IB Standard Level Type II
Math Portfolio

Sunset over New York

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I CERTIFY THAT THIS PORTFOLIO ASSIGNMENT IS ENTIRELY MY OWN WORK

Nam Vu Nguyen: _____

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Mathematics is a study of the concepts of quantity, structure, space and change. It is a type of science that draws conclusions and connections to the world around us. Mathematicians would call math a science of patterns and these patterns are discovered in numbers, space, science, computers, imaginary abstractions, and everywhere else. Mathematics is also found in numerous natural phenomena's that occurs around us. Today math is used all around us and is applied to many educational fields, through this people have become inspired to discover and make use of their mathematical knowledge which will then lead to entirely new disciplines. Math is present in wherever there are difficult problems that involve quantity, structure, space or change; such problems appear in various forms such as commerce, land measurement and especially astronomy.

The purpose of this paper is to examine the data on the times of the sunrise over New York, over a period of 52 weeks in one year. Sunrises are the beginning of a new day, when the first part of the sun appears over the horizon in the east. Since the dawn of mankind man himself have pondered on the mysteries of the Sun itself. Civilizations of the past have attempted to explain the reason why the sun rose in the morning and set in the night. This eventually led to creation of monuments around the world such as the Egyptian Pyramids, Stonehenge, and the Ancient Mayan Temples. By creating these monuments these people develop mathematical skills and knowledge in knowing how and where to strategically place these object in such a way that it will reflect or predict the rise and setting of the sun during the day. This involves predicting the times at which it rises and sets and the motion at which the sun moves throughout the day, and ultimately the entire year. This paper will take on a technological approach to analyzing the times at which sunrise occurs over New York, analyzing the data and determining the uses and advantages of applying the new found knowledge to the real world.

The following chart is a data chart, which shows the times at which sunrise occurs over the horizon of New York. The chart begins with week one of the year of 2003, and ends with the proposed last full week of 2003.

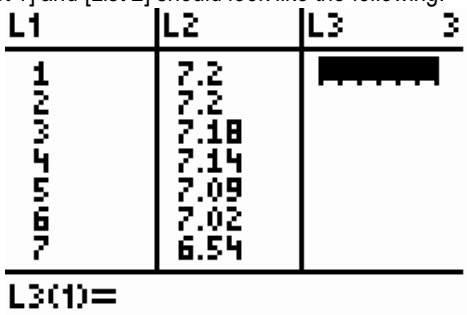
Week	Time	Week	Time	Week	Time
1	07:20:00	21	04:34:00	41	06:06:00
2	07:20:00	22	04:29:00	42	06:14:00
3	07:18:00	23	04:26:00	43	06:22:00
4	07:14:00	24	04:24:00	44	06:30:00
5	07:09:00	25	04:26:00	45	06:39:00
6	07:02:00	26	04:29:00	46	06:47:00
7	06:54:00	27	04:33:00	47	06:55:00
8	06:45:00	28	04:38:00	48	07:02:00
9	06:35:00	29	04:44:00	49	07:08:00
10	06:24:00	30	04:50:00	50	07:14:00
11	06:13:00	31	04:57:00	51	07:18:00
12	06:01:00	32	05:04:00	52	07:18:00
13	05:50:00	33	05:11:00		
14	05:38:00	34	05:17:00		
15	05:27:00	35	05:24:00		
16	05:16:00	36	05:31:00		
17	05:06:00	37	05:38:00		
18	04:56:00	38	05:45:00		
19	04:47:00	39	05:52:00		
20	04:40:00	40	05:59:00		

The table above shows the values of the time of sunrise. The time of sunrise in the beginning of the chart begins at 7:20am, and gradually begins to decrease up until the twenty-fourth week, with the lowest value for time being 4:24am. Following the twenty-fourth week the times begins to increase once again back to 7:20am. Therefore based on the data, we can safely assume that the times of sunrise have a period of 52 weeks, since the pattern of the times of sunrise seems to repeat after 52 weeks.

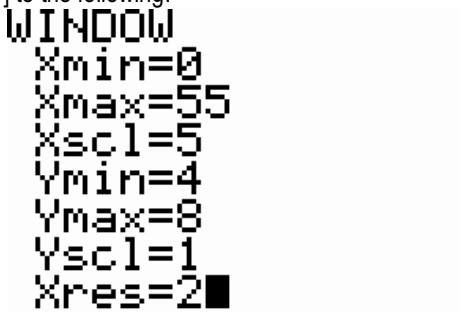
Using a Texas Instrument TI-83 PLUS Graphing Calculator, we can plot the data table into the calculator and later produce a graph of the data. To do so, we enter the numbers for the weeks into [List 1] of the calculator, and the times of the sunrise into [List 2] of the calculator.

To access [List 1], first press [STAT] on the calculator.
Then using the cursor buttons, move down to select [1: EDIT], press ENTER

Then begin to fill in the corresponding lists.
Once completed the beginning of [List 1] and [List 2] should look like the following:



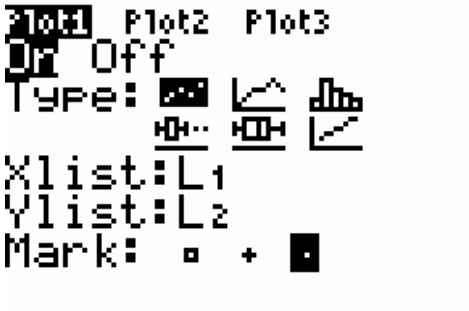
Before Graphing the Points, the WINDOWS of the calculator must be changed to fit the points.
To access the [WINDOWS], simply press [WINDOW] underneath the screen.
After doing so, change the [WINDOW] to the following:



Once the [WINDOW] has been changed, you must turn on [STAT PLOT]

To do so, press the [2nd] key function and then [Y=]
After accessing the [STAT PLOT] Menu, select [1]

After doing so, you should access a menu that looks like the following:



Edit your menu to look similarly like the above image.
After completing the above task, simply press the [GRAPH] button on your calculator, by doing so you should receive a screen that looks like the following:

The above image is what the points look like in the calculator, however, the way the image of the data is portrayed is not correct. This is due to the fact that the time of the sunrise that was entered into the calculator is in decimal form. Therefore since the data is entered into decimal form and not correctly in time form, the data graph points are fragmented because there are only 60 minutes in an hour, but the minutes are written as decimal points. There are sections containing points that appear to be part of the same function however the graph data points are broken this is because there are only 60 minutes in an hour and since the decimals only go up to a maximum of 0.59 a gap is noticeable from between decimal points 0.60 to 0.99. In order to correct the graph, we must change the time values in order to properly fit the scale.

In order to change the times to properly display the graph points we must first convert the time values from general decimal form to time decimal form.

Week: 1 – Time: 7:20am or 7 hours and 20 minutes

To convert the time, first take the minutes of the time and divide it by 60 minutes. The decimal answer given is the new minutes for the new time.

20 minutes / 60 minutes = 0.33 minutes

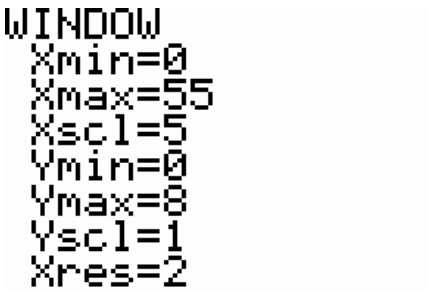
Therefore the new time values is 7.33 hours

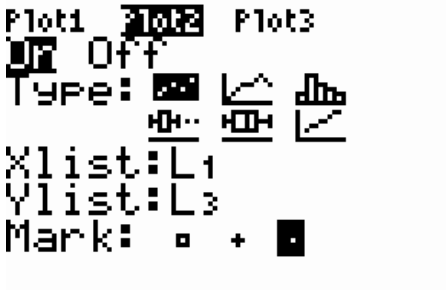
Once completed the new time values for the graph, the following new data table should look like the following:

Sunrise over New York

Week	Time	Week	Time	Week	Time
1	7.33	21	4.56	41	6.1
2	7.33	22	4.48	42	6.23
3	7.3	23	4.43	43	6.37
4	7.23	24	4.4	44	6.5
5	7.15	25	4.43	45	6.65
6	7.03	26	4.48	46	6.78
7	6.9	27	4.55	47	6.92
8	6.75	28	4.63	48	7.03
9	6.58	29	4.73	49	7.13
10	6.4	30	4.83	50	7.23
11	6.21	31	4.95	51	7.3
12	6.01	32	5.07	52	7.3
13	5.83	33	5.18		
14	5.63	34	5.28		
15	5.45	35	5.4		
16	5.28	36	5.52		
17	5.2	37	5.63		
18	4.93	38	5.75		
19	4.78	39	5.87		
20	4.66	40	5.98		

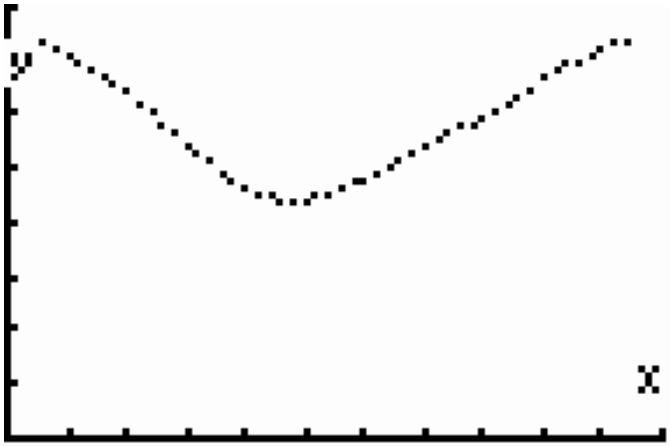
The data table when entered into your calculator as [LIST 3], changes should also be made to correctly display the graph. To display the correct graph, the following changes must be made and applied to [PLOT 2] of your calculator and also your [WINDOW] format.



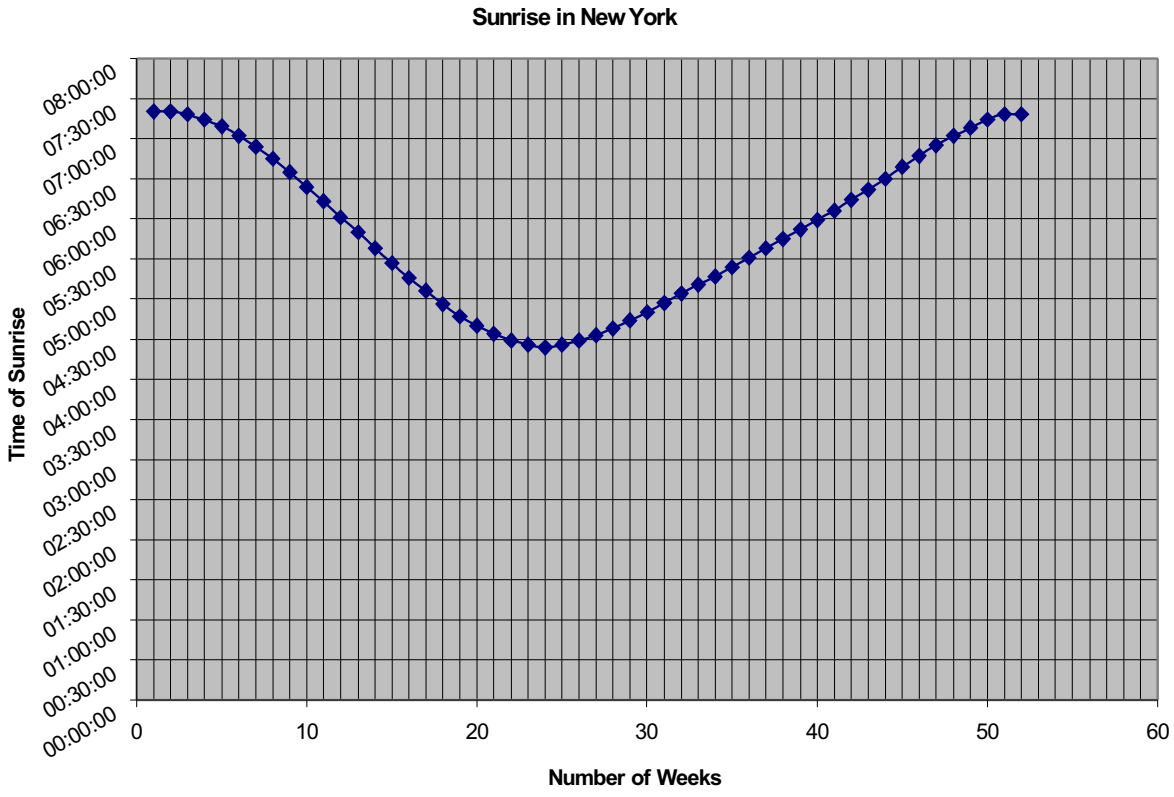


To access [STAT PLOT 2] Press the [2nd] key function and then [Y=]
After accessing the [STAT PLOT] Menu, select [2]

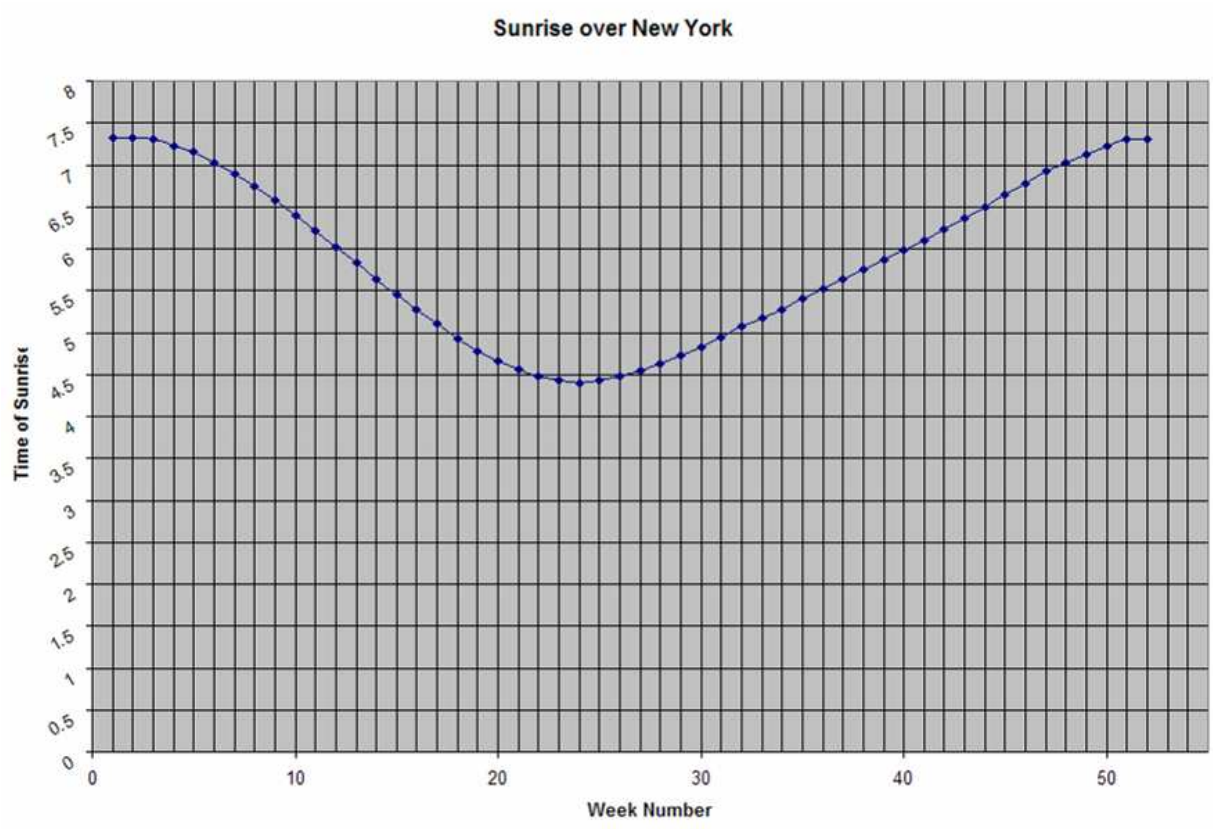
After completing the above tasks, the new graph should look similarly like the following:



Afterwards two more graphs were created using Microsoft Excel. One graph contains the old data, before the time value was changed to decimal time form.



D: { x|



The graphs above are the same graphs yet they use different time values, for the duration of this math portfolio, we shall continue to use the new time values, in decimal time format, for the values in the graphing calculator, whereas the original time data will be used to create graphs using Microsoft Excel. The above graphs show that the data clearly forms either a sine or cosine graph. The graph is similar to that of a parabola; therefore the graph appears to be somewhat as close to a quadratic graph as possible. However, since the graph is only a collection of data of sunrise in New York City in a one year time span, the graph would clearly repeat itself in not only the past years but also in the years to come. Therefore, the graphs represented are sine and cosine function graph in the following form respectively:

$$\boxed{f(x) = a\sin(bx + c) + d \quad OR \quad f(x) = a\cos(bx + c) + d}$$

To produce the equation for the two functions for the set of data, there are values and variables which are needed to be calculated first. These values that must be calculated first for the two functions are the values for a, b and c.

The variable "a" is the amplitude of the function. The amplitude is accountable for determining the vertical stretch of the trigonometric functions. Amplitude is the distance from the center line (equilibrium) of the function to either the maximum or minimum points. The value of a can be found out by the following formula, where the difference of the maximum value and the minimum value is divided by two.

$$a = \frac{|\max - \min|}{2}$$

The next variable "b" is responsible for the horizontal stretch of the sine function. It represents the number of cycles that a trigonometric graph has within a span of 2π . Therefore the formula to calculate the variable "b" is where 2π is divided by the period of the graph.

$$b = \frac{2\pi}{period}$$

The variable "c", determines the number of units of a function's horizontal translation. If $c \geq 0$ then there is a "c" unit's horizontal translation to the left. If the $c \leq 0$ then there is a "c" units horizontal translation to the right.

The variable "d" determine the "d" units of the function's vertical translation. If the $d \geq 0$ then there is a "d" units vertical translation up. If the $d \leq 0$ then there is a "d" units vertical translation down. It is determined by the formula where the sum of the maximum value and the minimum value is divided by two.

$$d = \frac{\max + \min}{2}$$

After discussing the purpose of the trigonometric values and how to calculate them, we will now set out to determine the functions that can model the data. The following values will now be calculated for a cosine function as follows:

"A" value = $a = \frac{|\max - \min|}{2}$ = $\frac{|7.20 - 4.24|}{2}$ = 1.48 or $\frac{37}{25}$

"B" value = $b = \frac{2\pi}{period}$ = $\frac{2\pi}{52}$ = $\frac{\pi}{26}$ or $\approx 0.120...$

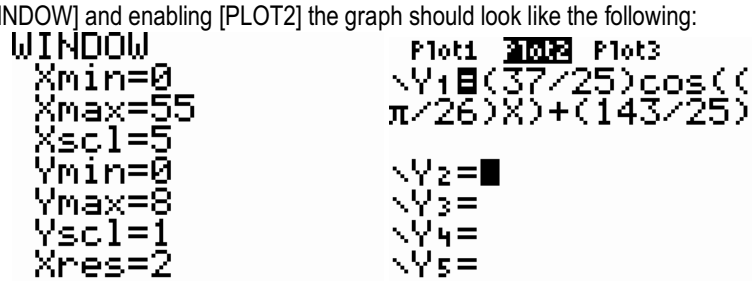
"C" value = 0, this is because the graphs have no phase shifts. Cosine functions have y-intercepts that begin at another number other than zero for their "x" value.

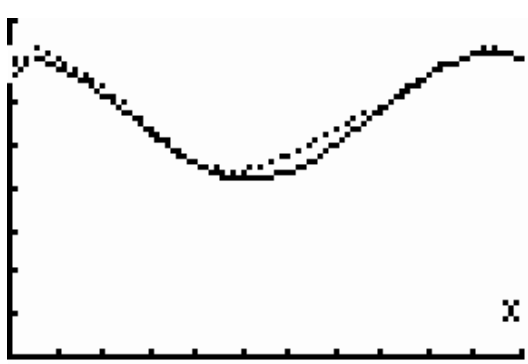
"D" value = $d = \frac{\max + \min}{2}$ = $\frac{7.20 + 4.24}{2} \approx 5.72$ or $\frac{143}{25}$

Now after the values for the cosine function is now completed we can now put together the function for the graph.

$$f(x) = a\cos(bx + c) + d \rightarrow f(x) = 1.48\cos(\frac{\pi}{26}x) + 5.72 \text{ Or } f(x) = \frac{37}{25}\cos(\frac{\pi}{26}x) + \frac{143}{25}$$

With the function created, we now enter it into our calculator to determine how well the function fits with our data.
We begin by entering the function into [Y1] of our graphing calculators. This is simply done by pressing the [Y=] button found underneath the graphing screen, and simply typing in the equation itself.
Keeping the same [WINDOW] and enabling [PLOT2] the graph should look like the following:





Now calculations will be made to determine the sine function of the data.

"A" value = $a = \frac{|\max - \min|}{2}$ = $\frac{|7.20 - 4.24|}{2}$ = 1.48 or $\frac{37}{25}$

"B" value = $b = \frac{2\pi}{period}$ = $\frac{2\pi}{52}$ = $\frac{\pi}{26}$ or $\approx 0.120...$

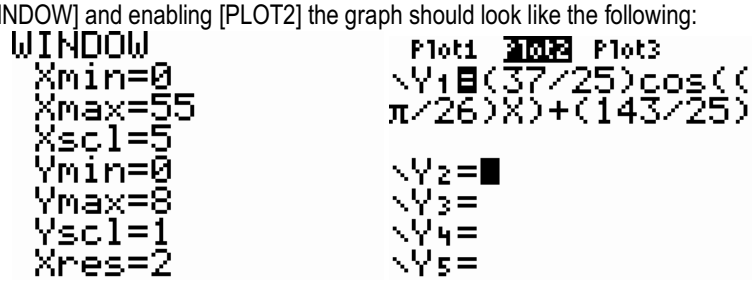
"C" value = $\frac{\pi}{2}$

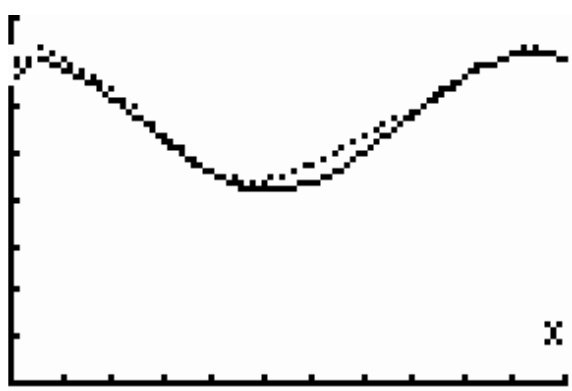
"D" value = $d = \frac{\max + \min}{2}$ = $\frac{7.20 + 4.24}{2} \approx 5.72$ or $\frac{143}{25}$

Now after the values for the cosine function is now completed we can now put together the function for the graph.

$f(x) = a\sin(bx + c) + d \rightarrow f(x) = 1.48\sin(\frac{\pi}{26}x + \frac{\pi}{2}) + 5.72$ Or $f(x) = \frac{37}{25}\sin(\frac{\pi}{26}x + \frac{\pi}{2}) + \frac{143}{25}$

Once the function is created and determined, we will now enter it into our calculators.
Keeping the same [WINDOW] and enabling [PLOT2] the graph should look like the following:





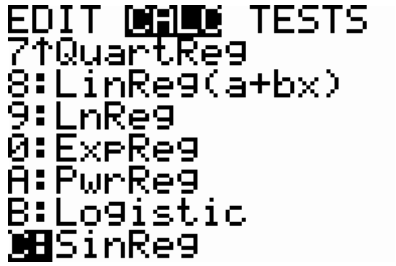
The graph of the equations above would both seem to be a suitable fit to the set of data; this is due to the fact that the functions generates a curve of fit that suits the data, to the point that it also intersects some points in the data. However, there is a part in both graphs where the equations miss the data points entirely. This is due to the fact that the graph is a perfect wave, whereas the data points are collected throughout the year and does not provide a clear enough data. The data themselves are not perfect, this is because the times could be slightly off, because we as humans cannot calculate the exact time of sunrise but rather an estimate, this will be regarded as a human error in the data.

Calculating for the functions manually without using calculator help could prove sometimes to be very bothersome, and errors could arise in the function themselves. This would create problems in the graphs themselves since the functions are not entirely accurate to an extent. Besides creating and formulating the functions manually there are also other ways of calculating for a function using a Regression Tool, most notably a Sinusoidal Regression Tool. This could be done by using the SinReg function tool, found in the TI-83.

To calculate a SinReg regression function on the TI-83, the following steps must be followed in order to achieve the correct results.

Firstly, access the Statistics menu by pressing the [STAT] button, your screen should look like the following:

Now press the right arrow once to move to the [CALC] menu. Then scroll down until you reach the last option labeled as [SinReg] for sinusoidal regression.



After selecting the function by pressing [ENTER]. The SinReg function is now present on the home screen. However you still need to perform four more steps in order to complete the command.

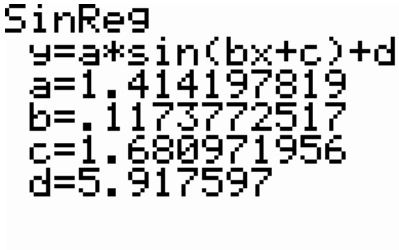
Step One: Enter in the [List] for the x values; simply by pressing [2nd] and [STAT], then select [L1], where our x values are located in. Now press [,].

Step Two: Enter in the [List] for the y values in this case the time values; simply by pressing [2nd] and [STAT], then select [L3], where our x values are located in. Now press [,].

Step Three: Enter in the value for our period, which are 52. Now press [,].

Step Four: Enter in [Y1], simply by pressing [VARS], then pressing the right arrow once to move to the [Y-VARS] menu, select [FUNCTION], then [Y1].

Now execute the SinReg function by pressing [ENTER]. You should then have a screen that looks like the following:



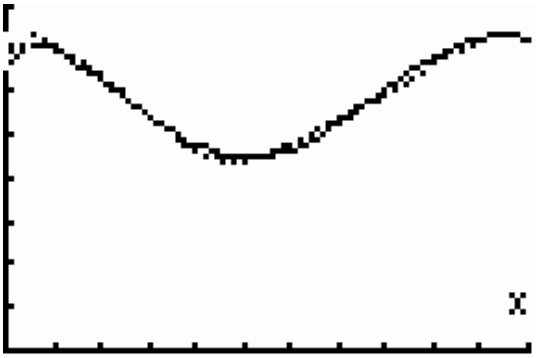
This tells you what the values are for the function, $f(x) = a\sin(bx + c) + d$ where:

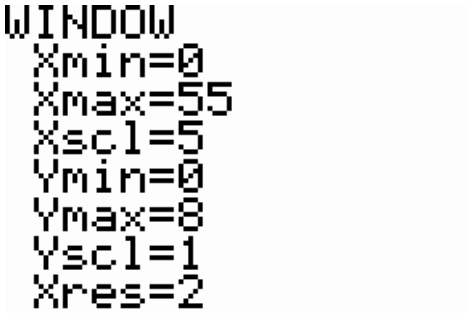
A≈ 1.41 B≈0.117 C≈1.68 D≈5.92

Therefore the function is written as $1.41\sin(0.117x + 1.68) + 5.92$.

When graphing the function, the function is already entered into your “y” functions list, your [Y1]

What is left to do is to simply press [GRAPH]. Where the following graph should appear, if the same [WINDOW] is kept:





After analyzing the data and graphs made, a conclusion should be able to drawn from the above evidence that the graph created by the calculator through the use of the SinReg function, portrays a much more accurate function then the one created manually by hand. This graph correctly connects or goes through all data points present on the graphing screen. As a result, the function created by the SinReg function tool is a more correct function, rather than the ones created manually. The two functions created by hand are different than the one created electronically due to the fact that the graphs have a slightly more off phase shift (different “c” value) and also a different period. Thus, the function $1.41\sin(0.117x+1.68)+5.92$ is much more sensible to use as the primary function for the model of “Sunrise over New York”.

Functions like these are used in ways that is not present to us in the real world. These functions that determines the times for the sunrise in the morning of a particular city, provides data for the city to plan when to start or end many of the city’s functions such as city lights, traffic, transportation means , store functionality and many more. Another situation is how the models are helpful for someone programming switching off street-lights. In a scenario such as determining the programming for switching off and on the city street lights, a function similar to one calculated for the “Sunrise over New York” is sensible to use. City street-lights are only required to turn on or off when there is no light present in the city such as during the night, or when there is enough visible light. Therefore, the usefulness of this equation for street-lighting schedules is very valued since it makes adjusting the time for powering on or off much easier then having to manually configuring it every time the sunrise time or sunset time is changed.

However, to a person planning a daybreak run, a light jog during the sunrise or early morning, the model for the “Sunrise over New York” is somewhat practical to use since the person is planning a run during the sunrise. The set of data could be proved useful to a person planning a run by providing the required information for the times for the days of the year so that they could plan according to their schedules with the sunrise times in mind.

The times of sunrise are variously different in many different parts of the year and also the world. A reason for this being would be the earth’s orbit and its titled axis. This affects how the sun shines on the earth’s surface and how the seasons are also affected. In the Northern Hemisphere, where the earth is titles more towards the sun the length of day ism much shorter due, because the sunrise comes up much earlier and then quickly setting after a few hours. In the Southern Hemisphere the opposite occurs, sunrise comes up somewhat later in the day, and sets later on at night, providing longer lengths of day. Therefore, as you travel further north, a later sunrise and earlier sunset could not only disrupts sleeping habits but also provides shorter days in winter and longer daylight hours in the summer. However, as you travel south further down an earlier sunrise and later sunset occurs, along with longer winter and shorter summer days. Even though the time of sunrise would be the same for places on the same latitude or plane, this would be only possible through if you are in the correct time zones.

For a city one thousand kilometers away from New York, the time of sunrise will vary. As mentioned before as you travel down south of New York by one thousand kilometers, to a city such as Charlotte, North Carolina. During a period known as the Winter solstice which occurs around December, the sun rises earlier and sets later creating a longer day length. In the period known as the summer solstice in the northern hemisphere, which happens June, the sunrise comes later and the sunset earlier creating a shorter daylight or day length.

Sunrise over Charlotte, North Carolina

Week	Time	Week	Time	Week	Time
1	07:32:00	21	05:15:00	41	06:29:00
2	07:32:00	22	05:12:00	42	06:35:00
3	07:31:00	23	05:09:00	43	06:42:00
4	07:29:00	24	05:08:00	44	06:48:00
5	07:25:00	25	05:09:00	45	06:55:00
6	07:20:00	26	05:10:00	46	07:02:00
7	07:13:00	27	05:13:00	47	07:09:00
8	07:05:00	28	05:16:00	48	07:15:00
9	06:57:00	29	05:21:00	49	07:21:00
10	06:48:00	30	05:25:00	50	07:26:00
11	06:39:00	31	05:30:00	51	07:29:00
12	06:29:00	32	05:36:00	52	07:32:00
13	06:19:00	33	05:41:00		
14	06:09:00	34	05:47:00		
15	06:00:00	35	05:52:00		
16	05:50:00	36	05:57:00		
17	05:42:00	37	06:02:00		
18	05:34:00	38	06:07:00		
19	05:26:00	39	06:13:00		
20	05:20:00	40	06:23:00		

As you travel north of New York by one thousand kilometers, to a city known as Quebec City, Quebec, the sun rises later and sets earlier creating a shorter day length. In the summer sunrise comes really early and sets later at night creating a much longer day length. A data table containing the times for Quebec City, contained missing information about the sunrise times for the city due to misinterpretation of the times by the US Navy Astronomical Association.

Chicago, Illinois located one thousand kilometers west of New York, is located on the same on latitude with New York, therefore the times of Sunrise for Chicago is relatively the same with New York.

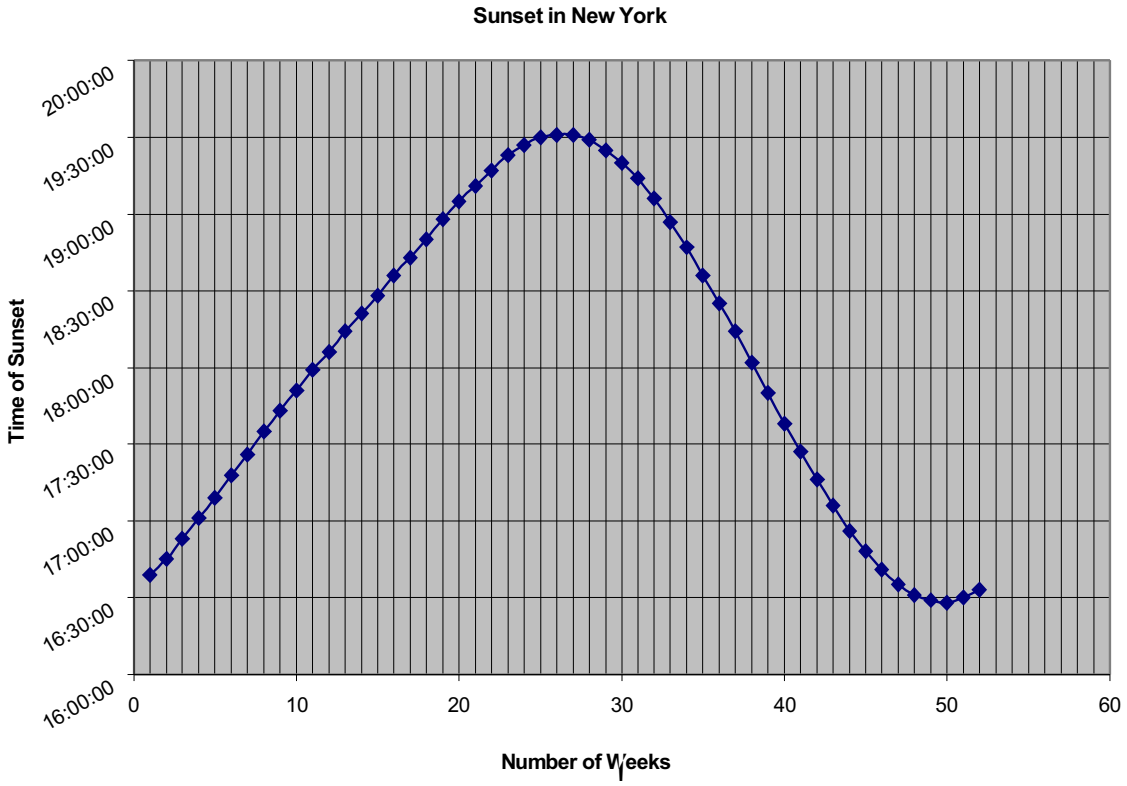
Sunrise Times for Chicago, Illinois

Week	Time	Week	Time	Week	Time
1	07:18:00	21	04:25:00	41	05:55:00
2	07:18:00	22	04:20:00	42	06:03:00
3	07:16:00	23	04:17:00	43	06:11:00
4	07:12:00	24	04:15:00	44	06:19:00
5	07:06:00	25	04:15:00	45	06:28:00
6	06:59:00	26	04:16:00	46	06:36:00
7	06:51:00	27	04:20:00	47	06:45:00
8	06:41:00	28	04:24:00	48	06:53:00
9	06:31:00	29	04:29:00	49	07:00:00
10	06:20:00	30	04:36:00	50	07:07:00
11	06:08:00	31	04:42:00	51	07:12:00
12	05:56:00	32	04:49:00	52	07:16:00
13	05:44:00	33	04:56:00		
14	05:32:00	34	05:04:00		
15	05:21:00	35	05:11:00		
16	05:09:00	36	05:18:00		
17	04:59:00	37	05:25:00		
18	04:49:00	38	05:33:00		
19	04:40:00	39	05:40:00		
20	04:32:00	40	05:47:00		

Along with the data table, on the "Sunrise over New York", a graph was also given on the sunset times of New York. The graph was then taken and turned into a data table and later a new graph was drawn up through Microsoft Excel.

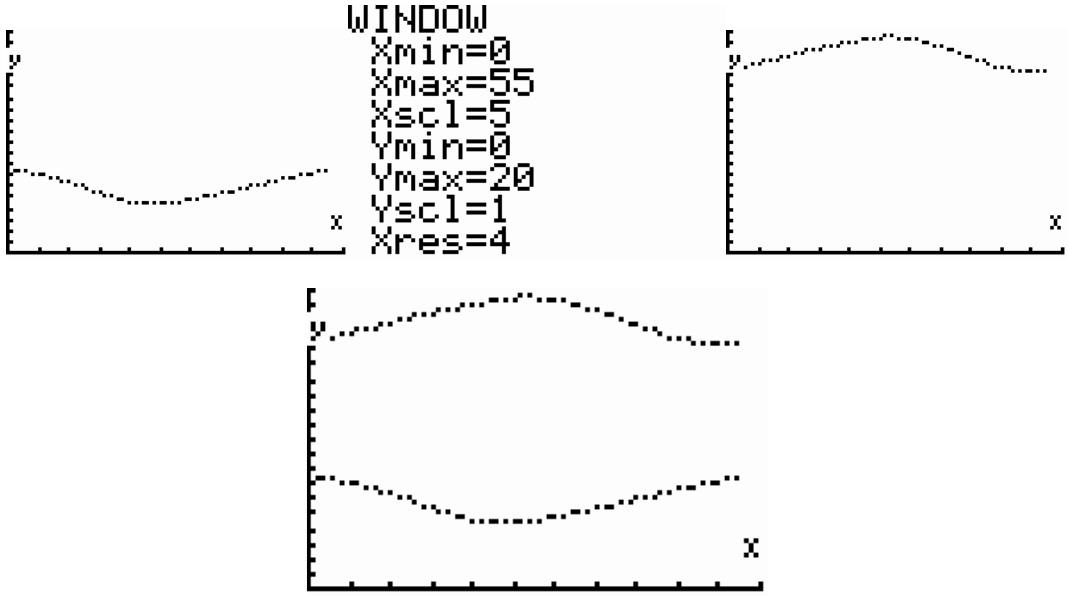
Sunset Times over New York

Week	Time	Week	Time	Week	Time
1	16.39	21	19.11	41	17.27
2	16.45	22	19.17	42	17.16
3	16.53	23	19.23	43	17.06
4	17.01	24	19.27	44	16.56
5	17.09	25	19.3	45	16.48
6	17.18	26	19.31	46	16.41
7	17.26	27	19.31	47	16.35
8	17.35	28	19.29	48	16.31
9	17.43	29	19.25	49	16.29
10	17.51	30	19.2	50	16.28
11	17.59	31	19.14	51	16.3
12	18.06	32	19.06	52	16.33
13	18.14	33	18.57		
14	18.21	34	18.47		
15	18.28	35	18.36		
16	18.36	36	18.25		
17	18.43	37	18.14		
18	18.5	38	18.02		
19	18.58	39	17.5		
20	19.05	40	17.38		



The above data charts and graph shows eth sunset times for New York. With this information derived from the data found in the graphs and tables, we can determine when the length of the shortest day of the year was. Along with that we can also determine the approximate dates in which the day was more than 12 hours long.

We can see when was the shortest day was by simply looking at a combined graph of both the sunrise and sunset times for New York, and determine when was the two graphs were very close to each other. The following graphs are the sunrise and sunset graphs separately and the third graph is when the two is combined. The [WINDOW] for these graphs are as follows



By simply looking at that graphs, the week where they are both at a very short distance from one another is the week where it has the shortest days. To determine the length of the shortest day, we subtract the time of the sunrise from the time of the sunset, which will give us the length of the shortest day, or rather week. The time of the sunset at that time was 16:39pm Eastern Time, and the time for the sunrise was at 7:20am Eastern Time. Therefore the difference between the two numbers is 9:19 hours or nine hours and nineteen minutes. Thus the shortest length of the day is nine hours and nineteen minutes. The same can be done to determine the dates when the days are more than twelve hours long. By drawing a line that runs along y=12, y=18 and y=6, we can see the where the weeks began to have more than twelve hours of sunlight. By looking at where the y=18 and y= 6 liens intersect we can determine the number of the week. The weeks after determining where they cross are at week 12 and ended on week 38. To determine the dates for we simply multiply the week number by 7 and we will get a number of the day. After doing so, the number of the days is Day 82 and Day 272 of the year of 2003. The by consulting a calendar and counting the days, we have found out that the dates are respectively March 23rd and September 29th.Therefore, the days when there was more than twelve hours of daylight are between March 23rd and September 29th of 2003.

To conclude the assessment, identified and discovered information from data given to us and solved the problems present to us by using technology that was granted to us. The purpose of the assignment is now complete as all necessary questions are resolved and steps were taken to correctly identify the problem, assess it and accomplish it. the portfolio demonstrated that all the data that was used and processed have the potential to solve no only have real life applications, but also the mysteries that surrounds them. The student using their knowledge of graphs, cosine and sine functions, transformations, and graphing on not only the TI-83 Plus calculator, but also the media present to them was successful in solving the task given to them on the Sunrise of New York.

Reference:

1.) Microsoft Office 2003 XP Professional
2.) http://www.cic-caracas.org/vanas/vanascontent/handouts/davis2.pdf
3.) http://aa.usno.navy.mil/data/docs/RS_OneYear.php
4.) http://www.xuru.org/rt/TOC.asp
5.) http://www.libs.uga.edu/ref/chicago.html
6.) Student based notes on Trigonometry and Functions.
7.) Texas Instruments TI-83 Plus
TI-GRAPH LINK USB
TI-Connect Disc