Introduction

The investigation given asks for the attempt in finding a rule which allows us to approximate the area under a curve (I.e. between the curve and the x-axis) using trapeziums (trapezoids).

The first task indicates us to find the approximation of the area of $x=0$ to $x=1$ under the curve of $gx=x2+3$.

$gx=x2+3$

First, consider the function of $gx=x2+3$.

Based on the graph, it shows that the area under the curve from $x=0$ to $x=1$ is approximated by the sum of the area of two trapeziums. Hence, in order to find the approximation, the formula to find the area of a trapezium has to be considered:

Trapezium = $12\times(a+b)\times h$

Where a = Length of one side

b = Length of parallel side

h = height of the trapezium.

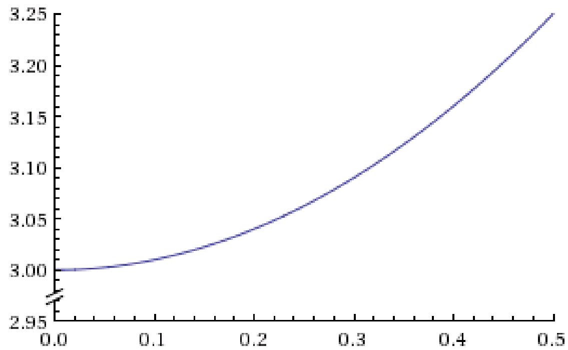
Since there are two trapeziums, it would be simpler to separate into two individual shapes, where the area will be approximated from is at x=0 and x=1. The sum of the area of both trapeziums should provide the overall area under the curve:

Trapezium A. X from 0 to 0.5

h = 0.5

b= 3.25

a = 3

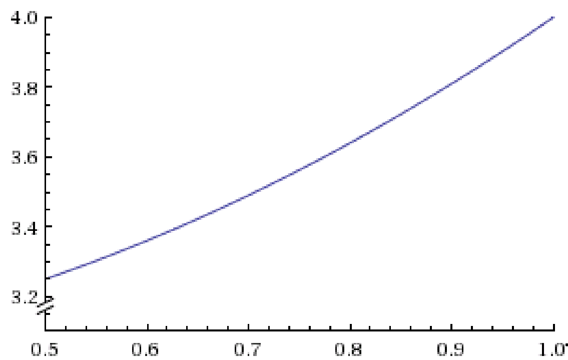


Trapezium B. X from 0.5 to 1.0

h = 0.5

b = 4

a = 3.2



Trapezium A. is represented from the first graph above. To calculate its area, we would then use the trapezium area formula:

$area=12\times3+3.25\times0.5$

$area=12\times6.25\times0.5$

$area=1.5625 \approx 1.56$

Trapezium B is represented from the second graph above. Using the same formula:

$area=12\times3.2+4\times0.5$

$area=12\times7.2\times0.5$

$area=1.8$

Therefore, the approximate of the function $gx=x2+3$ is given by
$Total\ area=1.56+1.8=3.36$

The trapezium extends past the curve, indicating that there will be an overestimation of the area. Through the values on the graph, one is able to determine the variables.

The second task assigned to this investigation is to increase the amount of trapeziums from the initial two trapeziums to five trapeziums, and then find a second approximation for the area.

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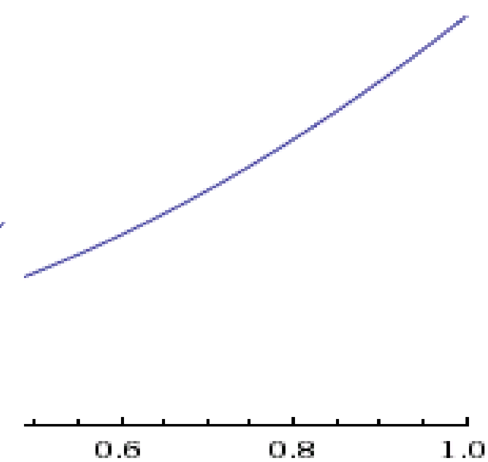
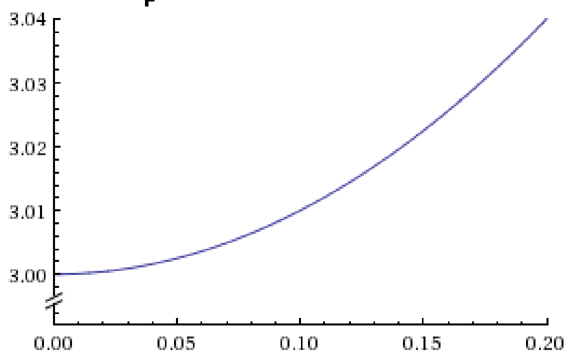




$area=12\times3.04+3.16\times0.2$

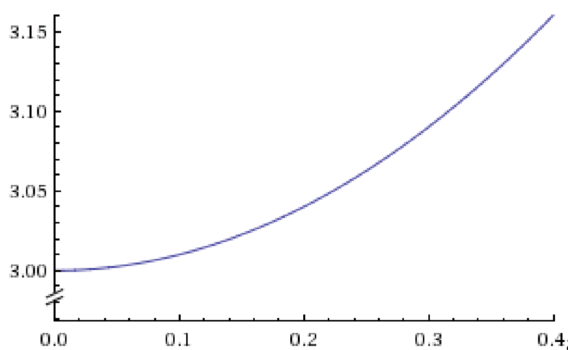
$area=12\times6.2\times0.2$

$area=0.62$

$area=12\times3.16+3.36\times0.2$

$area=12\times6.52\times0.2$

$area=0.652$



$area=12\times3.64+4\times0.2$

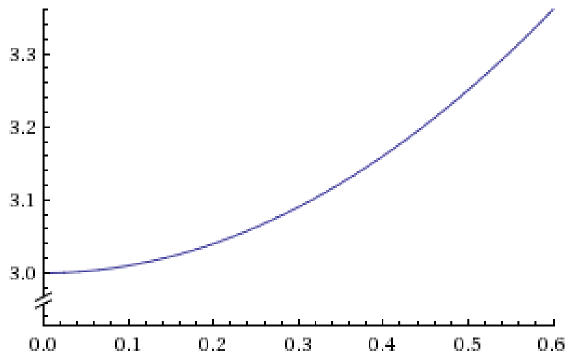
$area=12\times7.65\times0.2$

$area=0.765$

$area=12\times3.36+3.64\times0.2$

$area=12\times7\times0.2$

$area=0.7$

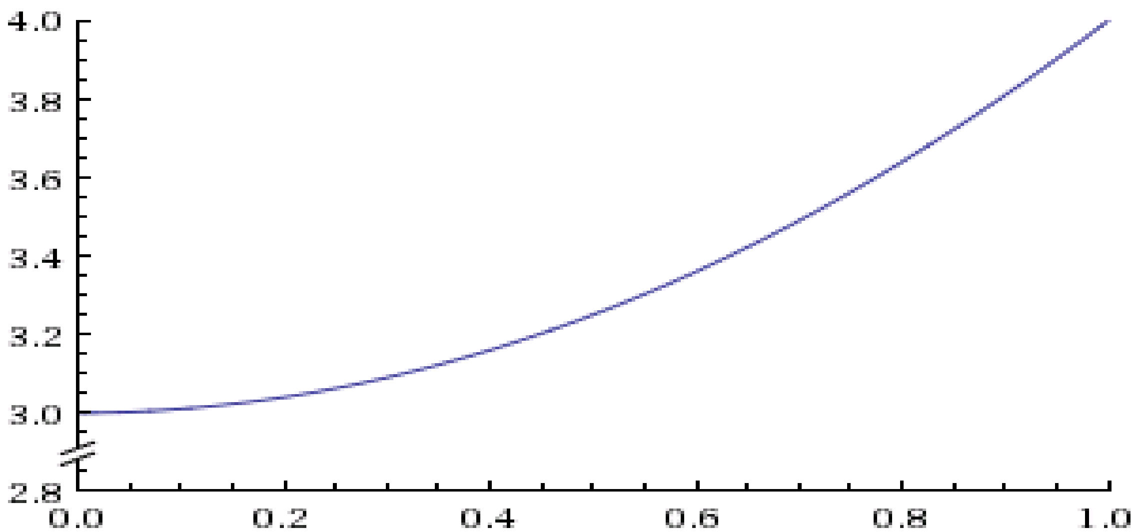


Adding all the areas together, we get the result

$Total\ Area=0.604+0.62+0.652+0.7+0.765$
$Total\ Area=3.341$

Based on the results, the approximation of each area is actually the results of both (a+b) added together. The area is approaching around the value of 3. However, more calculations with additional trapezoids are needed to determine a more precise value.

<u>Eight Trapezoids</u>



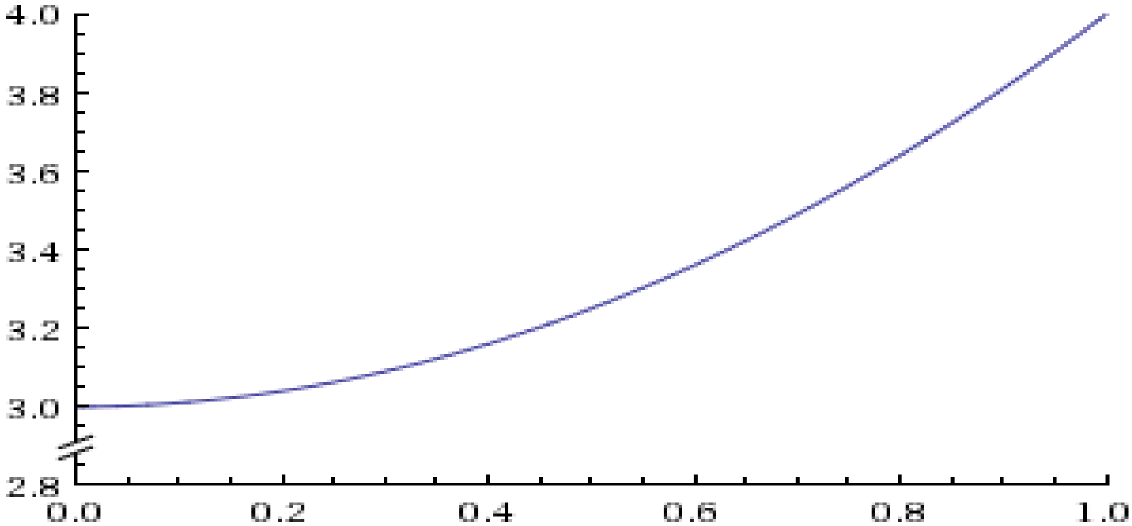
Trapezoid	h	a	b	Area
1	0.125	3	3.015625	0.375976562
2	0.125	3.015625	3.0625	0.379882812

3	0.125	3.0625	3.140625	0.387695312
4	0.125	3.140625	3.25	0.399414062
5	0.125	3.25	3.390625	0.415039062
6	0.125	3.390625	3.5625	0.434570312
7	0.125	3.5625	3.765625	0.458007812
8	0.125	3.765625	4	0.485351562

Total Area= 0.375976562+
0.379882812+0.387695312+0.399414062+0.415039062+0.434570312+0.458007812+0.485351562
Total Area=3.335937496≈3.336

The value shows that the area is approaching towards the value of around 3.3. However, more calculations will reveal a more accurate answer.

Twenty Trapezoids



Trapezoid	h	a	b	Area
1	0.05	3	3.0025	0.1500625
2	0.05	3.0025	3.01	0.1503125
3	0.05	3.01	3.0225	0.1508125
4	0.05	3.0225	3.04	0.1515625
5	0.05	3.04	3.0625	0.1525625
6	0.05	3.0625	3.09	0.1538125
7	0.05	3.09	3.1225	0.1553125
8	0.05	3.1225	3.16	0.1570625
9	0.05	3.16	3.2025	0.1590625
10	0.05	3.2025	3.25	0.1613125

11	0.05	3.25	3.3025	0.1638125
12	0.05	3.3025	3.36	0.1665625
13	0.05	3.36	3.4225	0.1695625
14	0.05	3.4225	3.49	0.1728125
15	0.05	3.49	3.5625	0.1763125
16	0.05	3.5625	3.64	0.1800625
17	0.05	3.64	3.7225	0.1840625
18	0.05	3.7225	3.81	0.1883125
19	0.05	3.81	3.9025	0.1928125
20	0.05	3.9025	4	0.1975625

Total Area=0.1500625+0.1503125+0.1508125+0.1515625+0.1525625+0.1538125+0.15538125+0.1570625+0.1613125+0.1638125+0.1665625+0.1695625+0.1728125+0.1763125+0.1800625+0.1840625+0.1883125+0.1928125+0.1975625

Total Area=3.3343125 ≈3.334

According to the integration function of Wolfram Mathematica Online, the actual value of the area under the curve would be x2+3 which equals to12xx2+9, hence, subbing the value of x = 1, the approximated area would be 3.333, re-curring. Evidently, by increasing the amount of trapezoids under the curve of the graph, leads to approximated values that were close to the actual value; in this case, the approximate area for twenty trapezoids were closer to the actual value than the approximate area of eight trapezoids. Additional trapezoids make the approximated area more accurate as the values decreases slowly, although very minimal. Possibly due to the fact that the graph is a positive curve, hence trapezoid does not extend much past the curve.

Furthermore, the distance between each trapezoid can be determined via the formula:

position of base ×Δxn+x value of first base

However, the position of the 1st base will be assumed to be in position 0, while the position of the 2nd base is assumed to be in position 1, etc. From the previous graphs, using the obtained values from the table, this seems to be a consistent pattern.

<u>General Expression from 0≤x≤1, g(x) =x^2+3</u>

Let n = number of trapezoids, and A = the approximation of area under the curve.

Based on the question, the general expression is to be found for the area under the curve of g, from x=0 to x=1.

The general expression is very similar to what was done previously, thus, it is simply a succinct form of the area formulas for *n* amount of trapezoids.

A=12hg0+gx1+12hgx1+gx2+…+12hgn-1+g1

Further factorizing the equation gives:

A=12hg0+gx1+gx1+gx2+…+(gxn-1+g(1))

$$h=\Delta xn=1-0n=1n$$

From the tables above, a very significant factor is how the first and last trapezoids only uses one side that is not shared; For example, only the lengths of 3 and 4 are by itself, while the values after and before are used again by the next trapezoids. Hence, from the expression above, all lengths beside the first and last are used twice, therefore, taking this into account, the expression obtained would now become:

$A=12ng0+2gx1+2gx2+\ldots+2g(xn-1+g(1)]$

From the previous analysis of the distance difference, where
$\text{position of base} \times \Delta xn+x \text{ value of first base}$

If the x value of first base = a, then the general expression of the area can be expressed by factorizing the pattern even more. By using sigma notation and having the number of base increasing by 1, there general expression would be:

$A=12n[g0+2i=1n-1fa+in+g(1)]$

<u>General Statement</u>

For the general expression, it is similar to the previous question to a certain extent. The essence of finding the approximate area from one point to other remains constant, however, the definite values of 0 to 1 is now replaced by unknowns a and b.

Allow A = approximation of the area under the curve, a = (x_0) first base, b = (x_n) last base and n = number of trapeziums.

Taking an example from the previous question, the general statement is basically a compacted form of the area formulas for n trapeziums hence:

$A=12hfa+fx1+12hfx1+fx2+\ldots+12hfxn-1+gb$

However, further simplifying the equation, and taking into account that first and lase base are only used twice the entire time, the expression would appear as follows:

$A=12hfa+2fx1+2fx2+\ldots+2f(xn-1+f(b)]$

Unlike previous time where$h=1n$, the difference between these expressions is that the value of *h* is changed. Since a and b are defined as the first and last bases, therefore, they would be the most extreme values on the x-axis.

$h=\Delta xn=b-an$

From the previous question, the values in both first and last based are used twice. Further outlined is the distance between each gap is h, and the x-value of the base is related to the relative position, h and the value of a. The pattern mentioned before was:

$\text{position of base} \times \Delta xn+x \text{ value of first base}$

The pattern now translates to= $a+i(h)$, since h is now changed to$h=b-an$, while ‘i’ remains as a variable of the sigma notation. The upper and lower limits of the summation sign are not required since it is only used once, hence, the general statement that estimates the area under any curve would be:

$A=12h[fa+2i=1n-1fa+ih+fb]$

And h is represented by$b-an$

Testing with General Statement

The general statement is:

$A=12h[fa+2i=1n-1fa+ih+fb]$

The statement is to be tested under three given curves, from points x=1 to x=3. Also required along the general statement are eight trapeziums.

1. The first function is $y1=x223$

To solve the function, the h must be calculated. Since the range is x=1 to x=3, therefore, a =1 while b=3. The difference would become$\Delta x=3-1=2$.

Since there will be eight trapeziums, therefore n = 8.

Hence,$h=28\approx 14$.

Subbing the values into the equation gives us:

$A=142[f1+2i=18-1f1+i14+f(3)]$

$A=18[f1+2i=17f1+i14+f(3)]$

Expanding the sigma notation allows for calculation, as the notation is merely a method to condense the sum of a series of numbers. Any ellipsis used it to represent a sum removed from the calculation in order to conserve space.

$A=18f1+2f1+14+2f1+24+2f1+34+\ldots+2f1+74+f3$

The values of x in f(x) are substituted into the original function, remaining consistency with the function notation, giving the equation:

$A=18[0.629960524+20.731004434+20.825481812+20.914826427+\ldots+21.236521861+1.310370697]$

$A=180.629960524+1.462008868+1.650963624+1.829652854+2+2.163374356+2.320794416+2.473043722+1.310370697$

$A=15.840169068=1.980021133\approx 1.980$

The answer is approximately 1.980. Based on Wolfram Mathematica online integrator, the integration formula is $x223=3x535\times 223$. to find the accurate value, x =3 minus x = 1 to get the approximate area. Hence,

$33535\times 223-31535\times 223$

2.358667255-0.377976315=1.98069094≈1.981

The answer from the general statement is almost identical to the accurate measured value. At this stage, the general statement appears to be accurate.

2. Similarly, the general statement is used to find the approximate area for the second given function, which isy2=9xx3+9.

Using the same method as the previous, h must be calculated. The range is x=1 to x=3, hence, a=1 and b=3. The difference then becomes Δx=3-1=2.

Since there are eight trapeziums, n=8.

h=28≈14

Subbing the values into the equation gives us:

A=142[f1+2i=18-1f1+i14+f(3)]

A=18[f1+2i=17f1+i14+f(3)]

The general statement is the same as the one used in the previous function. Therefore, the following steps can be followed from the previous question; only the function (f) is different, where f(x)=9xx3+9.

A=18f1+2f1+14+2f1+24+2f1+34+…+2f1+74+f3

Values of x are subbed into the equation, giving the values:

A=18[0.629960524+23.399253086+23.837612894+24.156356486+…+24.534086944+1.310370697]

A=182.846049894+6.798506172+7.675225788+8.312712972+8.731282501+8.968911824+9.068268993+9.068173888+4.5

A= 65.969132038=8.246141504≈8.246

The answer is approximately 8.246. Based on Wolfram Mathematica online integrator, the integration formula is

When we

$$\int_1^3 \frac{9\,x}{\sqrt{9+x^3}}\,dx = \frac{3}{2}\left(9\;{}_2F_1\!\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-3\right) - {}_2F_1\!\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{1}{9}\right)\right) \approx 8.25973$$

simplify the wolfram integration formula, we get y2=8.260. The answer gained from the general statement has 0.014 difference compared to the accurate value. This suggests that the general statement is precise, but not accurate. Although further confirmation is still required.

The last equation given would be y3=4x3-23x2+40x-18. To use the general statement, the variables have to be found out, hence:

Range is a=1 and b=3. The difference then becomes Δx=3-1=2.

Since there are eight trapeziums, n=8.

$h=28\approx14$

Substituting the values in would give:

$A=142[f1+2i=18-1f1+i14+f(3)]$

$A=18[f1+2i=17f1+i14+f(3)]$

The equation remains constant since all variables are constant.

Therefore, the expression can be repeated via the previous method since it is identical to previous expressions:

$A=18f1+2f1+14+2f1+24+2f1+34+\ldots+2f1+74+f3$

The values of function f(x) are different since $fx=4x3-23x2+40x-18$. Therefore, subbing into the equation gives the expression:

$A=18[3+23.875+23.75+23+\ldots+21.25+3]$

$A=183+7.75+7.5+6+4+2.25+1.5+2.5+3=37.58=4.6875\approx4.688$

The final answer is approximately 4.688. To determine the effectiveness of the general statement, it must be compared to the actual value. Based on the Wolfram alpha online integrator, $134x3-23x2+40x-18=4.66667\approx4.667$. Therefore, the different between the gained answer and the actual value is 0.021.

Comparison of Results

	Calculated Value	Actual Value	% Error
Function 1	1.980	1.981	**0.05**
Function 2	8.246	8.260	**0.169**
Function 3	4.688	4.667	**0.450**

Percentage Error = $\text{Calculated-ActualActual} \times 100\%$

Based on the obtained percentage errors, the differences between the actual value and calculated value are less than 1%, suggesting that the errors are negligible and the general statement provided is relatively accurate and precise for most graphs that contain functions to a power, functions with a square root value in the denominator as well as a cubic function.

Scopes and Limitations

To determine the scope and limitations of the general statement, functions with asymptotes, sine curves as well as points of inflection should be considered.

The follow graph represents y=1/x, where the function is a hyperbola, which means asymptotes is present.

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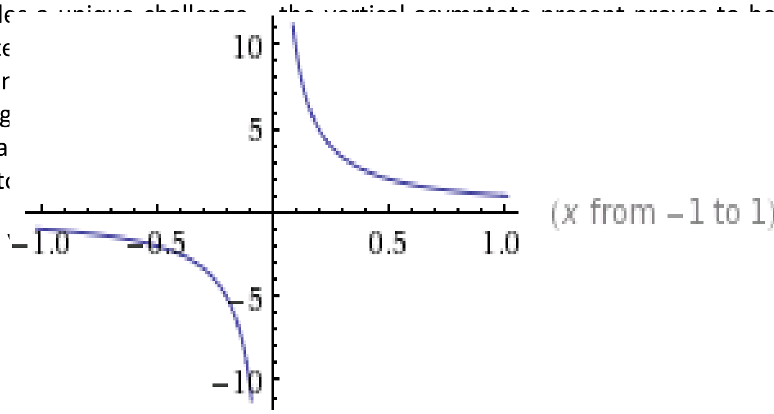
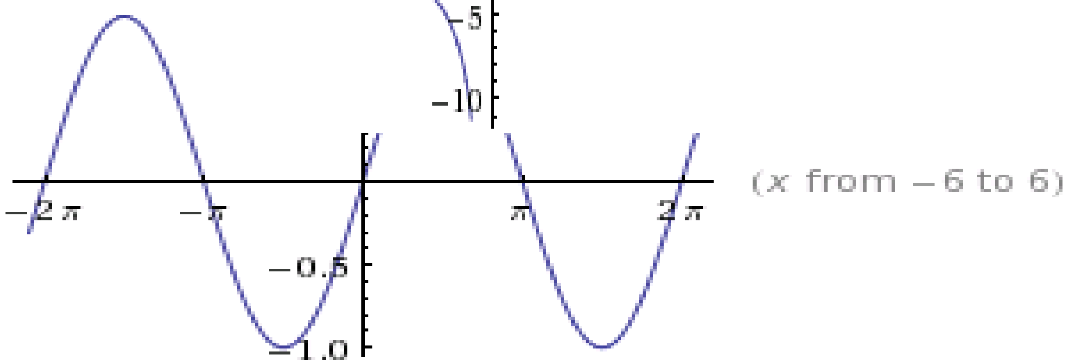


The second graph is …

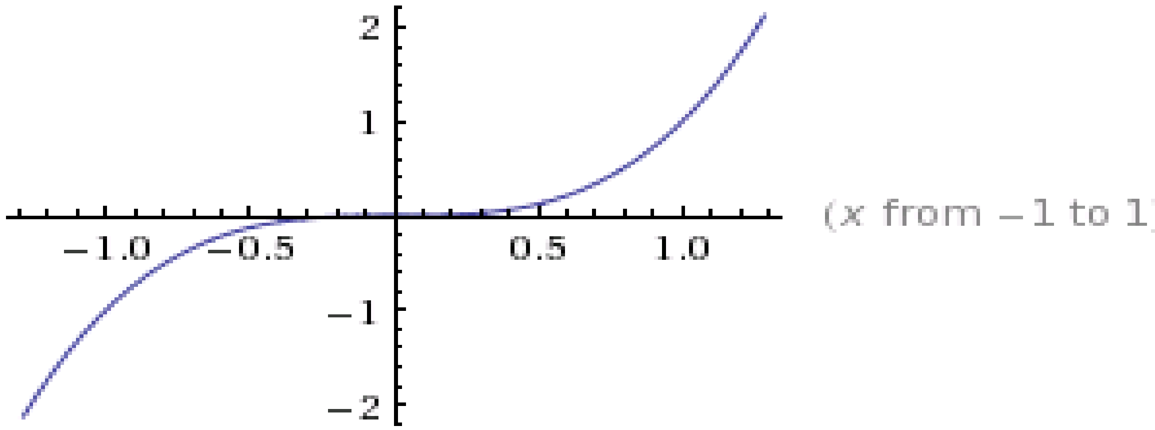


The approximate area and the actual value must be calculated in order to prove the effectiveness of the general statement. Hence, if a = 0, b =2π, and the amount of trapeziums used is 8, the general statement value becomes:

$A=18[f0+2i=17f1+i14+f(2\pi)]$
$A=0$

The general statement value is 0. And the actual value from Wolfram Alpha integrator Online gives the equation: $02\pi\sin x dx=0$. Areas from the sections above and below the curve would nullify each other, hence, giving the percentage error of 0%. This would mean that general statement is applicable to the sine function.

The third graph represents y=x^3:



The graph above represents a point of inflection at (0,0), hence, to find the effectiveness of the general statement, calculated values must be compared against the actual value. The variables of this equation includes:

h = $\Delta xn=1\text{-}02=12$

n= 2

a= 0

b=1

Substituting the values in:

$A=14[f0+2i=11f0+i12+f(1)]$

$A=140+0.5+1$

$A=0.375$

The answer gives an approximated value of 1, while the actual value of Wolfram Alpha integrator online gives the equation: $x3dx=x44=0.25$. The percentage error from the calculated area and the actual area gives = $0.375\text{-}0.250.25=50\%$. Therefore in this scenario, the general statement is ineffective to give the properly value since there is a 50% percentage error, which is significant.

Overall, the general statement is quite accurate. For the given functions, it has handle it efficiently with insignificant low percentage errors. Hence, the 'scope' of the general statement covers functions raised to a power to rational functions. From the previous segment, showing the scopes and limitations, the general statement is capable of being effecting on sine function, therefore the scope of the general statement is relatively big.

However, there are also limitations. The general statement will not work with functions that contain an asymptote or a point of inflection. This is due to the fact that there was a vertical asymptote present, hence, the trapezium cannot be drawn, also, there is no constant within the equation showing the point of inflection, therefore, the general statement had worked, but with a large percentage error.

Also, the general statement is very limited to the flat edges of the trapeziums, where the flat points does not allow for it to 'adapt' to the different curves, causing either overestimations or underestimations of the area. Therefore, functions with large curves will cause the approximated area to generate a vlue that is less accurate, in the example of the 3rd function given, where its steep curves shows the highest amount of percentage error – 0.450%. Since the general statement revolves around trapeziums, functions that have uneven curves will also cause fiddiculities. In this case, functions that have a parabola would be the most optical graph shape to accompany the general statement.

Although there statement has its limitations and will not always work, it will thoretically work within its boundaries.

<u>Conclusion</u>

Trapeziums can be used in integral calculus to find an approximate area under a curve of a graph, as long as the functions used alongside the general statement is not beyond its limitations.

The general statement which has been achieved in this assignment is:

$$A=\frac{1}{2}h\left[f(a)+2\sum_{i=1}^{n-1}f(a+ih)+f(b)\right]$$

And h is represented by $\frac{b-a}{n}$

The general statement obtained is able to work is due to its ability to deal with a wide variety of different functions. It uses the geometrical shape of a trapezium, which proves its uses better than either a rectange or square to suit and adjust to fit approximately around a shape of a curve. It is useful in this aspect since one would wish to minimize and sort of 'space' between the shape and the curve, as errors occur where the 'spaces' are at. Also, its methods are very simple, yet efficient, and it also have very few limitations, hence, the statement can be used over a wide range of functions.

In conclusion, the general statement would be a useful formula to used in calculating approximate area under curves, alongside the field of integral calculus.