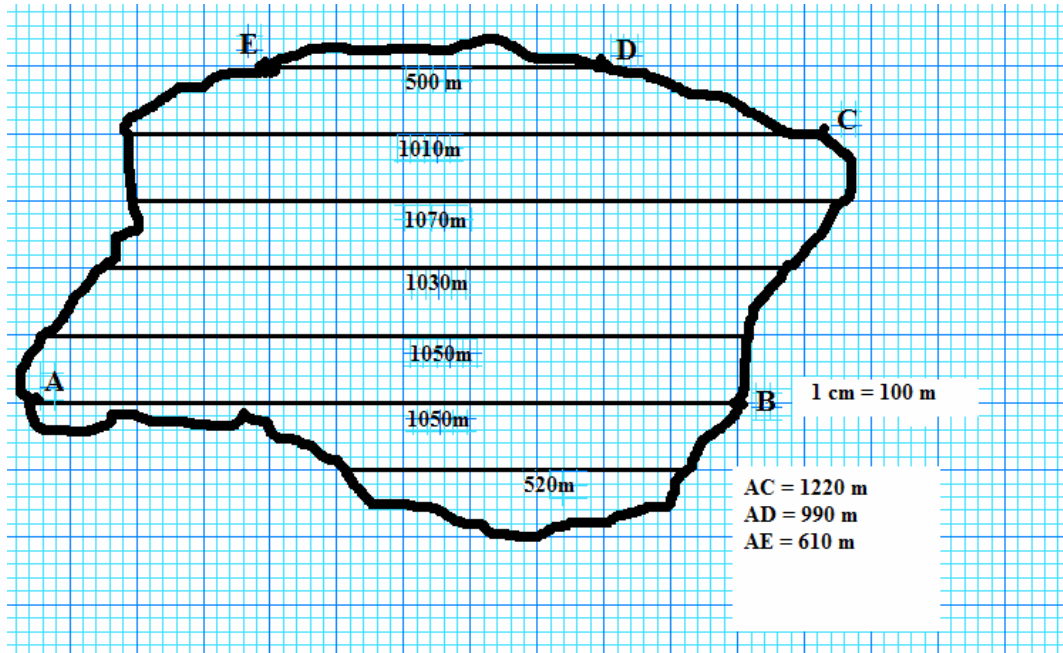


The Fish Pond

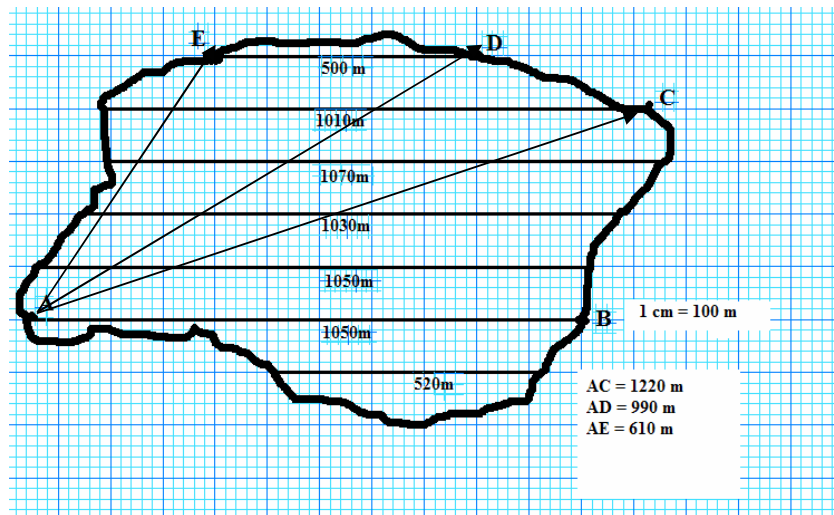


In this activity I will be using the information given to me about this specific fish pond, to create and solve an equation that will represent the amount of fish I will raise and afterwards sell at the local market. In order to do this in an efficient and cost effective manner, I will need to determine the pond's carrying capacity which can also be referred to as the maximum population that the pond is capable of sustaining in the long run. I will do this by using the scaled map of the pond and the given information:

- An average depth of 1.5 meters
- One fish requires 37 m^3 to grow to its maximum size and does so in one year.

1. The first step in solving for the maximum population would be to estimate the surface area of the pond by each of the following methods::

a) Use the sum of the areas of three different triangles: $\triangle ABC$, $\triangle AD$, $\triangle AE$



By calculating the area of each of these triangles and then adding them together we can estimate the pond's surface area.

Area of a triangle = $\frac{1}{2}bh$, where b is base and h is height

Area of $\triangle ABC$: Given that $AC=1220\text{ m}$ it can be used as the base of the triangle

Height can be calculated by finding a line perpendicular to the base passing through a vertex of the triangle, B.

$$AC \rightarrow B = 3.3\text{ cm}$$

$$3.3\text{cm} \bullet 100 = 330\text{ m}$$

Therefore, the $h=330\text{ m}$

$$\text{Area of } \triangle ABC = \frac{1}{2} (1220\text{ m})(330\text{ m}) = 201,300\text{ m}^2$$

Area of $\triangle AD$: $AC= 1220\text{ m}$ and is once again used as the base.

$$h = AC \rightarrow D = 2\text{ cm}$$

$$h = 2\text{ cm} \bullet 100 = 200\text{ cm}$$

$$\text{Area of } \triangle AD = \frac{1}{2} (1220\text{ m})(200\text{ m}) = 122,000\text{ m}^2$$

Area of $\triangle AE$: We are given that $AD = 990$

$$AD \rightarrow E = 2\text{ cm}$$

$$h = 2 \text{ cm} \cdot 100 = 200 \text{ m}$$

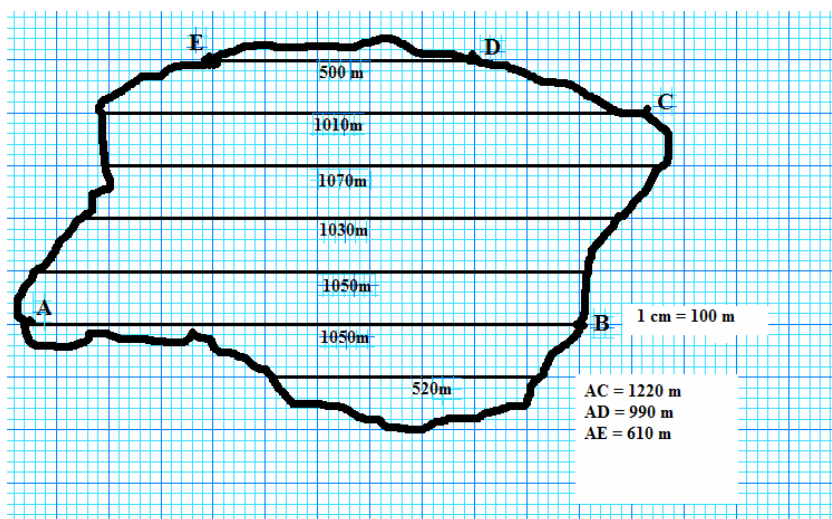
$$\text{Area of } \triangle AD = \frac{1}{2} (990 \text{ m})(200 \text{ m}) = 99,000 \text{ m}^2$$

Using these calculations we can estimate the overall surface area of the pond by adding the areas of the triangles:

$$\begin{array}{r} \text{Area of } \triangle ABC \quad 201,300 \text{ m}^2 \\ \text{Area of } \triangle AD \quad 122,000 \text{ m}^2 \\ \text{Area of } \triangle AE \quad + 99,000 \text{ m}^2 \\ \hline \end{array}$$

$$\text{Estimated area of the pond} = 422,300 \text{ m}^2$$

b) Divide the pond into rectangles and sum the areas.



By using the given information we can determine the lengths of the rectangles we use to divide the pond.

After measuring each rectangle for their width we can also determine that they are all have a width of 1 cm, or 100 m.

After finding the widths we can multiply by the lengths we obtain by measuring in centimeters.

$$1^{\text{st}} \text{ rectangle} = 100 \cdot 750 \text{ m} = 75,000 \text{ m}^2$$

$$2^{\text{nd}} \text{ rectangle} = 100 \text{ m} \cdot 1070 \text{ m} = 107,000 \text{ m}^2$$

$$3^{\text{rd}} \text{ rectangle} = 100 \text{ m} \cdot 1030 \text{ m} = 103,000 \text{ m}^2$$

$$4^{\text{th}} \text{ rectangle} = 100 \text{ m} \cdot 940 \text{ m} = 94,000 \text{ m}^2$$

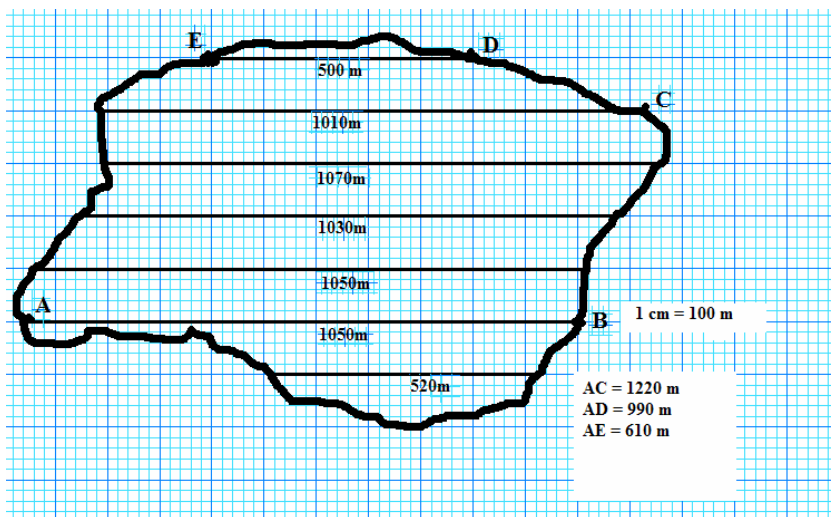
$$5^{\text{th}} \text{ rectangle} = 100 \text{ m} \cdot 1050 \text{ m} = 105,000 \text{ m}^2$$

$$6^{\text{th}} \text{ rectangle} = 100 \text{ m} \cdot 520 \text{ m} = 52,000 \text{ m}^2$$

$$7^{\text{th}} \text{ rectangle} = 50 \text{ m} \cdot 450 \text{ m} = 22,500 \text{ m}^2$$

When all areas are added, the estimated surface area of the pond = 558,500 m^2

c) Use the area of trapezoids.



After measuring the lengths of the parallel sides and the heights of the all the trapezoids, the values are plugged into the equation to solve for their areas.

Area of a trapezoid = $\frac{1}{2} (a + b) h$ where a and b are the parallel sides, h is the height

$$1^{\text{st}} \text{ trapezoid} = \frac{1}{2} (500 \text{ m} + 1010 \text{ m}) 100 = 75,500 \text{ m}^2$$

$$2^{\text{nd}} \text{ trapezoid} = \frac{1}{2} (1010 \text{ m} + 1070 \text{ m}) 100 = 104,000 \text{ m}^2$$

$$3^{\text{rd}} \text{ trapezoid} = \frac{1}{2} (1070 \text{ m} + 1030 \text{ m}) 100 = 105,000 \text{ m}^2$$

$$4^{\text{th}} \text{ trapezoid} = \frac{1}{2} (1030 \text{ m} + 1050 \text{ m}) 100 = 104,000 \text{ m}^2$$

$$5^{\text{th}} \text{ trapezoid} = \frac{1}{2} (1050 \text{ m} + 1050 \text{ m}) 100 = 105,000 \text{ m}^2$$

$$6^{\text{th}} \text{ trapezoid} = \frac{1}{2} (1050 \text{ m} + 500 \text{ m}) 100 = 77,500 \text{ m}^2$$

$$7^{\text{th}} \text{ trapezoid} = \frac{1}{2} (520 \text{ m} + 200 \text{ m}) 100 = 36,000 \text{ m}^2$$

After getting the area of each trapezoid and adding the areas up, the estimated surface area of the pond = $607,000 \text{ m}^2$

2. Which of the three above methods do you consider to be the most accurate? Why?

After looking at the results of the three different methods of finding the total surface area of the pond, the most accurate way seems to be the trapezoids. One reason would be that its shape fits best to fit into the overall shape of the pond. The areas that the trapezoid may go out of the pond, compensates for those areas that are not covered in the equation. Also the values used are mostly those that are given, and therefore can be more accurate since we know that they must be true.

3. What could be done in order to increase the accuracy of the approximation in #2?

In order to increase the accuracy of the approximation that the trapezoid is the best way to solve for the area of the pond, would be to use more than one shape. For instance with the use of trapezoids, triangles and rectangles all at once, it may be easier to fit into the shape of the pond and more accurately find the area.

4. a) Find the volume of water in the pond using your most accurate estimate.

$$\text{Volume} = \text{SA} \cdot \text{depth}$$

It is given that the average depth is 1.5 meters.

$$V = 607,000 \text{ m}^2 \cdot 1.5 \text{ m} =$$

$$V = 910,500 \text{ m}^3$$

b) What is the carrying capacity of your pond?

37 m^3 = space required for fish to grow to maximum size and does so in one year

$$\text{Carrying capacity} = V / 37 \text{ m}^3$$

$$\text{Capacity} = 910,500 \text{ m}^3 / 37 \text{ m}^3$$

$$\text{Capacity} = 246,081.0811 \text{ fish}$$

5. Let p represent the initial population of the pond, m represent the carrying capacity, $k=0.9/m$ represent the growth constant and t represent time in years.

a) Given the pond is stocked with 1000 fish initially, and using the carrying capacity obtained in #2 the exponential function that models the fish population of the pond in terms of time would be as follows,

To set up this equation we use a logarithmic growth model equation:

$$A(t) = \frac{m}{1 + Be^{-(0.9)t}}$$

$$A = \frac{246081.0811}{1 + Be^{-0.9t}}$$

To find B, we set $t = 0$, resulting in

$$A = \frac{m}{1 + B}$$

Since the pond is being stocked with 1,000 fish initially, we plug this in for A and then solve to find our B.

$$1000 = 246081.0811 / (1 + B)$$

$$1000 + 1000B = 246081.0811$$

$$B = 245.0810811$$

The reason we particularly choose a logistic growth model in this instance is the fact that it is a population growth that takes into account limitations on food and the environment. The initial population growth resembles exponential growth, but then at some point, due perhaps to food or space limitations, the growth slows down and eventually levels off. When this happens the population approaches an equilibrium level.

6. After approximately how many years will it take for your pond to reach its carrying capacity?

$$\text{a)} \quad A = \frac{246,081.0811}{1 + 245.0810811e^{0.9(30)}}$$

$$A = \frac{246081.0811}{1 + 2.29337635\dots}$$

$$A = \frac{246081.0811}{1}$$

$$A = 246081.0811$$

This proves that it will take approximately 30 years for the pond to reach its carrying capacity.

b) What would you do to halve the time in part (a)?

In order to halve the time in part (a) you would set up the equation so that t is divided by two, as seen in the following equation:

$$A = \frac{246081.0811}{1 + 245.0810811e^{-0.9(t/2)}}$$

$$A = \frac{246081.0811}{1 + 245.0810811e^{-13.5}}$$

$$A = 245998.4266$$

In order to halve the time in part (a), you would have to initially start out with 245,998 fish. This would lessen the amount of time would take to reach the carrying capacity.

d) Can you harvest 1000 fish each year for the first three years?

To figure this out you need to take the amount of fish that will be present after the first year and subtract 1,000 fishes.

$$A = 26081.0811 \div (1 + 245.0810811e^{-0.9}) \quad t = 1$$

$$A = 2445 - 1000 = 1445$$

Therefore for the 2nd year the equation would be $2445 + 1445 = 3890$

$$A = 3890 - 1000 = 2890$$

3rd year would be $2445 + 3890 + 2890 = 9225$

$$A = 9225 - 1000 = 8225$$

Yes you would be able to harvest 1000 fish each year for the first three years.

d) Can you harvest 2000 fish each year for the first three years?

$$1^{\text{st}} \text{ year } A = 2445 - 2000 = 445$$

$$2^{\text{nd}} \text{ year } A = 2445 + 445 = 2890$$

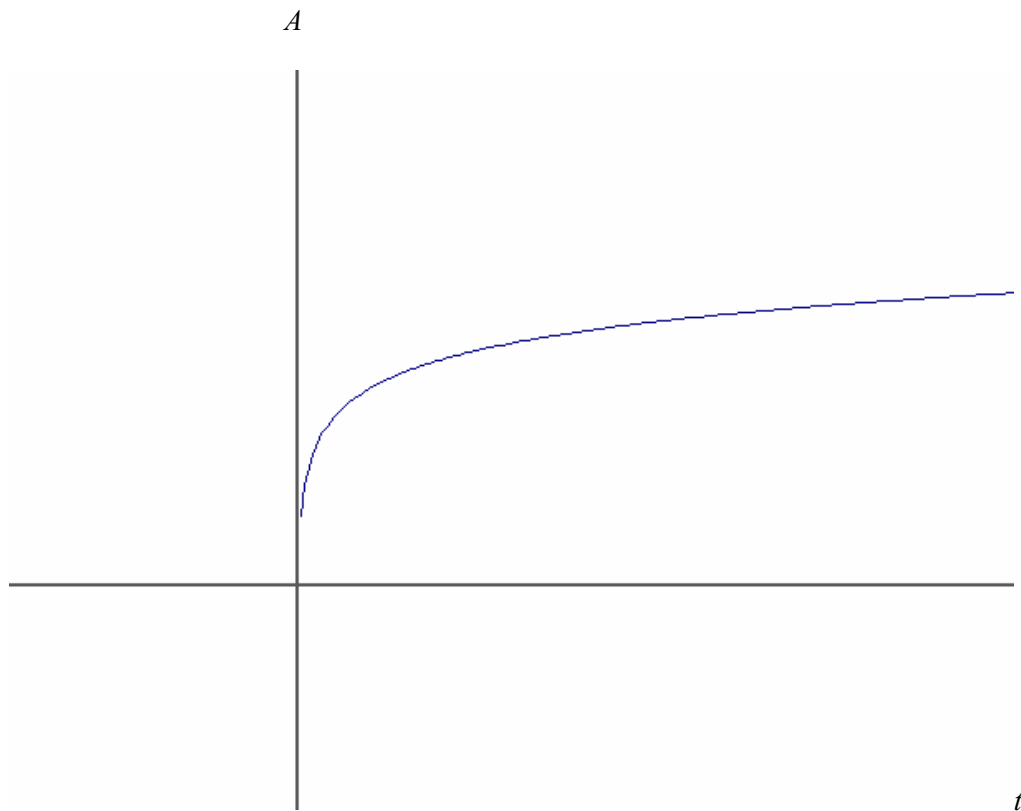
$$2890 - 2000 = 890$$

$$3^{\text{rd}} \text{ year } A = 2445 + 2890 + 890 = 6225$$

$$6225 - 2000 = 4225$$

Yes although barely making it at the beginning, you would be able to harvest 2000 fishes each year the first three years.

Logistic Model Graph



Equation Value Chart

$t =$	$A =$
3	14,085
6	116,796
9	229,042

12	244,856
15	245,998
18	246,075
21	246,080
24	246,081
27	246,081
30	246,081

7. At what time do you think the fish population will be growing the fastest?

I would have to say that the fish population would be growing the fastest before it reaches its carrying capacity at 246,081. One reason being that the growth rate of the fishes is increasing, as it is getting closer to the carrying capacity, and when it finally reaches it limited resources will inhibit further growth. As seen in the graph as time goes by the population continues to increase, until the limitations start to make an impact.

8. Some of the methods I will use to efficiently and effectively conduct business would be to uphold a certain minimum number of fish in the pond at all times. This way I will never be too low on fish and lose profit when there is not enough. I could do this by figuring out when I will have to restock to maintain a good amount of fish. Another way would be to make sure there is enough harvesting as well so that the fish pond does not meet its capacity too soon and overpopulate as well. Above all would be to pay attention to the patterns in the fish population and the factors that may cause it to change, like

weather for example. If I want to continue to harvest fish and be successful, only through these processes will I be able to do so.