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IB Student
DP1 HL Mathematics
Internal Assessment Type II

My Internal Assessment

In this internal assessment, I am going to create a recursive formula for a mathematic model. In this investigation, I use real life examples and uses real data to finish my investigation. Afterward, I place my model to real life case study. However, there are different limitations in different cases. Some of the limitations are dimensional plane, speed, boundaries and more. At last, I am going to present my recursive formula, $\frac{X(n)}{2(n)u - Y(n)} = \frac{X(n+1)}{2(n)u - Y(n+1)}$.

The speed of Ada and Fay

After reading the Internal Assessment question sheet, I had an idea on using a similar real life case study. Then, I invite my cousin, Ada and her dog, Fay, to have a running test and let me collect a set of data for my Internal Assessment. Therefore, I set a straight track with a distance of 100 meters and they need to run for 10 times. So, I can collect a set of accurate data. After each of them run for 10 times, I will take an average time for the 100 meters run and then uses the speed formula, $v = \frac{S}{T}$. In this formula, V stands for velocity, which is the speed; the S stands for distance, at last, T stands for the period

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of time the runner runs. Below is Ada's data for running 100 meters:

Ada's running records

Trails	*Time (s)*
1	16.6
2	16.7
3	16.8
4	16.7
5	16.6
6	16.5
7	16.6
8	16.8
9	16.5
10	16.7

After collecting Ada's set of data, then I need to define the average time of Ada running a 100 meters track, in the other word, the mean of the data. Therefore, I am going to use the mean equation from the statistic chapter, which is $\bar{X} = \frac{1}{n}\sum x$. The calculation will be as follow,

$$\bar{X} = \frac{1}{n}\sum x$$

$$= \frac{1}{10}(16.6+16.7+16.8+16.7+16.6+16.5+16.6+16.8+16.5+16.7)$$

$$= 16.7$$

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From the calculation, I define that the average time of Ada running 100 meters is 16.7 seconds. Furthermore, after finding the time of Ada running 100 meters, I need to define the speed Ada runs in the experiment. To find the velocity, I will apply an equation that I use both in Physic and in Mathematic, which is $v = \frac{s}{t}$. Therefore, by plugging the "s" and "t" values, then the result of the calculation will be the velocity. The calculation will be as follow,

$$\begin{aligned} v &= \frac{s}{t} \\ &= \frac{100}{16.7} \\ &= 5.988 \end{aligned}$$

After calculation, I found that the velocity of Ada is 5.988 m/s. Due to further easier calculation; I decide to round up Ada's speed to one significant figure. Therefore, her speed for running 100 meters is 6.0 m/s. Moreover, after defining the speed of Ada, it is time to investigate Fay's velocity of running 100 meters. The method of finding Fay's velocity is same as finding Ada's

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velocity. Below there will be table presenting Fay's record of running 100 meters for 10 trails.

Fay's running records

Trails	*Time (s)*
1	12.3
2	12.5
3	12.5
4	12.4
5	12.3
6	12.6
7	12.5
8	12.6
9	12.7
10	12.6

Then, I am going to define the average velocity of Fay's running time by using the mean formula.

$$\bar{X} = \frac{1}{n}\sum x$$
$$= \frac{1}{10}(12.3+12.5+12.5+12.4+12.3+12.6+12.5+12.6+12.7+12.6)$$
$$= 12.5$$

After finding Fay's average time, which is 12.5 seconds. Then, I am going to define the speed of Fay's runs, by the speed formula. The calculation will be as follow,

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$$v = \frac{s}{t}$$
$$= \frac{100}{12.5}$$
$$= 8$$

After the calculation, the outcome of Fay's running velocity is 8 m/s. Therefore, now I had define the velocity of Ada's running speed and Fay's speed. Furthermore, I am going to use the information I got above to construct a diagram out, which simulates the situation.

Diagram involve both runner position and direction

In this diagram, there will be direction indicate where the runner is running and how they run in actual situation. Furthermore, I also set Ada's velocity as "u m/s" and Fay's velocity is "v m/s". Ada is running in a straight line along the harbor path using the velocity of 6 m/s and Fay is using 8m/s running towards Ada, which its direction will change due to Ada's position. Therefore, "u" will appear on the Y-axis and "v" will be on the X-axis.

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Diagram simulate "Running with Ada and Fay"

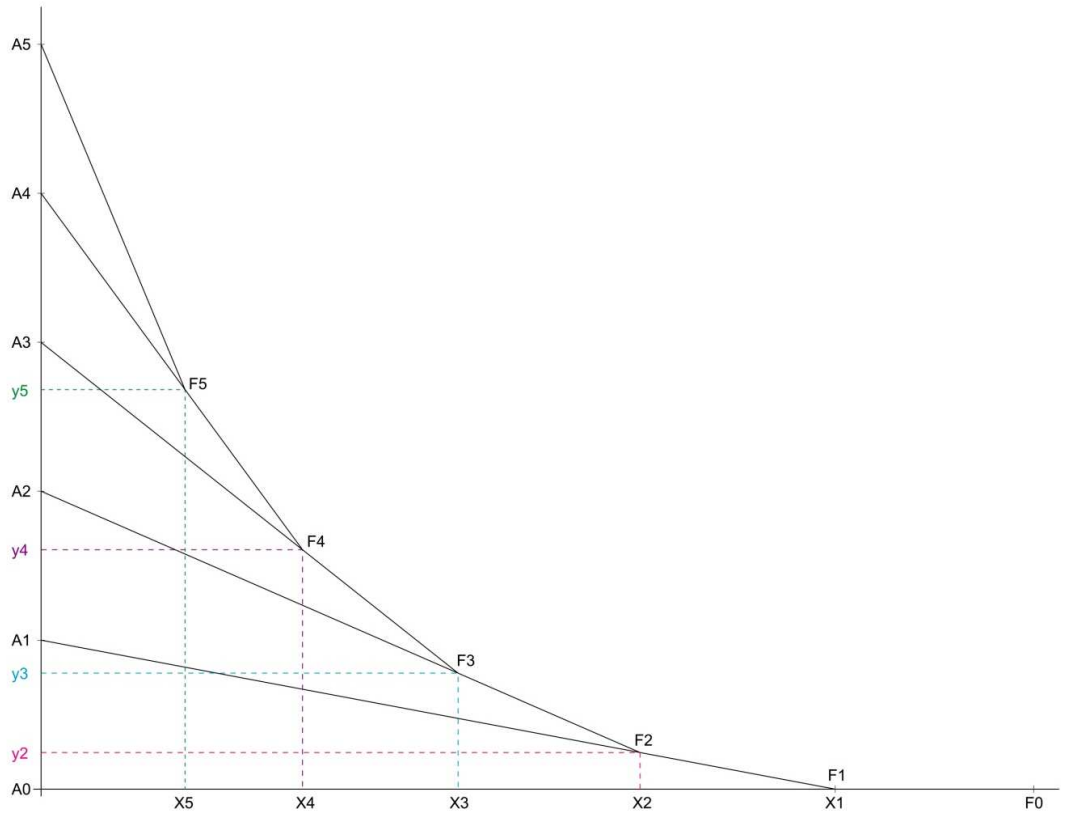

Diagram one: In this diagram it simulate the situation of Fay chasing Fay and the Y-axis is the direction Ada is running at and the broken curve with points named "F" in the front is the direct that Fay is running towards.

From the diagram above, on the Y-axis there are points "A0", "A1", and "A2", so on until it reaches "A5". Those points represent Ada's direction and her running distance. Due to the situation is every two seconds; Fay

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will look at Ada's new position. Therefore, the distance between the point "A1" and "A2" is 2u, which is 12 meters.

Furthermore, the points that represent Fay form different section in a line but not a curve. The reason is due to every 2 seconds Fay will change its direction according to Ada's new position. Taking an example from the diagram, the initial point of Fay position is at point "F0". After running for 2v, the distance Fay runs is 16 meters to point "F1". Then, Fay realizes that Ada has change her own position to "A1" and Fay changes it direction heading towards the point "A1". Therefore, Fay's route will construct triangle. This pattern will continue until Fay catches up with Ada.

Finding Fay's position in the diagram

According to the above paragraph, I have state that Fay's route will construct different size of triangle with the route it runs before and Ada's position. Therefore, finding Fay's coordinates I need to use trigonometry. I decide to use the method of Python's theorem and similar triangles to define the X and Y coordinates of Fay's position.

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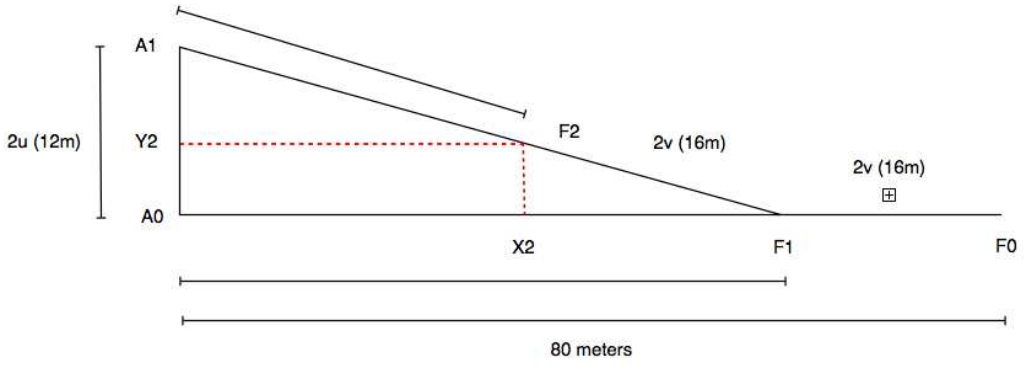
Diagram on finding Fay's position one



From this diagram, it shows several points that I need to find before finding the coordinate of Fay's position in "F1" and "F2". There are several lines "A1F2", "A0F1", "A1F1", "Y2F2" and "X2F2". After calculating the distance of these lines, then I will have a very clear and accurate coordinates for Fay's position.

There are different methods finding the distances of the lines. First, for lines "A1F2", "A0F1" and "A1F1", I am going to use the Python's theorem. The calculation of the lines will shown below,

$$\overline{A0F1} = 80 - 16$$
$$= 64$$

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After finding the distance of "A0F1", then I got enough information to use the Python's theorem to find the length of line "A1F1".

$$\begin{aligned} \overline{A1F1} &= \sqrt{A0F1^2 + A1A0^2} \\ &= \sqrt{64^2 + 12^2} \\ &= 65.115282 \end{aligned}$$

The length of "A1F1" is 65.115282 meters. Below will show the step to find "A1F2".

$$\begin{aligned} \overline{A1F2} &= A1F1 - 16 \\ &= 65.115282 - 16 \\ &= 49.1153 \end{aligned}$$

After calculation, I define the length of line "A1F2" is 49.1153 meters. After finding this lines' length, then I can use the propriety of similar triangle to define the length of "Y2F2" and "X2F2".

$$\begin{aligned} \frac{Y2F2}{A0F1} &= \frac{A1F2}{A1F1} \\ \frac{Y2F2}{64} &= \frac{49.1153}{65.115282} \\ Y2F2 &= 48.2740 \end{aligned}$$

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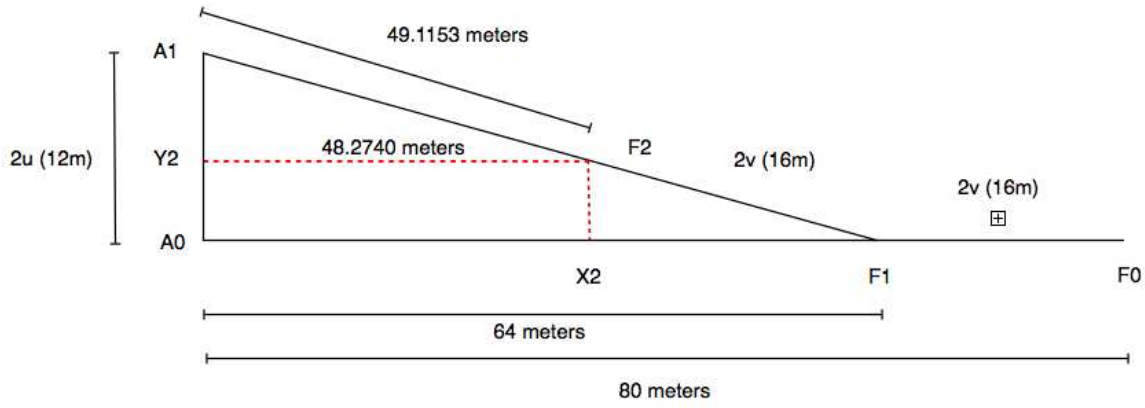
By finding "Y2F2", we can know the Y-coordinate of Fay's "F2" position.

$$\frac{F2X2}{A1A0} = \frac{F2F1}{A1F1}$$
$$\frac{F2X2}{12} = \frac{16}{65.115282}$$
$$F2X2 = 2.9486$$

After finish calculating the length of lines, "F2X2" and "Y2F2", now I got a full coordinate of Fay's position in point "F2", which is (48.2740, 2.9486).

Result of the diagram after finding all the unknowns



For other points, it is using the same methods to determinate their coordinates. I am going to show the calculation below to define the coordinate of point "F3". Afterward I will use the result to define a recursive

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formula for x- and y-coordinates of Fay's position to establish a discrete mathematical model.

Calculation for "F3" coordinate

$$\begin{aligned} A2Y2 &= A2A0 - Y2 \\ &= 24 - 2.9486 \\ &= 21.0514 \end{aligned}$$

Using Python's theorem, to find "A2F2",

$$\begin{aligned} A2F2 &= \sqrt{A2Y2^2 + Y2F2^2} \\ &= \sqrt{21.0514^2 + 48.2740^2} \\ &= 52.6644 \end{aligned}$$

Finding "A2F3",

$$\begin{aligned} A2F3 &= A2F2 - F3F2 \\ &= 52.6644 - 16 \\ &= 36.6644 \end{aligned}$$

Using the propriety of similar triangle to define the length of "Y3F3",

$$\begin{aligned} \frac{Y3F3}{Y2F2} &= \frac{A2F3}{A2F2} \\ \frac{Y3F3}{48.2740} &= \frac{36.6644}{52.6644} \\ Y3F3 &= 33.6078 \end{aligned}$$

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By using Python's theorem, to define the length of "a", which "a" is equals "F3X3-F2X2".

$$\begin{aligned} a &= \sqrt{F3X3^2 - (F2Y2 - F3Y3)^2} \\ &= \sqrt{16^2 - 14.6662^2} \\ &= 6.3955 \end{aligned}$$

Finding "F3X3",

$$\begin{aligned} a &= F3X3 - F2X2 \\ 6.3955 &= F3X3 - 2.9486 \\ F3X3 &= 9.3441 \end{aligned}$$

After finding length of line "F3X3" and "Y3F3", then I have calculated the coordinate of Fay's position after six seconds, which is (33.6078, 9.3441).

Define a recursive formula for x- and y-coordinates of Fay's position

After finding few coordinates on Fay position, I define there is a formula behind the whole calculation or a rule or a pattern I will say. I am going to form the formula by using the method of trigonometry. In the topic of trigonometry in our mathematic class, the teacher did mention that angle propriety, such as alternate angle,

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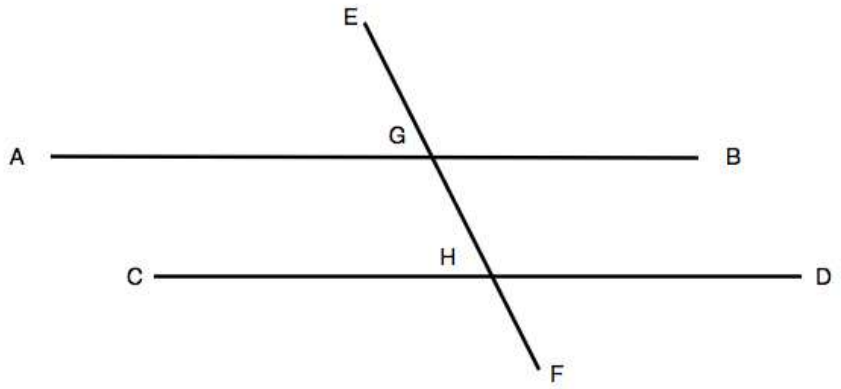
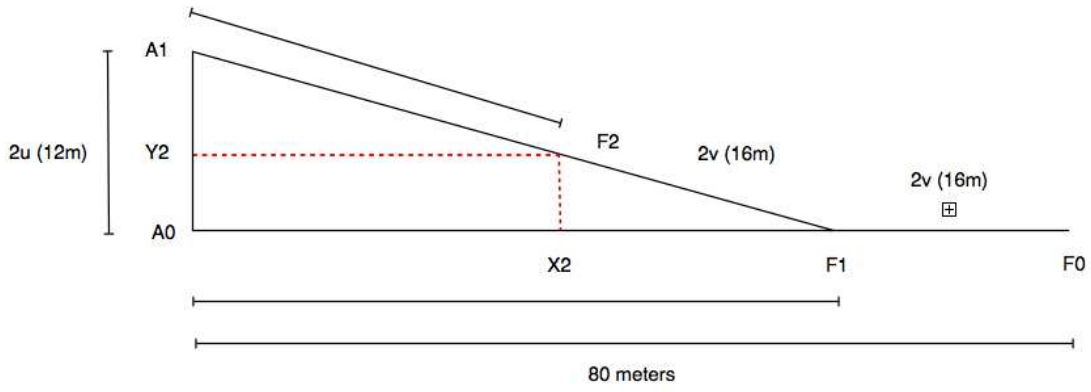
interior angle and corresponding angle. In the steps of finding the recursive formula, I am going to use the rule of corresponding angle.

Diagram of showing the corresponding angle rule



The rule of corresponding, says that the line AB and line CD need to parallel. Therefore, $\angle G$ will be equal to $\angle H$. By using the example of the first triangle, $\Delta F1A0A1$



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We know that Y2F2 is parallel with A0F1. Therefore, $\angle F1$ is an equal to $\angle F2$. By using the tangent formula in trigonometry,

For $\angle F1$ equals,

$$\tan\theta = \frac{2u}{X1}$$

For $\angle F2$ equals,

$$\tan\theta = \frac{2u - Y2}{X2}$$

By the corresponding angle rule, $\angle F1 = \angle F2$, which it means,

$$\begin{aligned} \angle F1 &= \angle F2 \\ \frac{2u}{X1} &= \frac{2u - Y2}{X2} \\ \frac{X1}{2u} &= \frac{X2}{2u - Y2} \end{aligned}$$

From the above calculation, I define the recursive rule for the mathematical model. After making improvement on the formula, the finial form of the recursive formula that simulate the model is,

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$$\frac{X(n)}{2(n)u - Y(n)} = \frac{X(n+1)}{2(n)u - Y(n+1)}$$

Plugging different value to the recursive formula

After finding the recursive formula, we need to prove it is correct and it can work as normal before calling it as a successful recursive formula. Therefore, now I am going to use the recursive formula to find the x-coordinate of the point "F2".

$$\frac{X1}{2(1)6 - Y1} = \frac{X(2)}{2(1)12 - Y2}$$

$$\frac{64}{12 - 0} = \frac{X2}{12 - 2.9480}$$

$$X2 = 48.2740$$

In this calculation, the result of X-coordinate is 48.2740. Referring back to the above, the result of the X-coordinate is also 48.2740. The result concludes that the recursive formula is successfully form and it is working normal. The formula can find any coordinates within this case not only the x-coordinate in the example; it can also define the y-coordinates also. Just remember to use the recursive formula, $\frac{X(n)}{2(n)u - Y(n)} = \frac{X(n+1)}{2(n)u - Y(n+1)}$ and then you can

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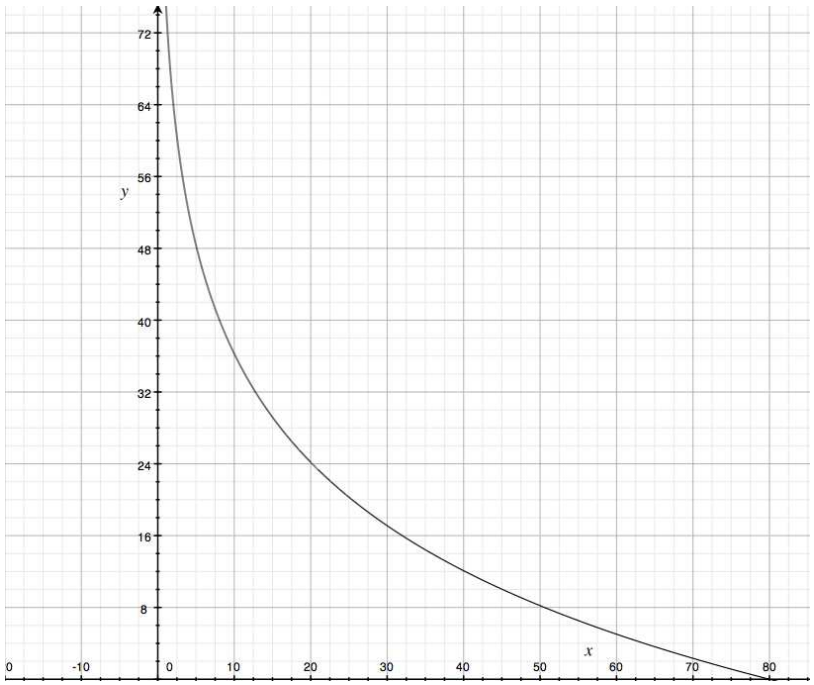
easily define the coordinates that you want to know about Fay's position.

Fay caught up with Ada

After finding few points, I plug the coordinates into my Graphic Display Calculator by using the statistic mode. After plugging the number, then I use the data to plot a graph and I chose to use logarithm function in the form of, $y = a+b\ln x$. Then, the calculator have to define the value of "a" and "b". At last, I use computer software; called "Grapher" to plot the function and the graph below is the one I plot with the computer software.



Graph One: Use "Grapher" to plot the graph with a logarithm function and it tells where will Fay caught up with Ada.

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From the graph, it tells when will Fay caught up with Ada, which is after Ada run for 140 meters. However, the graph also shows that Fay will only run parallel with Ada but not across Ada's path. Furthermore, the recursive can also be use in this situation to simulate when will Fay caught up with Ada. Due to the reason we do not know when, therefore, I am going to let "n" be "1", which is infinite. The expression will be as follow,

$$\frac{X(l)}{2(l)u - Y(l)} = \frac{X(l+1)}{2(l)u - Y(l+1)}$$

Assumption in "Running with Ada and Fay"

After working of the real life case study, "Running with Ada and Fay", which is similar from the case provide by the Internal assessment question sheet. I define that there are different real life cases, which also can be represent by the similar model provide by "Running with Ada and Fay". Furthermore, in the model represent "Running with Angie and Buddy", there are assumption create to model out the situation. Such as, both Angie and Buddy are running in their own constant speed, which is 6 m/s and 8 m/s representatively, Buddy does not like to get wet and will avoid taking a plunge into the lake and Buddy will looks up

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to notice Angie's new position every short period. These assumptions would not use in every single case.

Two different cases

Moreover, there are different cases in real world that can be model out by the same model. Below there will be two different cases present. The first case has related to an incident happen on me and the second one will be about fighter planes and missile. However, in different cases there will be assumptions and limitations.

"Chasing the private car for mobile"

Last Sunday, I went to have lessons in Tain Hau by my father's car. After I got off the car, I realize that I left my mobile in the car. Then I decide to chase the car and get back my mobile. Below there will be a diagram showing the route of the car and the way I should chase, if I am using the method of Buddy catches Angie in "Running with Ada and Fay". I will run behind the car and chase until my dad notices I am running after the car and he stopped the car.

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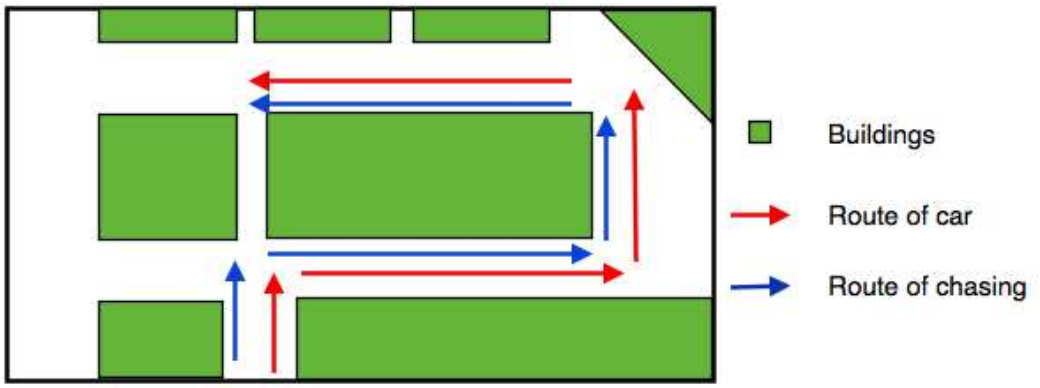
Chasing route using the method in "Running with Ada and Fay"



From the diagram, the total distance that I ran in this chase is about 250 meters. In the chase I sprit at a speed of 6.90 m/s, which is calculate by the speed formula $v = \frac{s}{t}$. However due to the affect of traffic, the car is moving slowly with a constant speed of 20 Km/h, in the other word 8.33 m/s. The unit is transfer by divide 30 km/h by 3.6 and we will get the answer of 8.33 m/s. Moreover, after 25 seconds of spiriting he realized I am running after the car, then he starts to reduce its speed to 10 km/h, which is 2.78 m/s and let me catch up with him. In additional, I chase up to my father's car after spiriting for 31.23 seconds. Furthermore, in every 5 seconds, I looked to my father's car new position and check whether that he changes his direction or not. There are assumptions set in this case, which is my father will

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reduce the speed of the car after realizing I am running behind him, I check the car's position every five seconds and I am running in a constant speed. There are also limitations, such as, I can only run on the path way, which is parallel with the road; I need to run in the same way as Buddy did in its case, "Running with Ada and Fay" and this is only a case in a 2-D dimension.

Unfortunately, on that day I did not chase my father's car with Buddy's method. I use a better and smart way, which is finding the shortest path to meet the car or run in a straight direction. They route I sprit was drawn below in the diagram. In this diagram, I sprit through the alley between two buildings and went to the back street and wait for my father, while he and the car still in the traffic.

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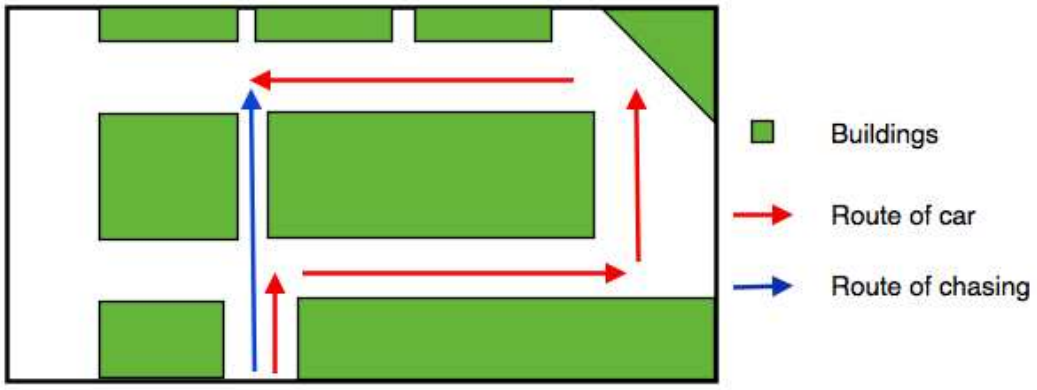
Chasing route usual human will use



"Fighter plane attack by missiles"

Another cases is "Fighter plane attack by missiles", which is about situation in war. In a war area, fighter planes are use fro air strikes and other an attacks. The main weapons the fighters planes will use while attacking enemy are missiles. When there is an enemy fighter plane in front, the pilot will uses missile to take down the enemy fighter planes. However, before the pilot fire out the missiles, they need to aim the target and send the message towards the computer chip located inside the missile and tells the missile where the target is. The computer chip runs a computer system in the missile, which will let the missile to re-route its track towards the target, if it changes its direction or increase in speed.

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In this example, the fighter plane called F/A-18E Super Hornet, which is a fighter plane build by Boeing from United States. Its maximum speed is Mach 1.8+, which is around 612.5 m/s.

Photo One: In this photo, it is the fighter aircraft F/A-18E Super Hornet

The F/A-18E Super Hornet carries missiles, which is called AIM-9 Sidewinder. AIM-9 Sidewinder is an air-to-air short-range heat-seeking missiles; in the NATO pilots the brevity code is "Fox Two".

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Photo Two: In this photo, it is the AIM-9 Sidewinder. In this case, the missile launch by the F/A-18E Super Hornet.

AIM-9 Sidewinder has a top speed of March 2.5 (850.7 m/s) after it launched by the F/A-18E Super Hornet. The F/A-18E Super Hornet fighter plane launched the missile, its target is Yak-141 Freestyle fighter plane, which is from Soviet Union, and the distance between the missile and plane is 2000 meters.

Photo Three: In this photo, it is the fighter aircraft Yak-141 Freestyle

Yak-141 Freestyle's top flying speed is 1800 km/h, which is 500 m/s. If we assume that all elements in this demonstration are flying in it's maximum speed and the computer chip in AIM-9 Sidewinder will re-calculate its track toward the target every 0.2 seconds. There are many limitations, which make us cannot fully illustrate the situation. The limitations that cause the situation can not

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be fully illustrate are the time of the attack will be very short, the missile can attack the Yak-141 Freestyle fighter plane in different angle, such as, from the top, under or side ways, due to the sky is a three dimensional plane.

Chasing by a missile

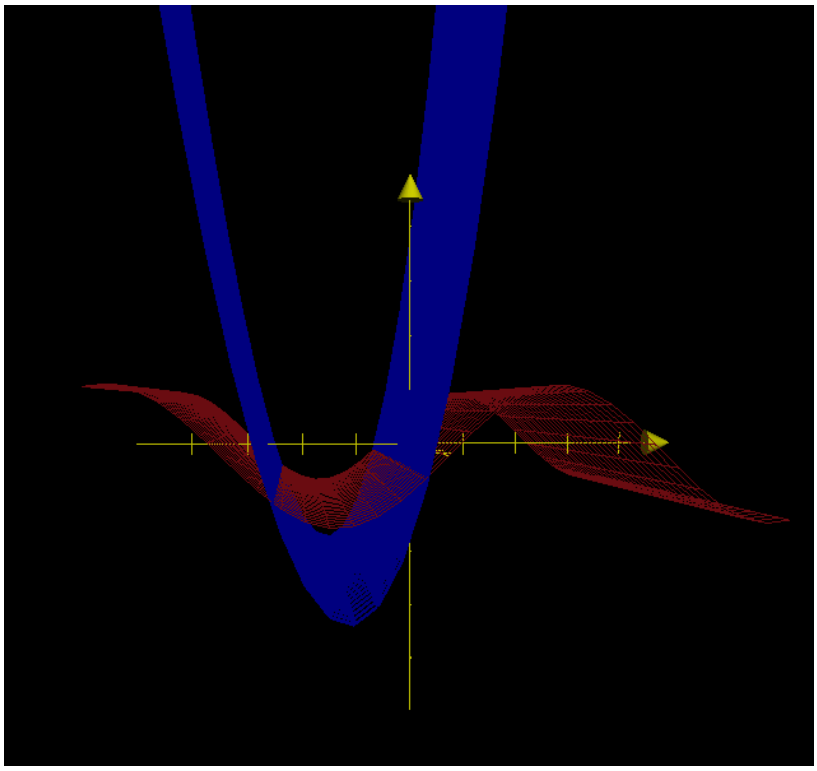

Diagram Two: In this diagram, there is the blue graph represents the missile; in the other word, the red graph represents the enemy fighter aircraft, which is being chase by the missile. From the diagram, it indicates the direction of both the missile and the aircraft. In conclusion, it simulates the situation whiling chasing in a three-dimension plane.

Conclusion

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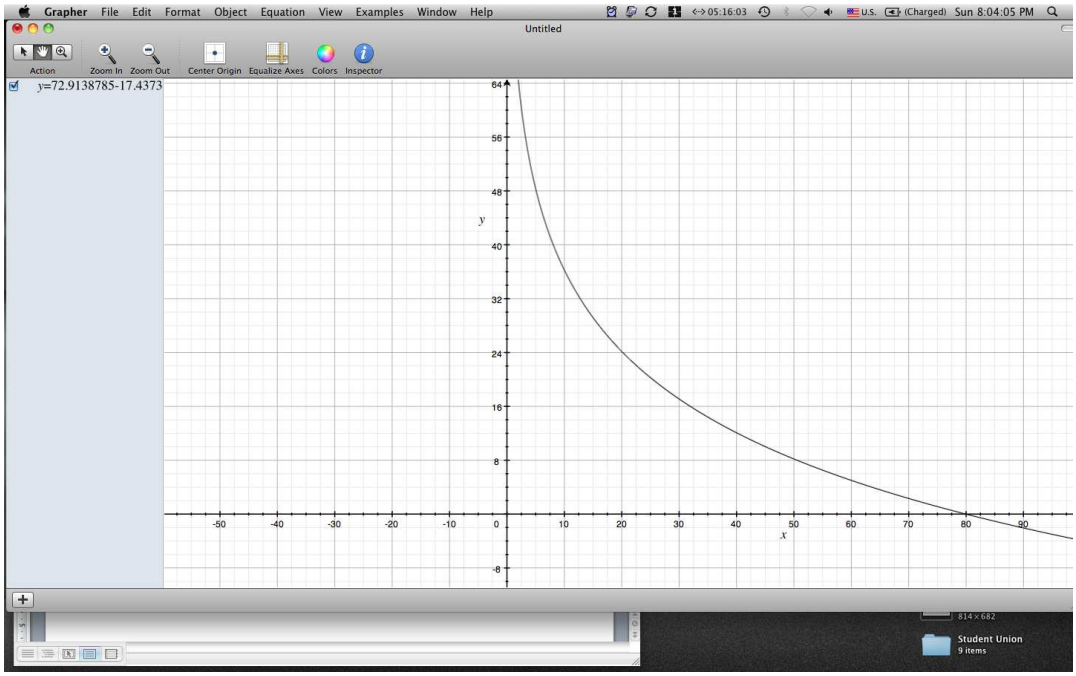
Throughout the internal assessment, I use real data instead of creating it. In this assessment, I include different methods, equation to help me to define the mean speed of Ada and Fay, to calculate the position of Fay and form a recursive formula and more.

In this assessment, I also use different kind of computer software to present my mathematic mode and finding solutions. I use "Grapher" in Mac to plot different graphs to solve problem and looked closely to the situation I got on my hand.



Snapshot One: Using the graphing software, Grapher by Apple

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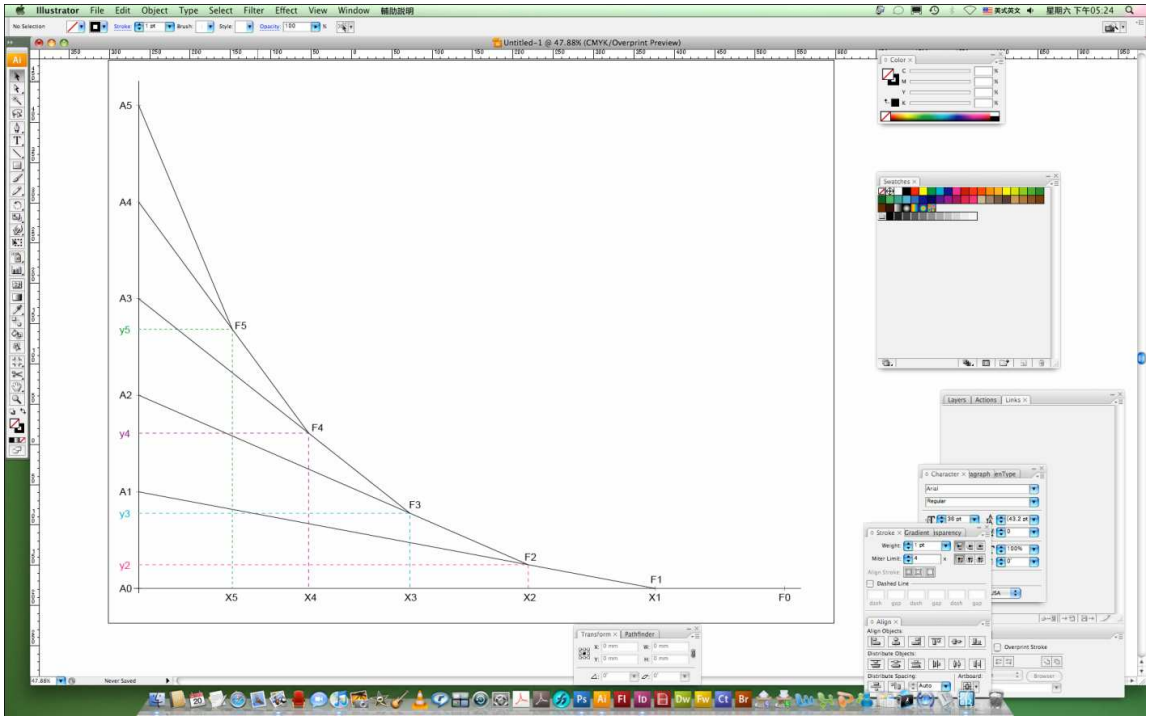
Except using "Grapher", I also use AI (Adobe Illustrator) to draw the out line of Fay's and Ada's route. Then, I printed out and use for calculation. This software gave me a very accurate diagram that makes my calculation much more accurate and faster.



Snapshot Two: Using Adobes Illustrator, to draw the diagram

At last, I use my friend while doing mathematics, which is my calculator, Casio fx-9860GII SD. The GDC help me to find the logarithm function for simulating the situation. In addition, it helps me to calculate different kinds of math problems while I am doing my internal assessment.

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Due to the help of technology and my mathematic textbook, which was written by Smythe. I finally form the recursive formula to the mathematical model in this internal assessment, which is $\frac{X(n)}{2(n)u - Y(n)} = \frac{X(n+1)}{2(n)u - Y(n+1)}$

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