

Stopping Distances

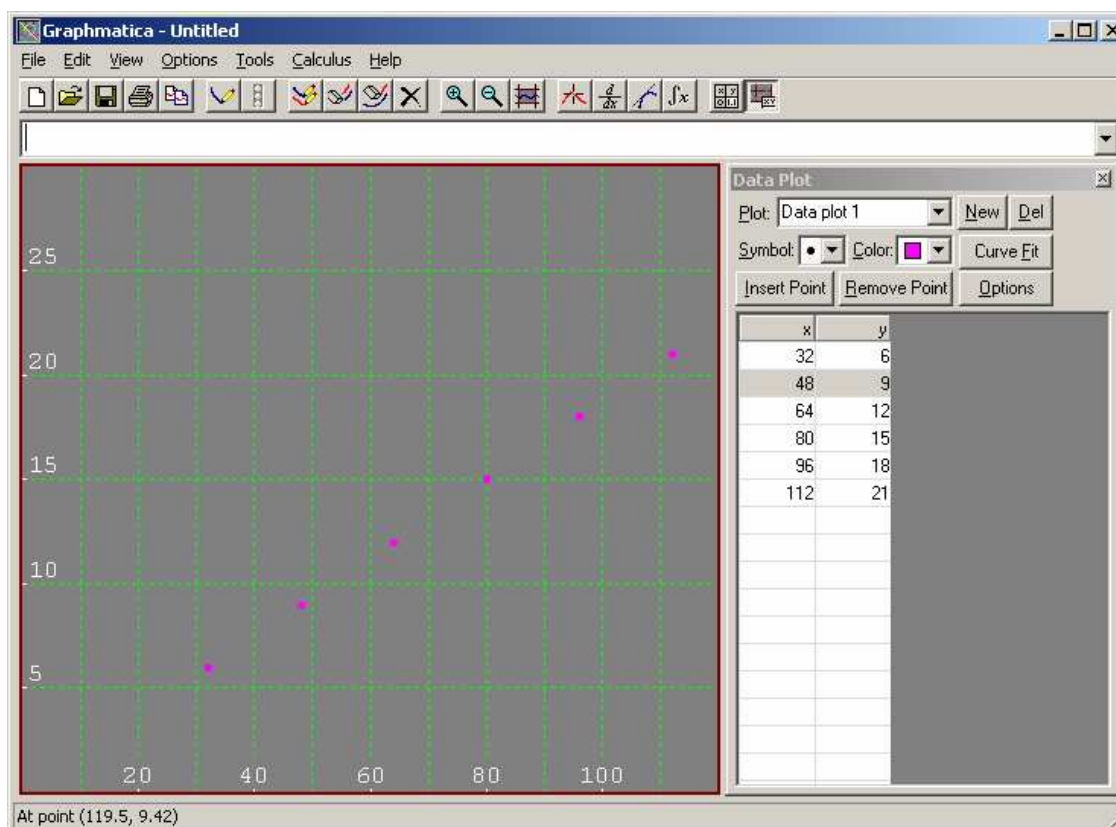
Portfolio

Mathematics SL Yr. 1
Period A

Speed (x) Vs. Thinking Distance (y)

Points plotted on the graph:

(32, 6)
 (48, 9)
 (64, 12)
 (80, 15)
 (96, 18)
 (112, 21)



As you can see in the graph above all six points seem to line up in a fairly straight line. As speed increases, so does the thinking distance, always by a similar amount. This means that the speed to thinking distance ratio is constantly increasing as speed increases.

Functions:

Slope:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

$$m = (12 - 6) / (64 - 32)$$

$$m = 6/32$$

$$m = 3/16$$

(32,6):

$$y - y_1 = m(x - x_1)$$

$$y - 6 = (3/16)(x - 32)$$

$$y - 6 = (3/16)x - 6$$

$$y = (3/16)x$$

(64, 12):

$$y - y_1 = m(x - x_1)$$

$$y - 12 = (3/16)(x - 64)$$

$$y - 12 = (3/16)x - 12$$

$$y = (3/16)x$$

Percentage of error"

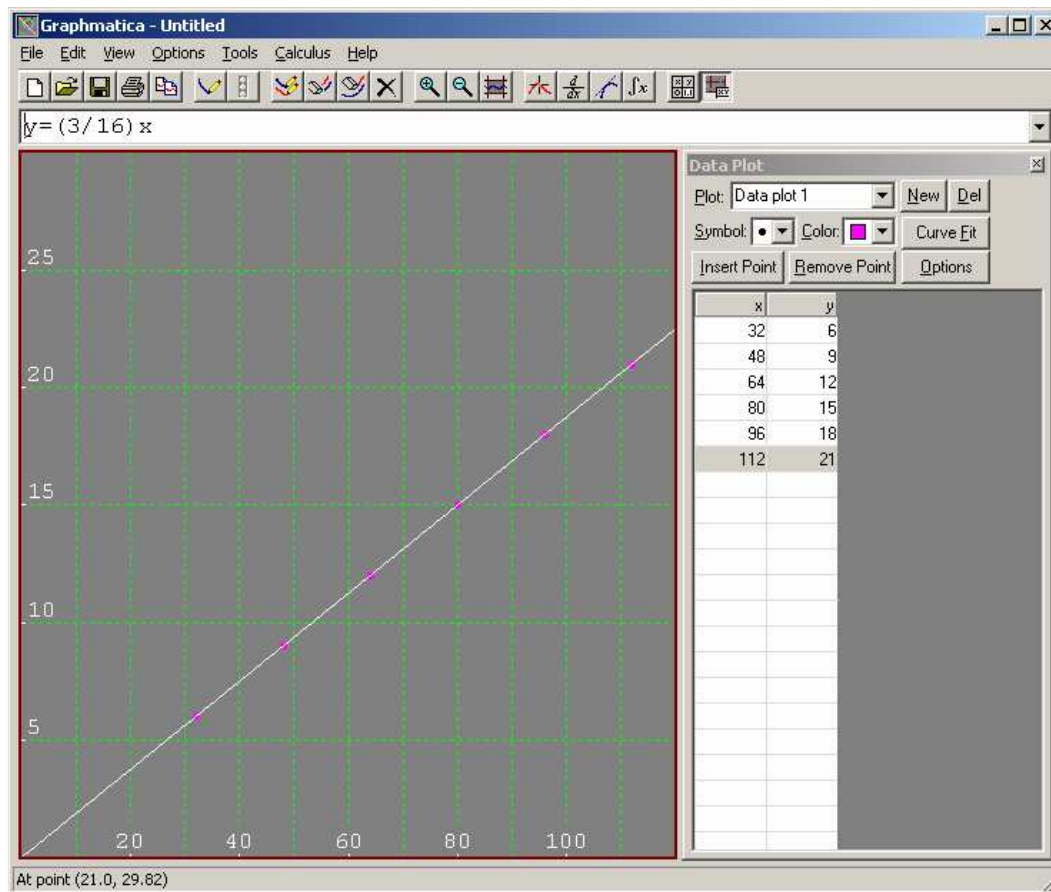
$$(64, 12)$$

$$y = (3/16)64$$

$$y = 12$$

$$(12 - 12) / 12$$

$$\approx 0 \%$$



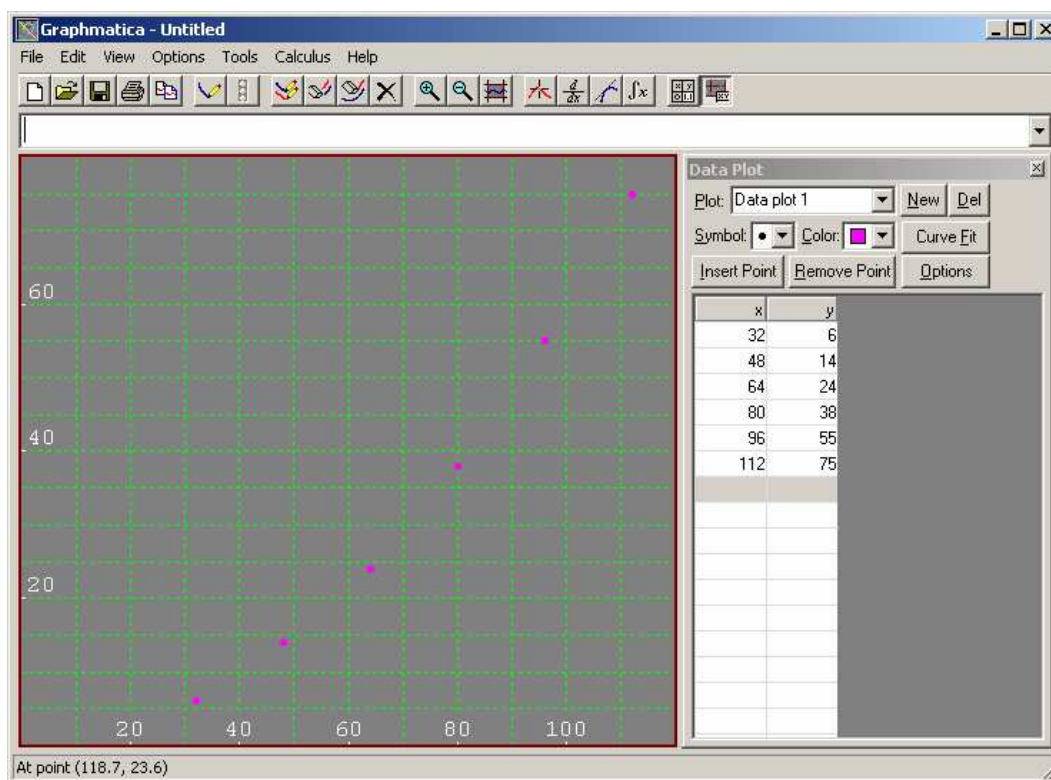
The function, $y = (3/16)x$, matches the points, plotted on the graph, almost perfectly. You can see this by the calculated percentage of error which equals 0%. The function goes through every single point graphed. In the real life situation this means that the function

represents the data recorded, while measuring how long it took for a person to think to apply the brakes, at different speeds, perfectly. The fact that a linear function resembles this situation in a very accurate manner means that the speed to thinking distance ratio is constant.

Speed (x) Vs. Braking Distance (y)

Points plotted on the graph:

(32, 6)
(48, 14)
(64, 24)
(80, 38)
(96, 55)
(112, 75)



Different from the first graph displayed, the points on this graph are not all lined up in a straight line. Here, the shape occurs to be a parabola. This graph however only includes the non-negative numbers, meaning it is only half of a parabola

Functions:

$$y = ax^2 + bx + c$$

1. (32, 6)
2. (48, 14)
3. (64, 24)

$$\begin{aligned} 1. \quad 6 &= (32)^2a + (32)b + c \\ 6 &= 1024a + 32b + c \end{aligned}$$

$$\begin{aligned} 2. \quad 14 &= (48)^2a + (48)b + c \\ 14 &= 2304a + 48b + c \end{aligned}$$

$$\begin{aligned} 3. \quad 24 &= (64)^2a + (64)b + c \\ 24 &= 4096a + 64b + c \end{aligned}$$

$$\begin{array}{r} 6 = 1024a + 32b + c \\ - 14 = 2304a + 48b + c \\ \hline 8 = 1280a + 16b \end{array}$$

$$\begin{array}{r} 14 = 2304a + 48b + c \\ - 24 = 4096a + 64b + c \\ \hline 10 = 1792a + 16b \end{array}$$

$$\begin{array}{r} 8 = 1280a + 16b \\ - 10 = 1792a + 16b \\ \hline 2 = 512a \end{array}$$

$$a \approx 0.003900$$

$$8 = 1280(0.0039) + 16b$$

$$8 = 4.992 + 16b$$

$$3.008 = 16b$$

$$b \approx 0.1880$$

$$6 = 1024(0.0039) + 32(0.188) + c$$

$$6 = 3.9936 + 6.016 + c$$

$$c \approx -4.0096$$

$$y = 0.0039x^2 + 0.188x - 4.0096$$

Percentage of error

$$(112, 75)$$

$$y = 0.0039(112)^2 + 0.188(112) - 4.0096$$

$$y = 48.9216 + 21.056 - 4.0096$$

$$y = 65.968$$

$$(65.968 - 75) / 75$$

$$\approx -12.04\%$$

$$(96, 55)$$

$$y = 0.0039(96)^2 + 0.188(96) - 4.0096$$

$$y = 35.9424 + 18.048 - 4.0086$$

$$y = 49.9818$$

$$(49.9818 - 55) / 55$$

$$\approx -9.124 \%$$

$$(80, 38)$$

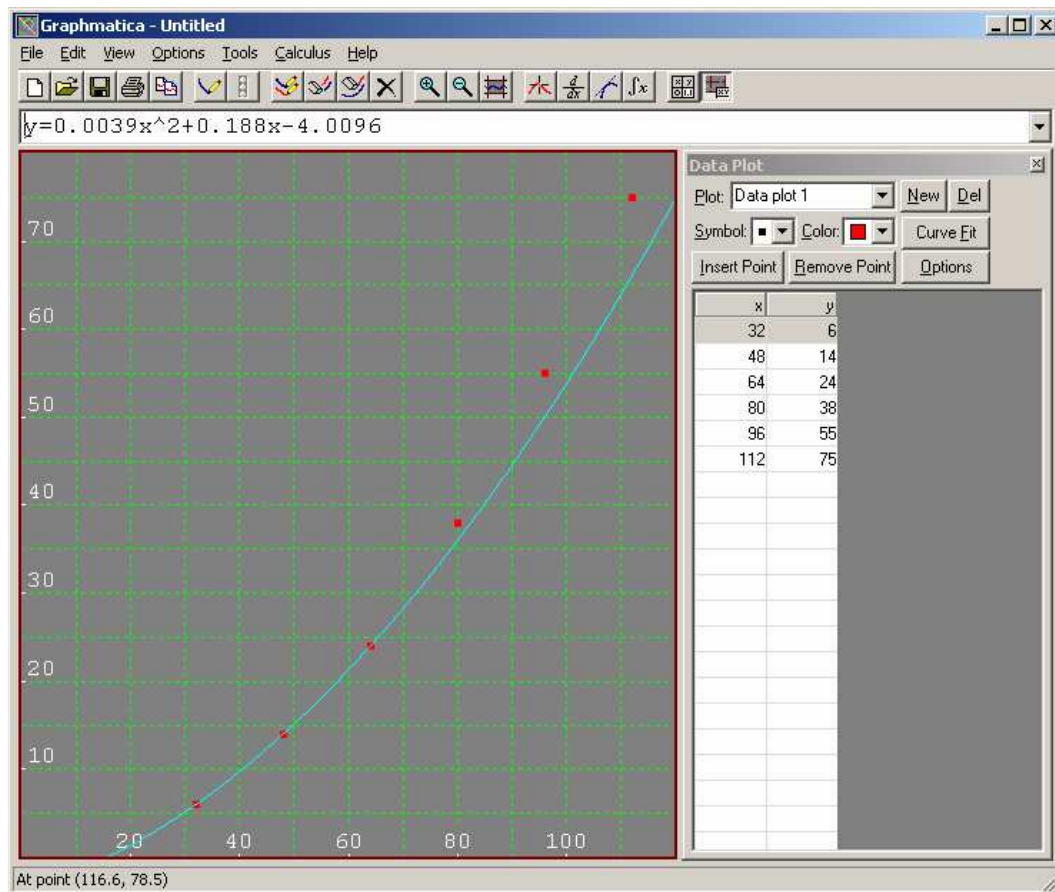
$$y = 0.0039(80)^2 + 0.188(80) - 4.0096$$

$$y = 24.96 + 15.04 - 4.0096$$

$$y = 35.9904$$

$$(35.9904 - 38) / 38$$

$$\approx -5.288 \%$$



The Function, $y = 0.0039x^2 + 0.188x - 4.0096$, used to represent these data points is a parabolic function. It is not quite as accurate as the function representing speed vs. braking distance. I used the first three data points to come up with an appropriate

function, thus it goes right through these points. The last three points however lay slightly above the function, with percentage errors of $\approx -12.04\%$, $\approx -9.124\%$ and $\approx -5.288\%$. This function has several other limitations aswell. First of all it only works for lower speeds rather than when the car travels at higher speeds. Also, it represents a parabolic function however the data points only includes the positive numbers, meaning half of a parabola.

$$y = ax^2 + bx + c$$

$$1. (80, 38)$$

$$2. (96, 55)$$

$$3. (112, 75)$$

$$1. 38 = (80)^2a + (80)b + c$$

$$38 = 6400a + 80b + c$$

$$2. 55 = (96)^2a + (96)b + c$$

$$55 = 9216a + 96b + c$$

$$3. 75 = (112)^2a + (112)b + c$$

$$75 = 12544a + 112b + c$$

$$38 = 6400a + 80b + c$$

$$- 55 = 9216a + 96b + c$$

$$17 = 2816a + 16b$$

$$55 = 9216a + 96b + c$$

$$- 75 = 12544a + 112b + c$$

$$20 = 3328a + 16b$$

$$17 = 2816a + 16b$$

$$- 20 = 3328a + 16b$$

$$3 = 512a$$

$$a \approx 0.005900$$

$$17 = 2816(0.0059) + 16b$$

$$17 = 16.6144 + 16b$$

$$0.3856 = 16b$$

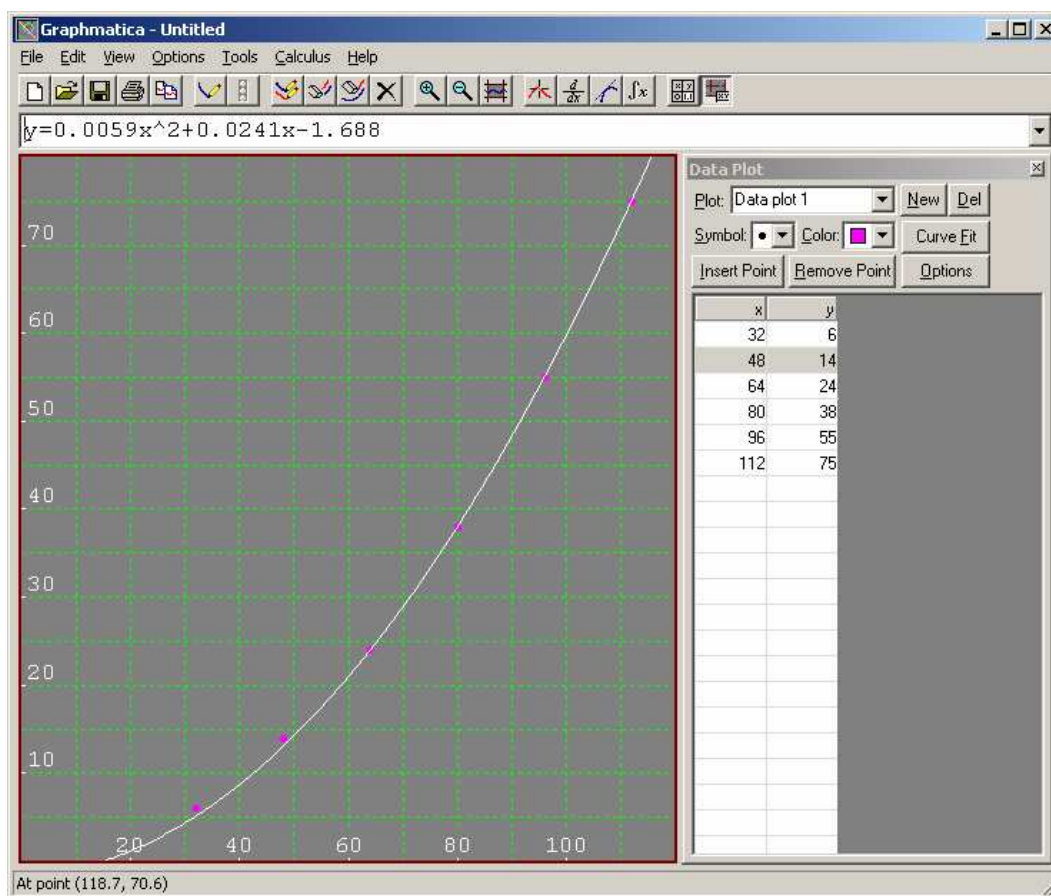
$$b \approx 0.02410$$

$$38 = 6400(0.0059) + 80(0.0241) + c$$

$$38 = 37.76 + 1.928 + c$$

$$c \approx -1.688$$

$$y = 0.0059x^2 + 0.0241x - 1.688$$



The points used to find a new function were the last three points. Different from the previous function, this one works only for cars at higher speed, considering it goes right through the last couple of point however is slightly below the first two.

Percentage of error

(32, 6)

$$y = 0.0059(32)^2 + 0.0241(32) - 1.688$$

$$y = 6.0416 + 0.7712 - 1.688$$

$$y = 5.1248$$

$$(5.1248 - 6) / 6$$

$$\approx -14.59\%$$

1. (32, 6)

2. (80, 38)

3. (112, 75)

$$1. \quad 6 = 1024a + 32b + c$$

$$2. \quad 38 = (80)^2a + 80b + c$$

$$38 = 6400a + 80b + c$$

$$3. \quad 75 = 12544a + 112b + c$$

$$\begin{array}{r} 6 = 1024a + 32b + c \\ - 38 = 6400a + 80b + c \\ \hline 32 = 5376a + 48b \end{array}$$

$$\begin{array}{r} 38 = 6400a + 80b + c \\ - 75 = 12544a + 112b + c \\ \hline 37 = 6144a + 32b \end{array}$$

$$\begin{array}{l} (32 = 5376a + 48b) \times 2 \\ 64 = 10752a + 96b \end{array}$$

$$\begin{array}{l} (37 = 6144a + 32b) \times 3 \\ 111 = 18432a + 96b \end{array}$$

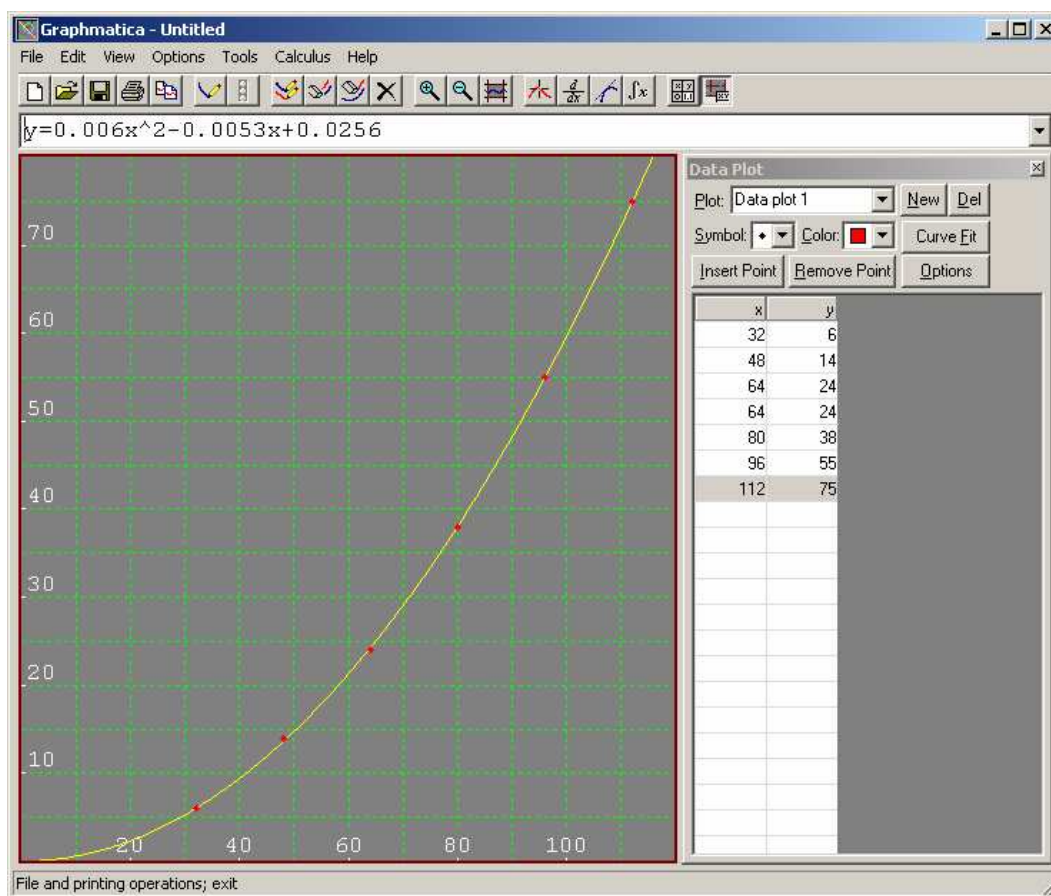
$$\begin{array}{r} 64 = 10752a + 96b \\ - 111 = 18432a + 96b \\ \hline 47 = 7680a \end{array}$$

$$a \approx 0.006$$

$$\begin{array}{l} 32 = 5376(0.006) + 48b \\ 32 = 32.256 + 48b \\ -0.256 = 48b \\ b \approx -0.0053 \end{array}$$

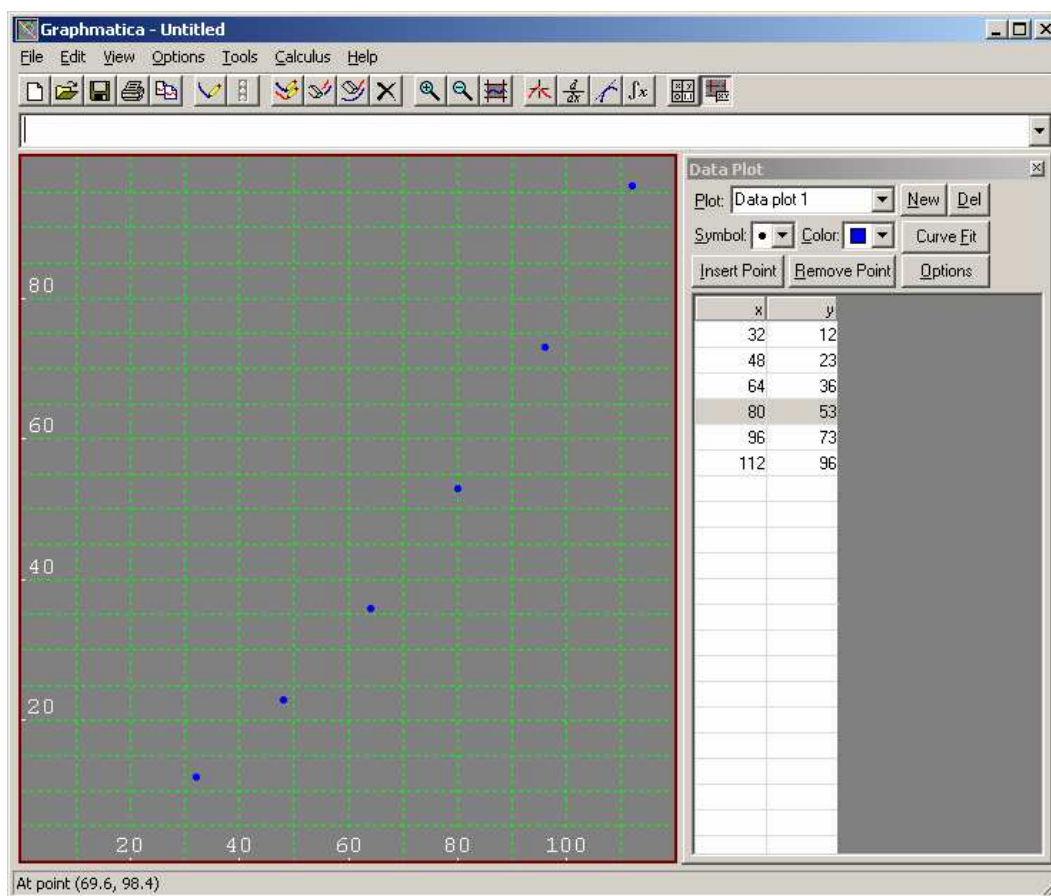
$$\begin{array}{l} 6 = 1024(0.006) + 32(-0.0053) + c \\ 6 = 6.144 - 0.1696 + c \\ 6 = 5.9744 + c \\ c \approx 0.0256 \end{array}$$

$$y = 0.006x^2 - 0.0053x + 0.0256$$



This parabolic function is combining the previous two functions, $y = 0.0059x^2 + 0.0241x - 1.688$ and $y = 0.0039x^2 + 0.188x - 4.0096$. It seems to fit quite well for almost all functions however still passes through the center of some points whereas others it barely touches the dot. A limitation for this graph, as for the previous two, is that it only includes positive numbers, meaning only half the parabola. This is because in the real life situation it would be impossible for the distance to be of negative amount. The calculated function however represents both positive and negative, creating a small restraint while representing speed vs. braking distance.

Speed (kmh ⁻¹) (x-axis)	Stopping distance (m) (y-axis)
32	12
48	23
64	36
80	53
96	73
112	96



As you can see, as the speed of the car increases, so does the braking distance. These two variables however don't increase at a constant ratio. This is apparent by studying the points on the graph and determining that they do not form a straight line however turn slightly upwards as they continue.

1. (32, 12)
2. (80, 53)
3. (112, 96)

1. $12 = (32)^2a + (32)b + c$
 $12 = 1024a + 32b + c$

2. $53 = (80)^2a + (80)b + c$
 $53 = 6400a + 80b + c$

3. $96 = (112)^2a + (112)b + c$
 $96 = 12544a + 112b + c$

$$\begin{array}{r} 12 = 1024a + 32b + c \\ - 53 = 6400a + 80b + c \end{array}$$

$$41 = 5376a + 48b$$

$$\begin{array}{r} 53 = 6400a + 80b + c \\ - 96 = 12544a + 112b + c \end{array}$$

$$43 = 6144a + 32b$$

$$\begin{array}{r} (41 = 5376a + 48b) \times 2 \\ 82 = 10752a + 96b \end{array}$$

$$\begin{array}{r} (43 = 6144a + 32b) \times 3 \\ 129 = 18432a + 96b \end{array}$$

$$\begin{array}{r} 82 = 10752a + 96b \\ - 129 = 18432a + 96b \end{array}$$

$$47 = 7680a$$

$$a \approx 0.0061$$

$$41 = 5376(0.0061) + 48b$$

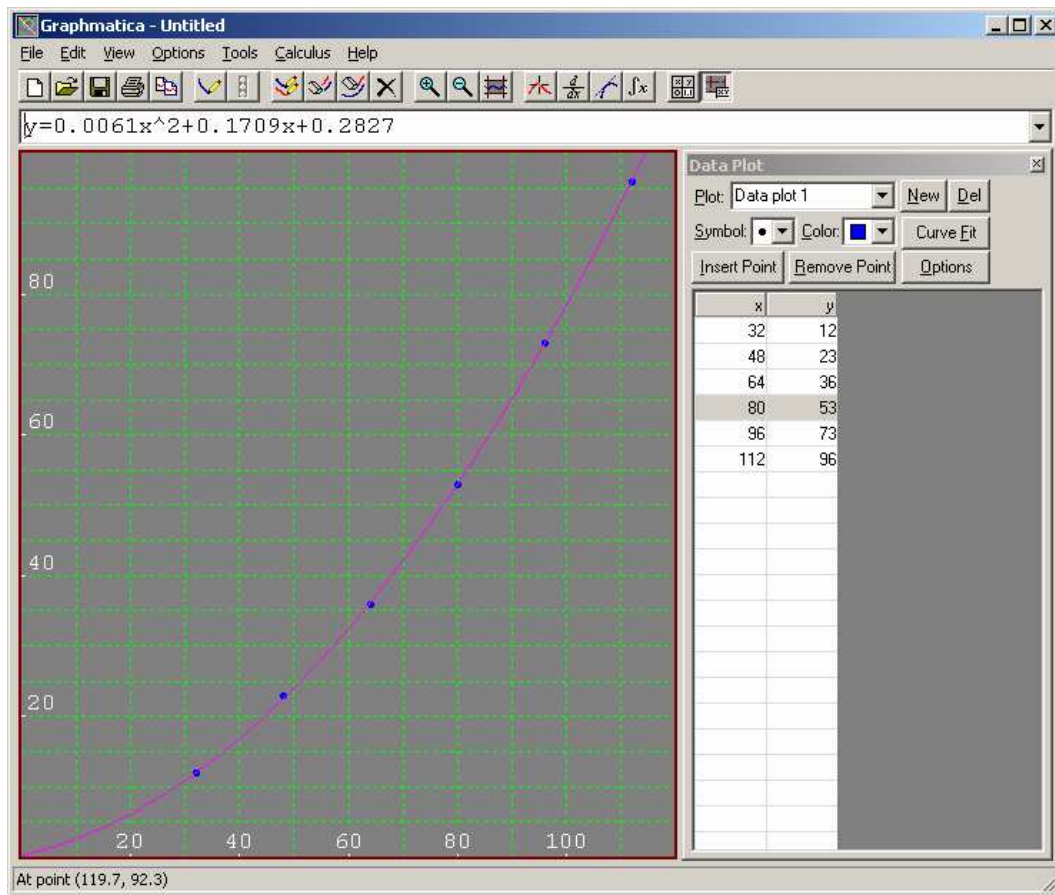
$$b \approx 0.1709$$

$$12 = 1024(0.0061) + 32(0.170967) + c$$

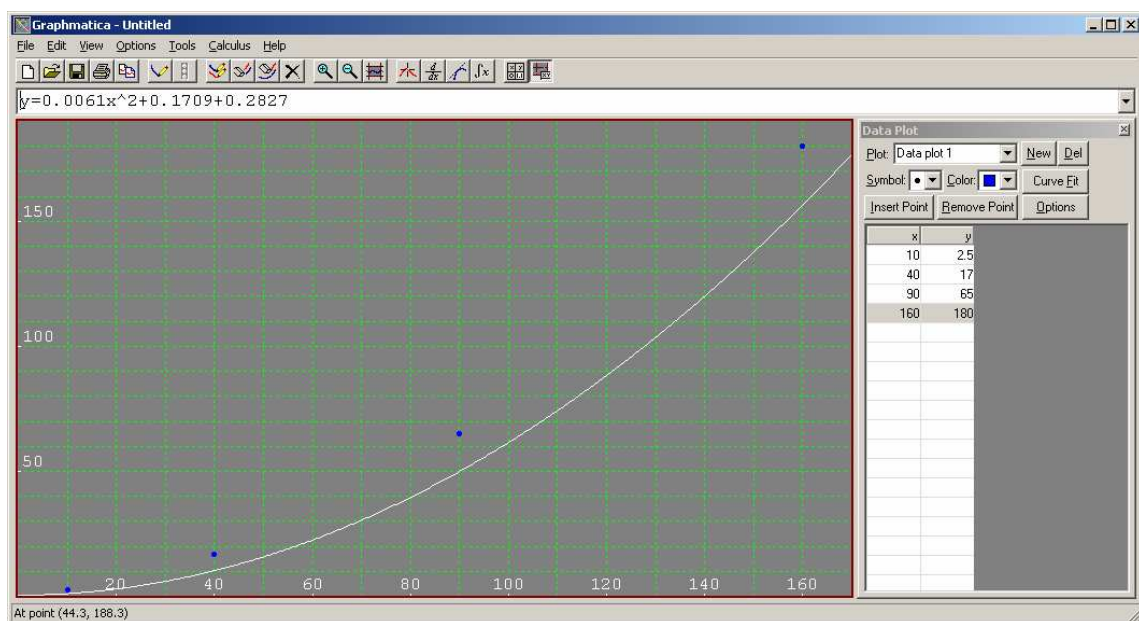
$$12 = 6.2464 + 5.470944 + c$$

$$c \approx 0.2826$$

$$y = 0.0061x^2 + 0.1709x + 0.2827$$



This is also a parabolic function. It works for all of the points and is thus a very good function to represent the speed vs. stopping distance. This function, $y = 0.0061x^2 + 0.1709x + 0.2827$, is very similar to the previous function, $y = 0.006x^2 - 0.0053x + 0.0256$, representing speed vs. braking distance. Both are parabolic functions and intersect all points of the graph. IN addition, both have the same limitation which is the fact that the function represents an entire parabola, including both positive and negative points on the graph. The situation however only includes the positive numbers due to the real life situation in which it is physically impossible to have negative meters. This function is different to the function representing speed vs. thinking distance, $y = (3/16)x$, because it is a linear function representing one straight line. Also this function barely has any limitations when compared to the real life situation and the data recorded.



Percentage of error

(10, 2.5)

$$y = 0.0061(10)^2 + 0.1709(10) + 0.2827$$

$$y = 2.6017$$

$$(2.6017 - 2.5) / 2.5$$

$$\approx 4.068 \%$$

(40, 17)

$$y = 0.0061(40)^2 + 0.1709(40) + 0.2827$$

$$y = 16.8787$$

$$(16.8787 - 17) / 17$$

$$\approx -2.376 \%$$

(90, 65)

$$y = 0.0061(90)^2 + 0.1709(90) + 0.2827$$

$$y = 65.0739$$

$$(65.0739 - 65) / 65$$

$$\approx 0.1134 \%$$

(160, 180)

$$y = 0.0061(160)^2 + 0.1709(160) + 0.2827$$

$$y = 183.7867$$

$$(183.7867 - 180) / 180$$

$$\approx 2.104 \%$$

My model, $y = 0.0061x^2 + 0.1709x + 0.2827$, does not fit the data of overall stopping distances for other speeds. It barely intersects the first point and is extremely off the next couple. Although the type of function (parabolic) fits the graph, several points