

INTERNATIONAL BACCALAUREATE

MATHEMATICS SL

PORTFOLIO TYPE ONE

LOGARITHM BASES

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LOGARITHM BASES

INTRODUCTION

The reasoning behind conducting this investigation is to identify patterns in logarithmic sequences. Furthermore, after identifying the patterns, one must produce a general statement expressing the general trend within the sequence. Once forming a general statement, one must be able to rewrite logarithms in the form of a fraction. This investigation tests the acquired knowledge of logarithms by testing the general statement using various values, forming limitations, and proving scopes. In addition, further investigation may be conducted through the use of graphs, tables of values, and charts.

This assignment is looking for a formula that will give the n^{th} term for the sequence.

This table gives the next two terms of each sequence.

n^{th}	<i>sequence(1)</i>	<i>sequence(2)</i>	<i>sequence(3)</i>	<i>sequence(4)</i>
1	$\log_2 8$	$\log_3 81$	$\log_5 25$	$\log_m m^k$
2	$\log_4 8$	$\log_9 81$	$\log_{25} 25$	$\log_{m^2} m^k$
3	$\log_8 8$	$\log_{27} 81$	$\log_{125} 25$	$\log_{m^3} m^k$
4	$\log_{16} 8$	$\log_{81} 81$	$\log_{625} 25$	$\log_{m^4} m^k$
5	$\log_{32} 8$	$\log_{243} 81$	$\log_{3125} 25$	$\log_{m^5} m^k$
6	$\log_{64} 8$	$\log_{729} 81$	$\log_{15625} 25$	$\log_{m^6} m^k$
7	$\log_{128} 8$			

I created the last two terms in this sequence, terms 6 and 7 (5 & 6), simply by doubling the base of the logarithm for each term.

By following these sequences a pattern can be shown. The base of each term in the sequences changes but the index is constant.

Finding an expression for the n^{th} term for each sequence:

To find the n^{th} term we need to solve the logarithms by using the change of base law.

$$\log_a b = \frac{\log_k b}{\log_k a}$$

$$1. \log_2 8 = \frac{\log_{10} 8}{\log_{10} 2} = 3 \text{ (using GDC)}$$

$$\log_4 8 = \frac{\log_{10} 8}{\log_{10} 4} = 1.5 \text{ or } \frac{3}{2}$$

$$\log_8 8 = \frac{\log_{10} 8}{\log_{10} 8} = 1 \text{ or } \frac{3}{3}$$

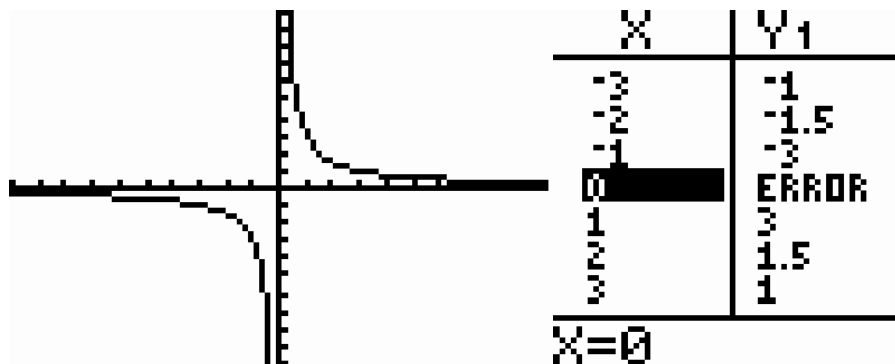
$$\log_{16} 8 = \frac{\log_{10} 8}{\log_{10} 16} = .75 \text{ or } \frac{3}{4}$$

$$\log_{32} 8 = \frac{\log_{10} 8}{\log_{10} 32} = .6 \text{ or } \frac{3}{5}$$

$$\log_{64} 8 = \frac{\log_{10} 8}{\log_{10} 64} = .5 \text{ or } \frac{3}{6}$$

Thus, the n^{th} term is $\frac{3}{n}$ where p is 3 and q is n.

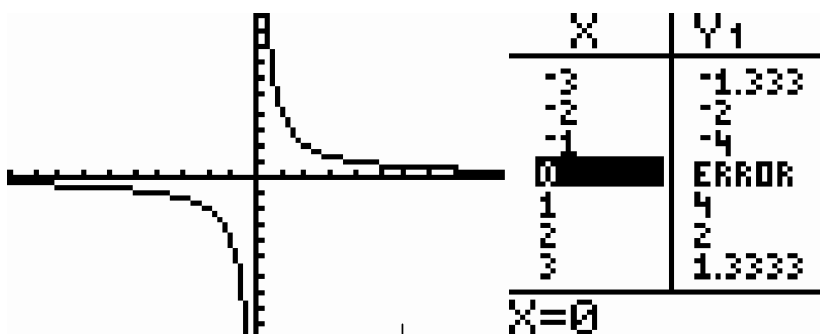
Justify using technology:



In this manner, I created a formula to find the numerical equivalence for the nth term of the sequence in the form, where both p and q are integers.

$2. \log_3 81 = 4$	$\log_8 81 = 1 \text{ or } \frac{4}{4}$
$\log_9 81 = 2 \text{ or } \frac{4}{2}$	$\log_{25} 81 = 0.8 \text{ or } \frac{4}{5}$
$\log_{27} 81 = 1.667 \text{ or } \frac{4}{3}$	$\log_{27} 81 = 2/3 \text{ or } \frac{4}{6}$

Thus the n^{th} term is $\frac{4}{n}$ where p is 4 and q is n.



$$3. \log_5 25 = 2$$

$$\log_5 25 = 1$$

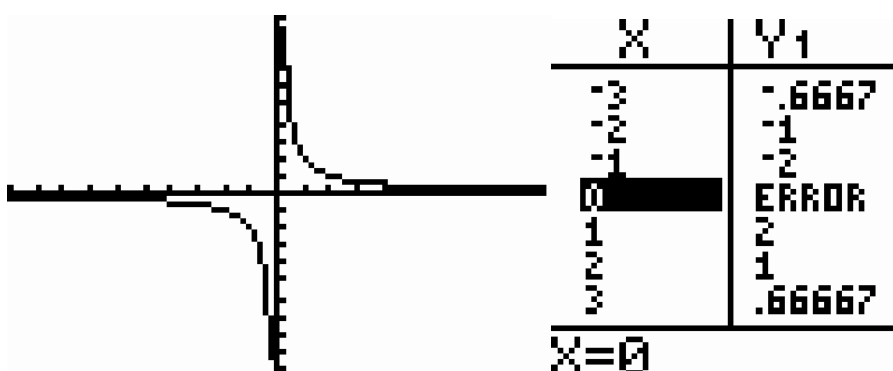
$$\log_{25} 25 = 2/3$$

$$\log_{25} 25 = 0.5$$

$$\log_{325} 25 = 0.4$$

$$\log_{625} 25 = 1/3$$

Thus the n^{th} term is $\frac{2}{n}$ where p is 2 and q is n.



$$4. \log_m m^k = k$$

$$\log_{m^4} m^k = k/4$$

$$\log_{m^2} m^k = k/2$$

$$\log_{m^5} m^k = k/5$$

$$\log_{m^3} m^k = k/3$$

$$\log_{m^6} m^k = k/6$$

Thus the n^{th} term is $\frac{k}{n}$ where p is k and q is n.

Describing how we obtain $\log_x(64)$ from $\log_4(64)$ and $\log_8(64)$.

In order to obtain the answer of the third logarithm, the product of the first and second logarithms must be divided by the sum of the answers of the first and second logarithms.

$$\log_4(x), \log_8(x) \rightarrow \log_x(x)$$

We use the formula:

$$\log_a b = \frac{1}{\log_b a}$$

$$\log_4(x) = \frac{1}{\log_x 4} \rightarrow \log_x 4 = \frac{1}{\log_4 x} \quad [1]$$

$$\log_8(x) = \frac{1}{\log_x 8} \rightarrow \log_x 8 = \frac{1}{\log_8 x} \quad [2]$$

Add [1] and [2]:

$$\log_x 4 + \log_x 8 = \frac{1}{\log_4 x} + \frac{1}{\log_8 x}$$

And we have:

$$\log_x 32 = \frac{\log_8(x) + \log_4(x)}{\log_4(x) \log_8(x)}$$

Therefore the formula we obtain:

$$\log_x(x) = \frac{\log_4(x) \log_8(x)}{\log_4(x) + \log_8(x)} \quad \text{or} \quad \log_a(x) = \frac{\log_b(x) \log_c(x)}{\log_b(x) + \log_c(x)}$$

By replacing the value of $x = 64$ we get,

$$\log_x(64) = \frac{\log_4(64) \log_8(64)}{\log_4(64) + \log_8(64)}$$

$$\log_4(64) = 3 \text{ \& } \log_8(64) = 2(\text{using GDC})$$

$$\text{Thus, } \log_x(64) = \frac{6}{5}$$

Describing how we obtain $\log_{35}(49)$ from $\log_7(49)$ and $\log_5(49)$.

Thus by using the above formula above:

$$\log_a(x) = \frac{\log_b(x) \log_c(x)}{\log_b(x) + \log_c(x)}$$

$$\log_{35}(49) = \frac{\log_7(49) \log_5(49)}{\log_7(49) + \log_5(49)}$$

Solving the equations,

$$\log_4 (49) = 1$$

$$\log_7 (49) = 2$$

Therefore, $\log_{38} (49) = \frac{2}{3}$

Describing how we obtain $\log_{\frac{1}{5}} (125)$ from $\log_{\frac{1}{25}} (125)$ and $\log_{\frac{1}{65}} (125)$.

Thus, by using the formula

$$\log_a (x) = \frac{\log_b (x) \log_c (x)}{\log_b (x) + \log_c (x)} ,$$

$$\log_{\frac{1}{65}} (125) = \frac{\frac{\lg 125}{\lg \frac{1}{5}} \frac{\lg 125}{\lg \frac{1}{25}}}{\frac{\lg 125}{\lg \frac{1}{5}} + \frac{\lg 125}{\lg \frac{1}{25}}}$$

Solving the equations

$\log_{\frac{1}{5}} (125)$ & $\log_{\frac{1}{25}} (125)$ by change of base log

$$\frac{\lg 125}{\lg 1/5} = -3 \quad \frac{\lg 125}{\lg 1/25} = -1$$

Therefore, $\log_{\frac{1}{8}}(125) = -\frac{3}{4}$

Describing how we obtain $\log_6 (512)$ from $\log_8 (512)$ & $\log_2 (512)$.

Thus, by using the formula

$$\log_a(x) = \frac{\log_b(x) \log_c(x)}{\log_b(x) + \log_c(x)}$$

We obtain,

$$\log_6 (512) = \frac{\log_8 (512) \log_2 (512)}{\log_8 (512) + \log_2 (512)}$$

Solving the equations,

$$\log_8 (512) = 3, \quad \log_2 (512) = 9$$

Therefore,

$$\begin{aligned} \log_6 (512) &= \frac{(3)(9)}{(3)+(9)} \\ &= 2.25 \end{aligned}$$

Example 1:

$$1^{\text{st}} \text{ term} - \log_3 \sqrt{3}$$

$$2^{\text{nd}} \text{ term} - \log_7 \sqrt{3}$$

$$3^{\text{rd}} \text{ term} - y$$

Therefore,

$$y = \frac{\log_b(x) \log_c(x)}{\log_b(x) + \log_c(x)}$$

$$y = \frac{\log_3(\sqrt{3}) \log_7(\sqrt{3})}{\log_3(\sqrt{3}) + \log_7(\sqrt{3})}$$

$$y = \log_8 \sqrt{3}$$

$$y = \frac{1}{8}$$

Example 2:

$$1^{\text{st}} \text{ term} - \log_9 0.01$$

$$2^{\text{nd}} \text{ term} - \log_{10} 0.01$$

$$3^{\text{rd}} \text{ term} - y$$

Therefore,

$$y = \frac{\log_b(x) \log_c(x)}{\log_b(x) + \log_c(x)}$$

$$y = \frac{\log_9(0.01) \log_{10}(0.01)}{\log_9(0.01) + \log_{10}(0.01)}$$

$$y = \log_{90} 0.01$$

$$y = -0.8692$$

Expressing, $\log_b(x)$ in terms of c and d

Let $\log_a(x) = c$ and $\log_b(x) = d$

Formula:

$$\log_N(M) = \frac{1}{\log_M N} \quad \log_a(x) + \log_b(x) = \log_x(ab)$$

$$\frac{1}{\log_a x} + \frac{1}{\log_b x} = \frac{1}{\log_{ab} x}$$

Substituting c and d

$$\frac{1}{c} + \frac{1}{d} = \frac{1}{\log_{ab} x}$$

$$\frac{d}{c+d} = \log_{ab}(x)$$

OR

$$\log_a(x) = c \iff \frac{\ln(x)}{\ln(a)} = c$$

$$\log_b(x) = d \iff \frac{\ln(x)}{\ln(b)} = d$$

Therefore,

$$\log_{ab}(x) = \frac{\ln(x)}{\ln(a) + \ln(b)}$$

$$\frac{\ln(a)}{\ln(x)} = \frac{1}{c}$$

$$\frac{\ln(b)}{\ln(x)} = \frac{1}{d}$$

$$\frac{\ln(a) + \ln(b)}{\ln(x)} = \frac{1}{c} + \frac{1}{d}$$

$$\frac{cd}{c+d} = \log_b(x)$$

Thus, the general statement is $\frac{cd}{c+d}$.

To test the validity of the statement we need to by putting different values of a, b and x in the equation.

Values of a, b and x	$\log_b(x) = \frac{ad}{c+d}$	
	$\log_b(x)$	$\frac{ad}{c+d}$
a = 2 b = 5 x = 7	$= \log_{2(5)} 7$ $= \log_v 7$ $= 0.8451$	$\frac{\lg 7}{\lg 2} \times \frac{\lg 7}{\lg 5}$ $= \frac{\lg 7}{\lg 2} + \frac{\lg 7}{\lg 5}$ $= 0.85451$
a = 4 b = 2 x = 16	$= \log_8 16$ $= \frac{\lg 16}{\lg 8}$ $= \frac{4}{3}$	$\frac{\lg 16}{\lg 4} \times \frac{\lg 16}{\lg 2}$ $= \frac{\lg 16}{\lg 4} + \frac{\lg 16}{\lg 2}$ $= \frac{4}{3}$
a = $\sqrt{5}$ b = $\sqrt{8}$ x = 16	$= \log_{\sqrt{40}} 16$ $= \frac{\lg 16}{\lg \sqrt{40}}$ $= 1.5032$	$\frac{\lg 16}{\lg \sqrt{5}} \times \frac{\lg 16}{\lg \sqrt{8}}$ $= \frac{\lg 16}{\lg \sqrt{5}} + \frac{\lg 16}{\lg \sqrt{8}}$ $= 1.5032$
a = π b = 12 x = 9	$= \log_{12\pi} 9$ $= \frac{\lg 9}{\lg 12\pi}$ $= 0.6054$	$\frac{\lg 9}{\lg \pi} \times \frac{\lg 9}{\lg 12}$ $= \frac{\lg 9}{\lg \pi} + \frac{\lg 9}{\lg 12}$ $= 0.6054$

$a = \frac{1}{2}$ $b = 2$ $x = 6$	$= \log_1 6$ $= \frac{\lg 6}{\lg 1}$ $= \text{Not Defined}$	$\frac{\lg 6}{\lg 1/2} \times \frac{\lg 6}{\lg 2}$ $= \frac{\lg 6}{\lg 1/2} + \frac{\lg 6}{\lg 2}$ $= \text{Not Defined}$
$a = 12$ $b = 0$ $x = 2$	$= \log_0 2$ $= \frac{\lg 2}{\lg 0}$ $= \text{Not Defined}$	$\frac{\lg 2}{\lg 12} \times \frac{\lg 2}{\lg 0}$ $= \frac{\lg 2}{\lg 12} + \frac{\lg 2}{\lg 0}$ $= \text{Not Defined}$
$a = -12$ $b = 6/5$ $x = 18$	$= \log_{-2/5} 18$ $= \frac{\lg 18}{\lg (-2/5)}$ $= \text{Not Defined}$	$\frac{\lg 18}{\lg 6/5} \times \frac{\lg 18}{\lg (-12)}$ $= \frac{\lg 18}{\lg 6/5} + \frac{\lg 18}{\lg (-12)}$ $= \text{Not Defined}$
$a = 9$ $b = 6$ $x = 1$	$= \log_9 1$ $= \frac{\lg 1}{\lg 9}$ $= 0$	$\frac{\lg 1}{\lg 9} \times \frac{\lg 1}{\lg 6}$ $= \frac{\lg 1}{\lg 9} + \frac{\lg 1}{\lg 6}$ $= 0$
$a = 25$ $b = \frac{1}{5}$ $x = 0$	$= \log_5 0$ $= \frac{\lg 0}{\lg 5}$ $= \text{Not Defined}$	$\frac{\lg 0}{\lg 25} \times \frac{\lg 0}{\lg 1/5}$ $= \frac{\lg 0}{\lg 25} + \frac{\lg 0}{\lg 1/5}$ $= \text{Not Defined}$

$a = 2$ $b = 8$ $x = -5$	$= \log_2 -5$ $= \frac{\lg(-5)}{\lg 2}$ $= \text{Non real answer}$	$\frac{\lg(-5)}{\lg 2} \times \frac{\lg(-5)}{\lg 8}$ $= \frac{\lg(-5)}{\lg 2} + \frac{\lg(-5)}{\lg 8}$ $= \text{Non real answer}$
$a = 1$ $b = 1$ $x = 1$	$= \log_1 1$ $= \frac{\lg 1}{\lg 1}$ $= \text{Not defined}$	$\frac{\lg 1}{\lg 1} \times \frac{\lg 1}{\lg 1}$ $= \frac{\lg 1}{\lg 1} + \frac{\lg 1}{\lg 1}$ $= \text{Not defined}$
$a = 0$ $b = 0$ $x = 0$	$= \log_0 0$ $= \frac{\lg 0}{\lg 0}$ $= \text{Not defined}$	$\frac{\lg 0}{\lg 0} \times \frac{\lg 0}{\lg 0}$ $= \frac{\lg 0}{\lg 0} + \frac{\lg 0}{\lg 0}$ $= \text{Not defined}$
$a = -5$ $b = -11$ $x = -15$	$= \log_{-5} -15$ $= \frac{\lg(-15)}{\lg(-5)}$ $= \text{Not Defined}$	$\frac{\lg(-15)}{\lg(-5)} \times \frac{\lg(-15)}{\lg(-11)}$ $= \frac{\lg(-15)}{\lg(-5)} + \frac{\lg(-15)}{\lg(-11)}$ $= \text{Not Defined}$

Scope/Limitations

After testing the validity of the general statement by putting numerous values of a , b and x we can say that some values are still undefined.

The base of the log must be positive and not equal to one so we can say: $a > 0$, $b > 0$, $a \neq 1$, $b \neq 1$, they both can't be negative because each of them is shown separately in the logarithm. Also $ab > 1$ and $a \neq \frac{1}{b}$.

Finally, the argument (x) must be positive, so:
 $x > 0$

General Statement

The general statement was arrived by two ways:

1st way:

By using change of base log formula $\log_N(M) = \frac{1}{\log_M N}$.

We simplified $\log_{ab}(x)$ and used $\log_{10} ab = \log_{10} a + \log_{10} b$ in the denominator.

Now by simplifying $\log_a x = c$ and $\log_b x = d$ by changing the base logs by the formula above and making $\log_a x$ and $\log_b x$ as the subject of the formula and substituting in the first part we get the general statement as $\frac{c \times d}{c+d}$.

2nd way:

We simplified the statement $\log_{ab} x$ by making the bases similar.

Now in the equations $\log_a x = c$ and $\log_b x = d$ we used the formula to make the bases similar i.e. $\frac{\log_c b}{\log_c a}$ and making $\log_a x$ and $\log_b x$ as the subject of the formula.

Then we substituted it in the simplified version of $\log_{ab} x$. By substituting the values we get the general statement.

Through this general statement we can find out the value of the third term in a sequence when the first two terms are given.

Conclusion

In conclusion, after applying the given knowledge of logarithms, a general statement was formed to prove the validity of various sequences. Also through the use of technology, general statements were proven and theories were confirmed.