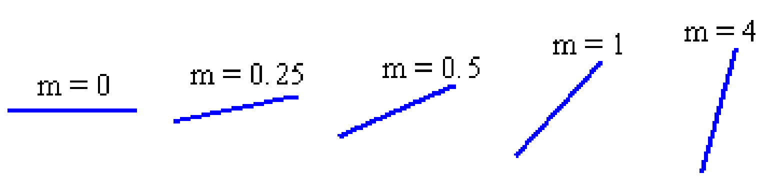
When you are given an actual line instead of an equation the question is slightly different, but then again it isn't, as finding the y-intercept of a straight line doesn't require any sort of calculation either.

In order to find the intersection you simply follow the line with your finger within the coordinate system (Cartesian Coordinate System) until you cross over the y axis. The grid number where the line crossed the y axis is the y-intercept of the line.

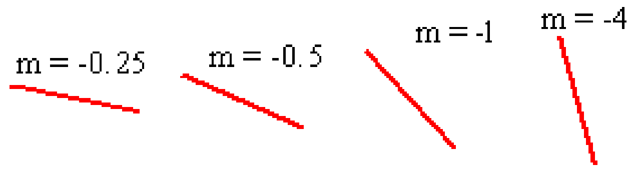
Thus the answer to the question must be '2' as that is where the line meets the y axis.

Slope/Gradient

The slope (gradient) ultimately determines the 'steepness' or incline of a line, the higher the slope, the steeper the incline will be. For example, a horizontal line has a slope equal to zero while a line with an angle of 45° has a slope equal to one.



The sign (positive or negative) of the slope is very important, as it determines whether the line slopes uphill or downhill. Positive slopes (like the ones above) mean that the line slopes uphill, from left to right. Meanwhile if negative, the line slopes downhill, from right to left.



The slope of a line is usually represented as the letter 'a', and is defined as being the change in the y coordinate (rise) divided by the corresponding change in the x coordinate (run).

$$a = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

(The Greek symbol 'Δ', is commonly used in mathematics to indicate 'difference' of 'change')

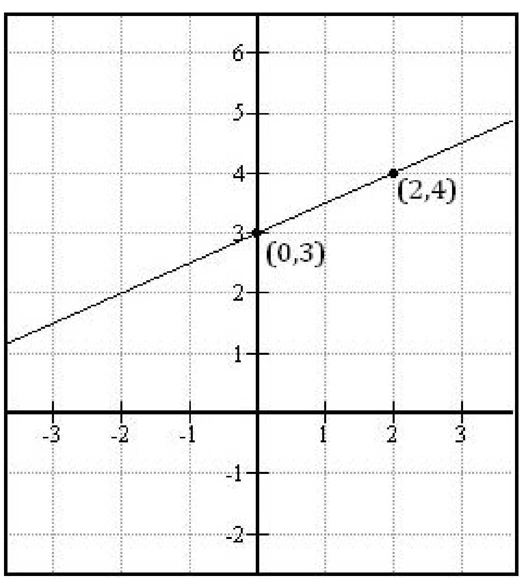
Given two points (x_1, y_1) and (x_2, y_2), the change in the x coordinate would be the same as $x_2 - x_1$. Meanwhile, in order to find the change in the y coordinate one would have to subtract y_1 from y_2. Therefore, by simply substituting both quantities into the above equation, we are left with the slope formula.

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

Knowing this formula we can calculate the slope of any straight line, which can become very helpful if you need to find the actual equation of a line.

Examples

1. *Given two points, (2,4) and (0,3), find the slope of the following straight line.*



Finding the slope of any straight line is really quite simple, that is if you remember the slope formula. Basically all you do is substitute the letters of the equation with the points we were given:

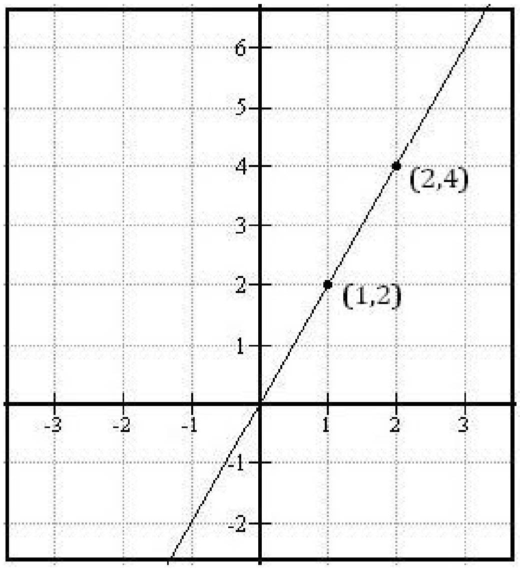
$$a = \frac{4-3}{2-0}$$

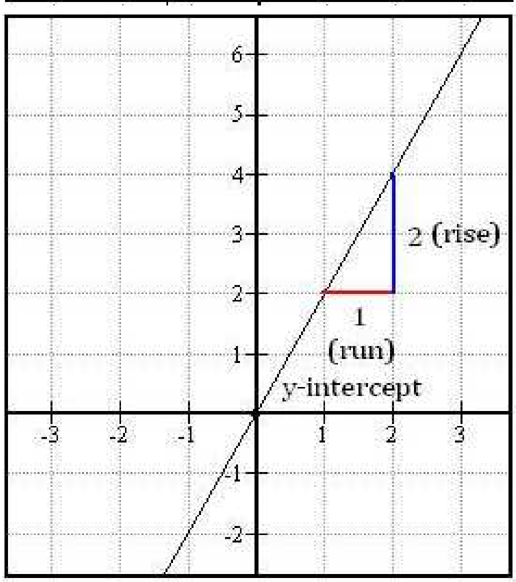
$$a = \frac{1}{2}$$

$$a = 0.5$$

So the slope of the straight line on the right is '0.5'.

2. *Given two points, (2,4) and (1,2), find the equation of the following straight line.*





This question can be sub-divided into two steps, first one being finding the slope of the line.

Finding the slope of a line like this with two known points is really quite simple, that is if you remember the slope formula.

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

O[illegible]:

$$a = \frac{rise}{run}$$

All you do is substitute the given points, (2,4) and (1,2), with the $(y_2 - y_1)$ and the $(x_2 - x_1)$ in the formula. This leaves us with the following:

$$a = \frac{4-2}{2-1}$$

$$a = \frac{2}{1}$$

$$a = 2$$

Now that we have got the slope we can move on to the next step which is to find the y-intercept, the place where the line crosses the y axis. Then we simply have to replace the letters in the slope-intercept form with the

Vertical Line

A vertical line is a line of which is parallel to the y-axis, which simply means that all points on the line will have the same x-coordinate. A vertical line is a special case as it has no slope. Or put another way, for a vertical line the slope is undefined. The equation of a vertical line will therefore be:

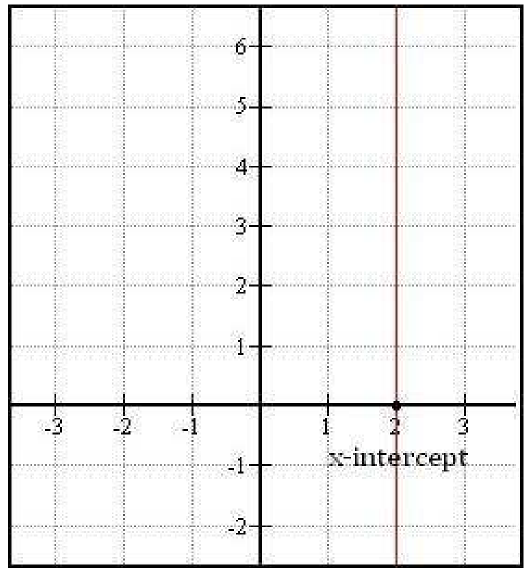
$$x = a$$

Where:

$x = the\ x\ coordinate$

$a = the\ x\ intercept\ (where\ the\ line\ croses\ the\ x\ axsis)$

Notice that the equation is independent of y. Any point on the vertical line satisfies the equation.



Perpendicular Lines

Perpendicular lines are straight lines of which intersect to form a 90° angle (right angle). Take two different lines:

$$L: y = ax + b$$

$$M: y = cx + d$$

Then, in this case 'a' and 'c' are the slopes of the two lines. The lines 'L' and 'M' are only perpendicular if and only if the product of their slopes is -1. In this case:

$$ac = -1$$

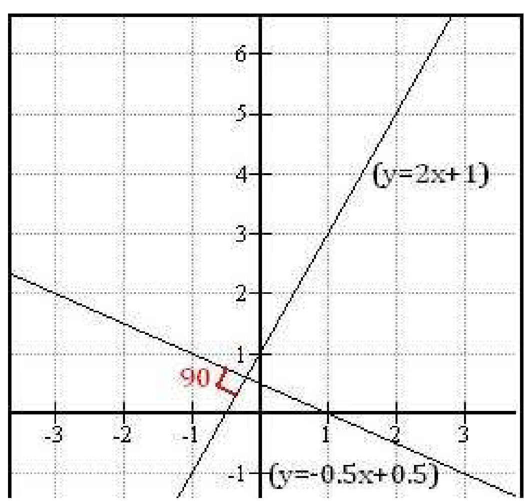
Examples

1. *Are the following two linse perpendicular?*
   - $y = 2x + 1$
   - $y = -0.5x + 0.5$

   In order to figure out whether the lines are perpendicular or not, we need to find their slopes.

$$y = \mathbf{2}x + 1$$

$$y = \mathbf{-0.5}x + 0.5$$



As you can see in the graph above the angle of intersection is 90°

Now that we have got the slopes

$$2 \times -0.5 = -1$$

If the sum of the slopes are equal to '-1' the lines are perpendicular, thus the answer to the question is yes.

2. Do the following two lines intersect to form a 90º angle?

- $y = 2x + 1$
- $y = -2x + 1$

Although 'worded' differently, the question asks for the same thing, whether or not the above lines are perpendicular. Thus the first step is to find 'a', their slopes.

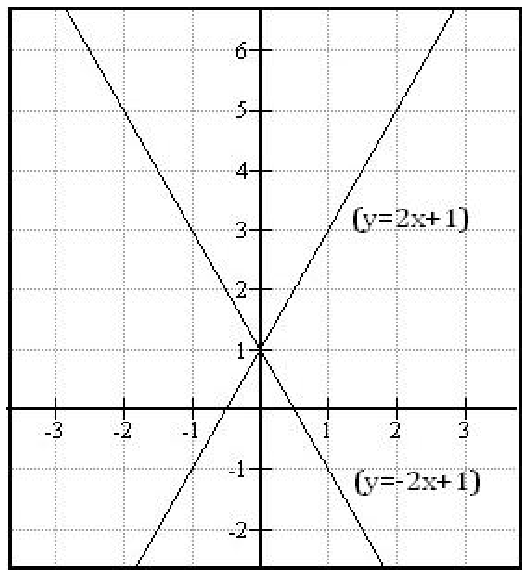
$$y = \mathbf{2}x + 1$$

$$y = \mathbf{-2}x + 1$$

Now all we need to do is to multiply the values we got for the slope:

$$2 \times -2 = -4$$

The answer to the question will therefore be no, as the sum of the lines slopes did not give '-1'.



As you can see in the graph above, unlike the other example, the angle of intersection is not 90º, thus the lines aren't perpendicular

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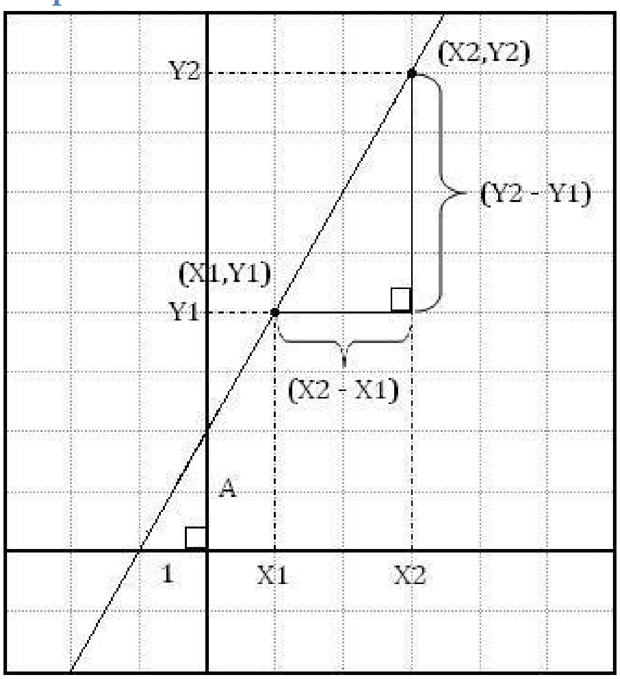
Appendix

Proof

Slope Formula:



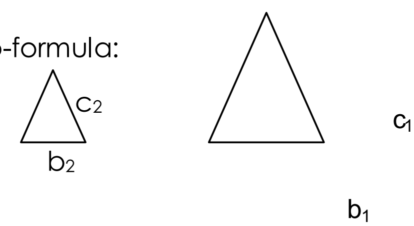
Take two unknown points, (x_1,y_1) and (x_2,y_2). We know that in order to calculate the difference in height (rise) between the two different points we have to subtract one 'y' value from the other:

$$(y_2 - y_1)$$

As for the length (run) we will have to subtract the 'x' values by each other:

$$(x_2 - x_1)$$

Using our help-formula:



$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

We know that the triangle, a and 1, is rationally the same as the triangle, $(y_2 – y_1)$ and $(x_2 – x_2)$. This means that by simply using the same principles as the above equation we get the slope formula:

$$\frac{1}{x_2 - x_1} = \frac{a}{y_2 - y_1}$$

$$\frac{1(y_2 - y_1)}{x_2 - x_1} = \frac{a(y_2 - y_1)}{y_2 - y_1}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = a$$

$$y_2 - y_1$$

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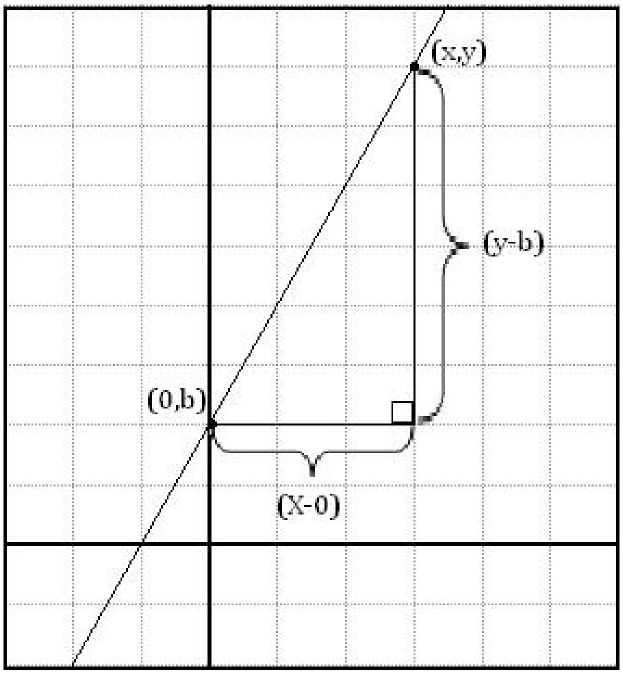
Slope-Intercept Form:



Take two points, (x,y) and (0,b), (0,b) because of the fact that 'b' represents the y-intercept of a line (b is in this case the y value) and '0' because the 'x' value of this specific point is indeed '0'.

We know that in order to calculate the difference in height (rise) between the two different points we have to subtract one 'y' value from the other:

$$(y - b)$$

As for the length (run) we will have to subtract the 'x' values by each other:

$$(x - 0)$$

Our help-formula:

Plays a big role when it comes to getting the actual slope-intercept form as we, by substituting our points with the ones from the above help-formula, we get:

$$a = \frac{y - b}{x}$$

$$ax = y - b$$

Now all we need to do is rearrange the formula and 'voila', the slope-intercept form:

$$\boldsymbol{y = ax + b}$$

Notation

Different Countries teach different "notation".

US, Canada, Egypt, Mexico, and Philippines:

$\boldsymbol{y = mx + b}$

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UK, Australia, Bahamas, Bangladesh, Belgium, Brunei, Cyprus, Germany, Ghana, India, Indonesia, Ireland, Jamaica, Kenya, Kuwait, Malaysia, Malawi, Malta, Nepal, Netherlands, New Zealand, Nigeria, Pakistan, Singapore, Solomon Islands, South Africa, Sri Lanka, Turkey, UAE, Zambia and Zimbabwe:

$$y = mx + c$$

Albania, Brazil, Czech Republic, Denmark, Ethiopia, France, Lebanon, Holland, Kyrgyzstan and Vietnam:

$$y = ax + b$$

Azerbaijan, China, Finland, Russia and Ukraine:

$$y = kx + b$$

Greece:

$$\psi = \alpha\chi + \beta$$

Italy:

$$y = mx + q$$

Japan:

$$y = mx + d$$

Latvia:

$$y = jx + t$$

Romania:

$$y = gA + c$$

Sweden:

$$y = kx + m$$

Slovenia:

$$y = kx + n$$

The point is that it does not matter whether the 'slope/gradient' is defined as an 'm', 'a' or a 'b', as all three letters ultimately represent the same initial thing.