Type II

IB Mathematics SL Portfolio

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Stopping Distances

When a driver stops her car, she must first think to apply the brakes. Then the brakes must actually stop the vehicle.

The table below lists the average times for these processes at various speeds.

Table 1. average times for these processes at various speeds

Speed (kmh^{-1})	Thinking distance (m)	Braking distance (m)	Stopping distance (m)
32	6	6	12
48	9	14	23
64	12	24	36
80	15	38	53
96	18	55	73
112	21	75	96

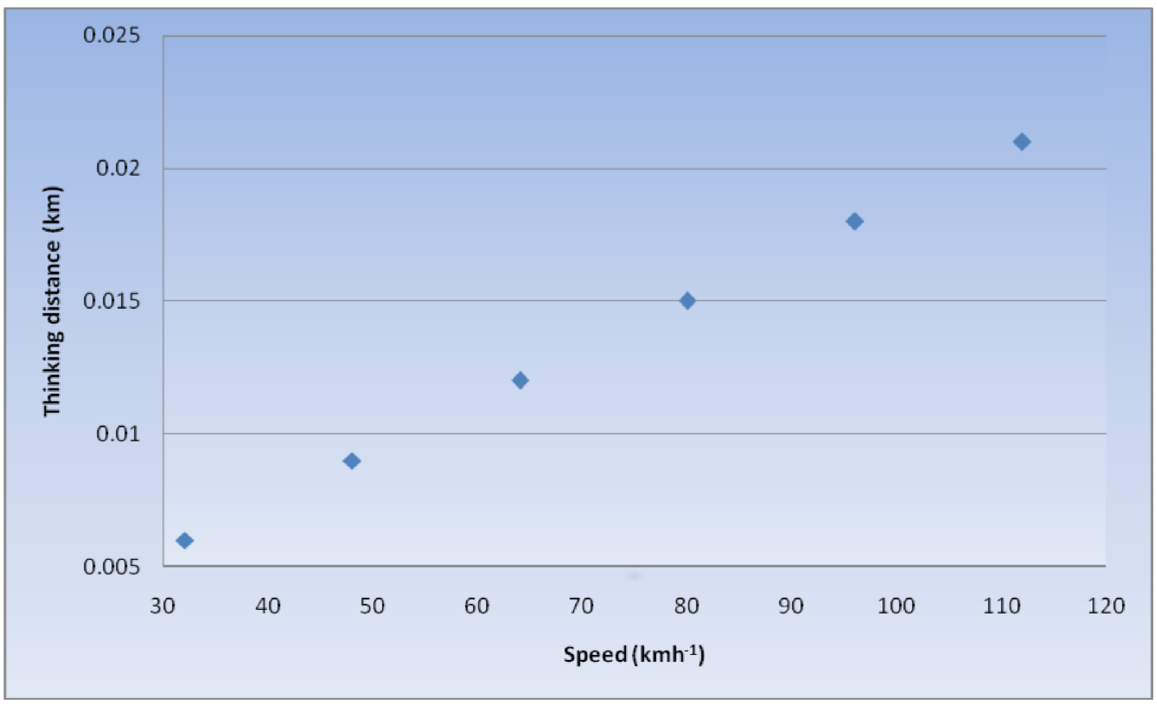
Using this table we can graph two data plots: (i) Speed versus Thinking distance, (ii) Speed versus Braking distance, and (iii) Speed versus Stopping distance

(i) Speed versus Thinking distance

Table 2. Speed versus Thinking distance

Speed (kmh^{-1})	Thinking distance (km)
32	0.006
48	0.009
64	0.012
80	0.015
96	0.018
112	0.021

Graph 1. Speed versus Thinking distance



This is clearly a linear graph since we can see a straight line. This shows us that the correlation between speed and thinking distance is directly proportional, meaning that as speed increases the thinking distance will also increase. In other words, as the speed of a car increases it takes a longer time for the driver to think about applying the breaks.

Since this graph is linear we can develop a model to fit the data using the equation $y = mx + b$ where m stands for gradient and b stands for the y-intercept.

Steps taken:

1. First we find the gradient m by taking any two points of coordinates and calculating m using the following equation $m = \frac{y_2 - y_1}{x_2 - x_1}$

   The two points taken: P_1 (48,0.009)
   P_2 (112,0.021)

   ∴ $x_1 = 0.009$
   $x_2 = 0.021$
   $y_1 = 48$
   $y_2 = 112$

   $$m = \frac{0.021 - 0.009}{112 - 48}$$

   $$m = \frac{0.012}{64}$$

   $$m = 1.875 \times 10^{-4}$$

2. The value of b is the y-intercept but since the car is at rest the speed is 0 and therefore the breaking distance is also 0.

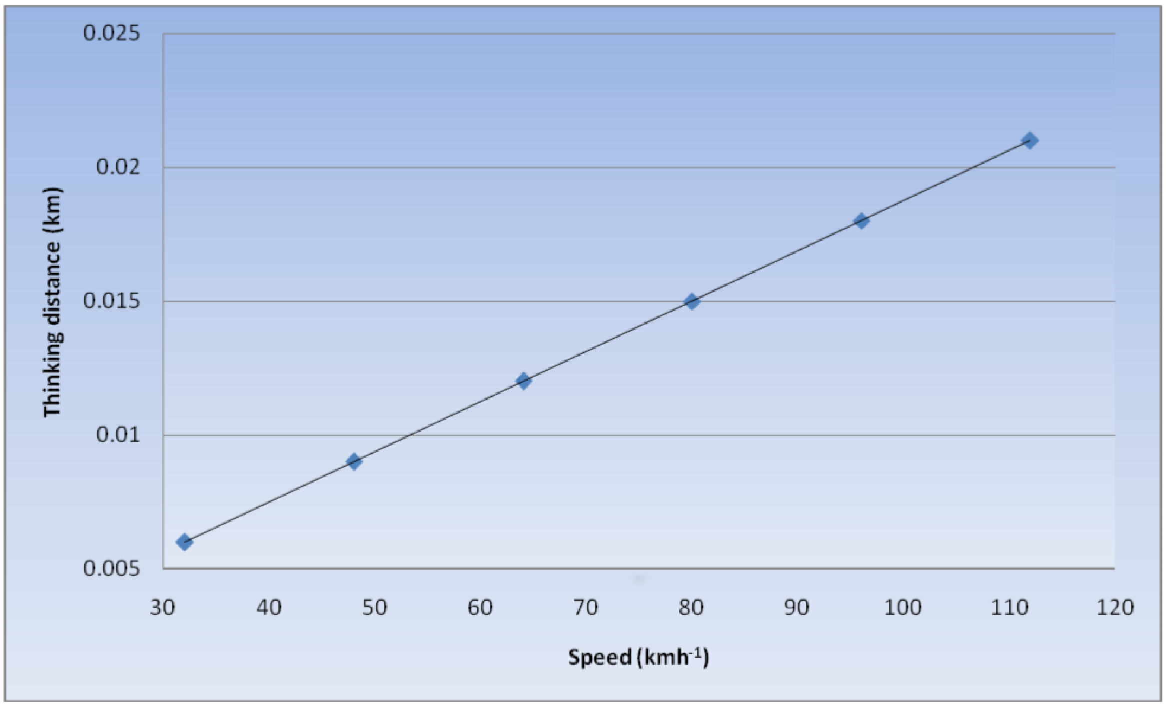
   $\therefore\ b = 0$

3. Equation obtained:

   $y = mx + b$
   $y = 1.875 \times 10^{-4} x$

```
LinReg
 y=ax+b
 a=1.875E-4
 b=0
 r²=1
 r=1
```

Graph 2. Model of Speed versus Thinking distance



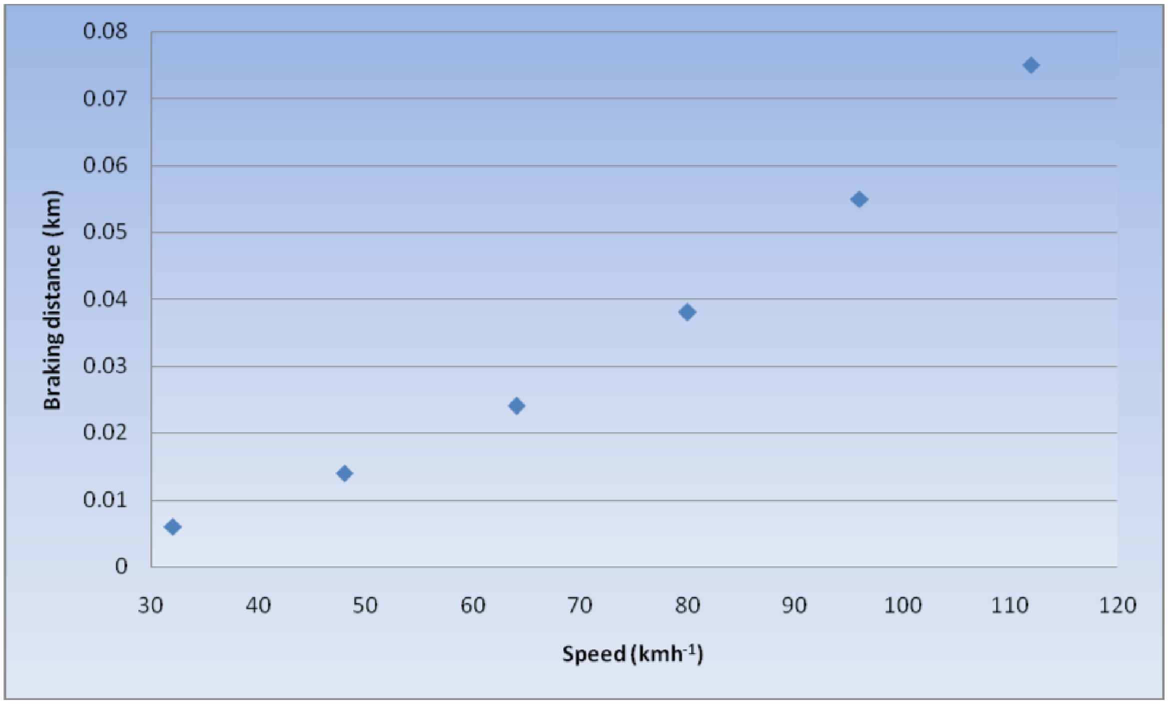
This model is a very good and accurate fit for the graph because the correlation coefficient, r, is 1 which means that it is a very strong correlation. The coefficient of determination, r^2, is 1 which means that 100% of the total variation in y can be explained by the relationship between x and y. However, since we cannot have negative values of speed or thinking distance we can eliminate all negative possibilities with this function. $y = |1.875 \times 10^{-4}x|$

(ii) Speed versus Braking distance

Table 3. Speed versus Braking distance

Speed (kmh^{-1})	Braking distance (km)
32	0.006
48	0.014
64	0.024
80	0.038
96	0.055
112	0.075

Graph 3. Speed versus Braking distance



This graph is, from observation, a quadratic or a semi-parabola in which the y-value (braking distance) increases exponentially.

Since it is a quadratic we can develop a model to fit a data using the equation $y = a(x - \alpha)^2$ also known as $y = ax^2 + bx + c$, however, there is another model that would fit the data is by using a power function. First I will develop a quadratic model.

Steps taken to develop quadratic model:

1. The value of α is the y-intercept and since the car is at rest the speed is 0 and so is the braking distance.

$\therefore \quad \alpha = 0$

$\therefore \quad y = ax^2$

2. To solve for a, we can use a pair of coordinates and plug them into the equation $y = ax^2$

   P_1 (64,0.024)

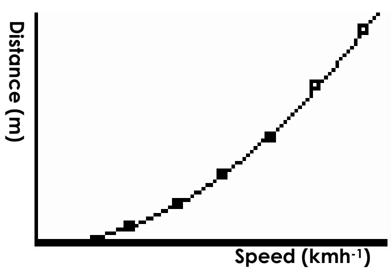
   $$\therefore 0.024 = a(64)^2$$
   $$0.024 = a(4096)$$
   $$a = \frac{0.024}{4096}$$
   $$a = 5.859375 \times 10^{-6}$$

3. Equation obtained:

   $$\boldsymbol{y = 5.859375 \times 10^{-6} x^2}$$

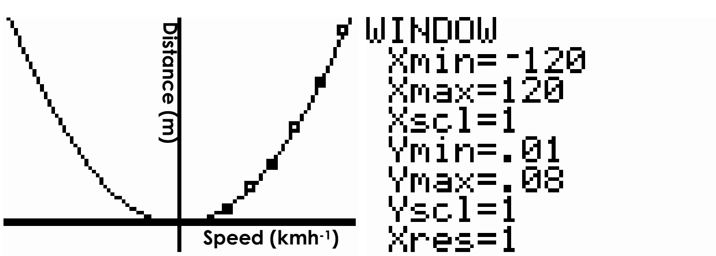
When graphed on a GDC this is what it looks like below using this and the function
$y = 5.859375 \times 10^{-6} x^2$

Graph 4. Quadratic model for Speed versus Braking distance



However, because it is a quadratic we have to evaluate whether the negatives will be a good fit to represent the data. Below is a graph showing the same graph as above but with an extended window frame.

Graph 5. Quadratic model for Speed versus Braking distance with enlarged window frame



Here we can see that the plots match well on the right side. However since we cannot have negative speed the model is not a good fit despite that it is a good fit to represent the data on the right.

Having that said, the other option is the power function and it was chosen because it is polynomial and we can eliminate all negative values since the domain is within positive values.

Steps taken to develop power model using GDC:

1. Insert data into GDC table

L1	L2
32	.006
48	.014
64	.024
80	.038
96	.055
112	.075

L2(7) =

L1 – Speed
L2 – Braking distance

2. Use implemented Power Regression for variables L1 and L2

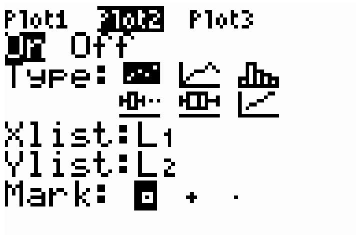
```
PwrReg L1,L2

PwrReg
 y=a*x^b
 a=5.7605396E-6
 b=2.007560819
 r²=.9997972583
 r=.999898624
```

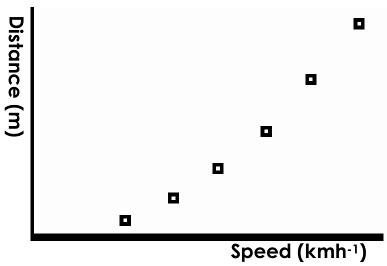
3. Insert the information into STAT PLOT

```
Plot1 Plot2 Plot3
\Y1=5.7605396088
924E-6X^2.007560
8192828
```

4. Plot data from table

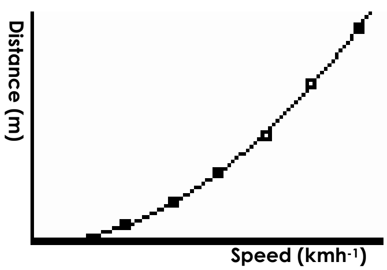


Graph 6. Speed versus Braking distance



5. Implement power function into the graph

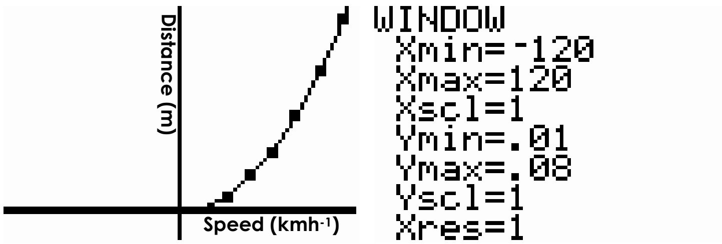
Graph 7. Power model for Speed versus Braking distance



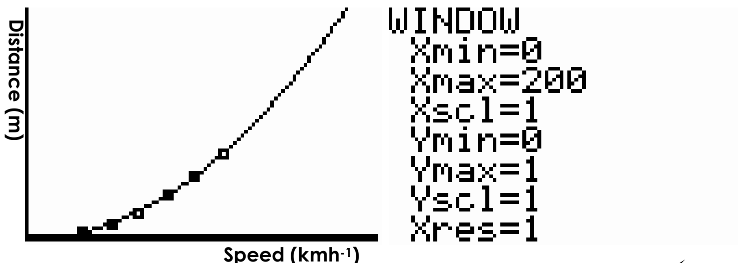
In the end we get a function: $y = 5.7605396088924 \times 10^{-6} x^{2.0075608192828}$

Visually, the model is a good fit to represent the data and by looking at the coefficient of determination, r^2, we can tell that the model is a good fit to represent the data. This is because 0.9997972583 is very close to 1. However, we cannot base the question of whether this model is a good fit by looking at the coefficient of determination. We have to look at whether the model fits real life situations. But first we have to expand the window frame of the graph.

Graph 8. Power model for Speed versus Braking distance with enlarged window frame (1)



Graph 9. Power model for Speed versus braking distance with enlarged window frame (2)



Unlike the quadratic model we can see from [illegible] that all negatives have been eliminated which means that it eliminates all possibilities of having a negative speed or braking distance, which is good for real life situations because we cannot have negative speeds or braking distance. As we can see from [illegible] the model is infinite and therefore we can predict the breaking speed for all speeds.

This power model is much better than the quadratic model simply because it is better suited for real life situations. However, that does not mean the quadratic model cannot be used. We can add absolute into the function to eliminate the negative values.

So in the end we end up with two functions that can represent the data.

$$y = 5.7605396088924 \times 10^{-6} x^{2.0075608192828}$$

$$y = |5.859375 \times 10^{-6} x^2|$$

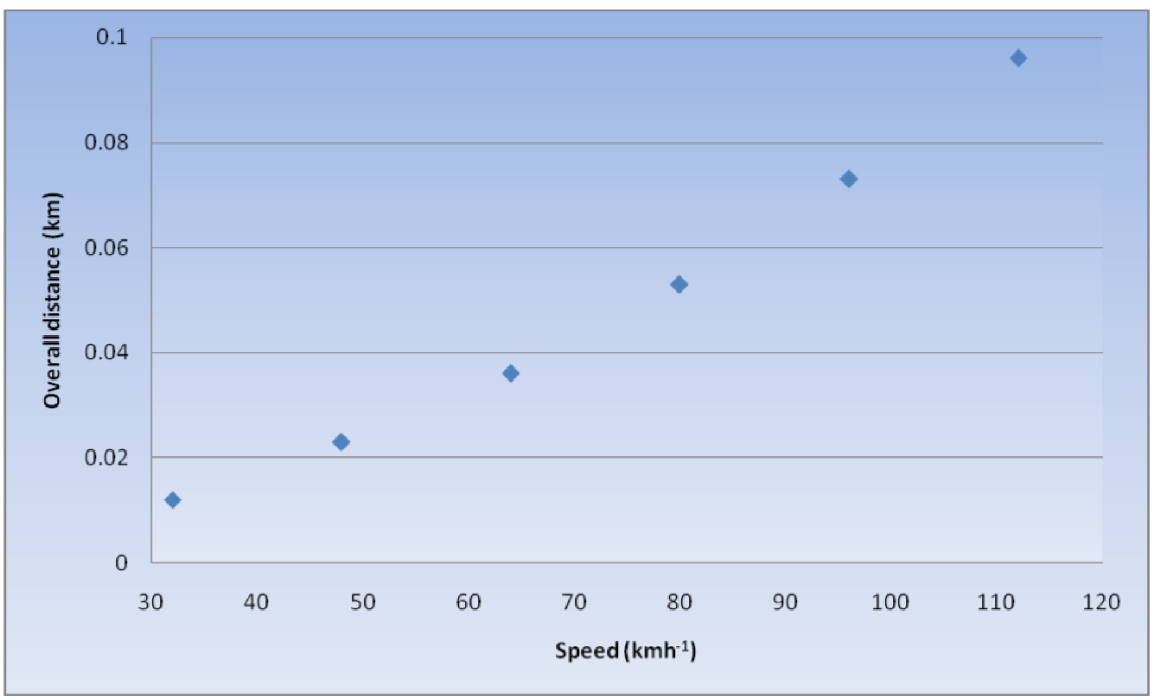
(iii) Speed versus Stopping distance

The stopping distance is obtained from adding the thinking distance to the breaking distance.

Table 4. Speed versus Thinking distance, Braking distance, and Stopping stopping distance

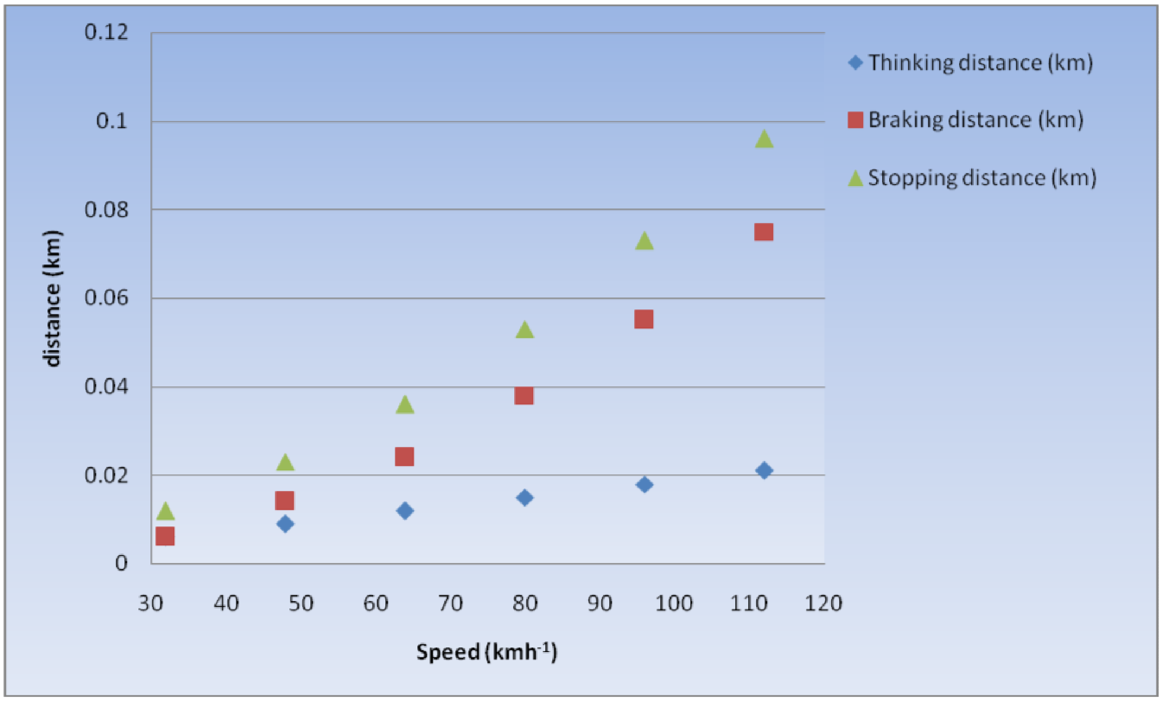
Speed (kmh^{-1})	Thinking distance (km)	Braking distance (km)	Stopping distance (km)
32	0.006	0.006	0.012
48	0.009	0.014	0.023
64	0.012	0.024	0.036
80	0.015	0.038	0.053
96	0.018	0.055	0.073
112	0.021	0.075	0.096

Graph 10. Speed versus Stopping distance



The graph is, from observation, a quadratic of semi-parabola where the y-value (Stopping distance) increases exponentially. Compared with [redacted], this graph is a curve and not a straight line and the two graphs share no characteristics other than an increase in the y-value. This graph is, however, similar to [redacted]3 because the y-values for both increase exponentially against the x-value and therefore they both have the same characteristics of an exponential increase in the y-value.

Graph 11. Speed versus Thinking distance, Braking distance, and Stopping distance



Since it is a semi-parabola, we can apply the quadratic and power function since it was the two best fit for the previous model – Speed versus Braking distance.

Steps taken to develop quadratic model:

$y = a(x - \alpha)^2$

1. The value of α is the y-intercept and since the car is at rest the speed is 0 and so is the braking distance.

$\therefore \quad \alpha = 0$

$\therefore \quad y = ax^2$

2. To solve for a, we can use a pair of coordinates and plug them into the equation $y = ax^2$

   P_1 (80,0.053)

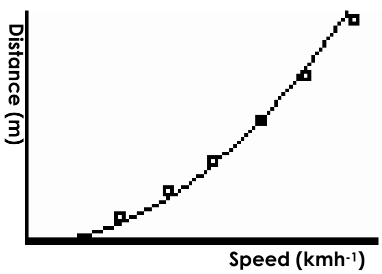
   $$\therefore 0.053 = a(80)^2$$
   $$0.053 = a(6400)$$
   $$a = \frac{0.053}{6400}$$
   $$a = 8.28125 \times 10^{-6}$$

3. The final equation will then be:

   $$\boldsymbol{y = 8.28125 \times 10^{-6} x^2}$$

4. Graph model on GDC

Graph 12. Quadratic model for Speed versus Stopping distance



Steps taken to develop power model:

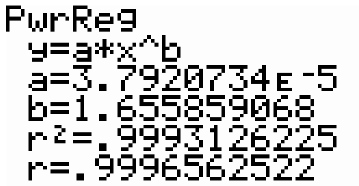
1. Insert data into GDC table

L1	L2	L3 2
32	.012	------
48	.023	
64	.036	
80	.053	
96	.073	
112	.096	
------	------	

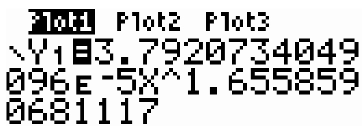
L2(2) =.023

2. Use implemented Power Regression for variables L1 and L2

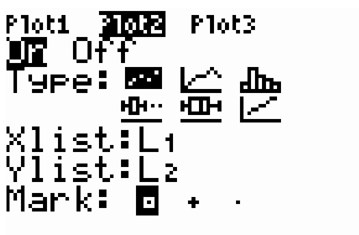
PwrReg L1,L2



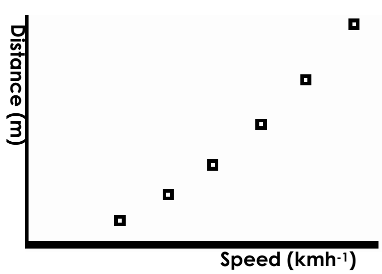
3. Insert information into STAT PLOT



4. Plot data from table

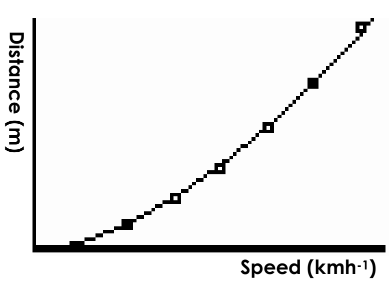


Graph 13. Speed versus Stopping distance



5. Implement power function into graph

Graph 14. power model for Speed versus Stopping distance



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In the end we get the function $\mathbf{y = 3.7920734 \times 10^{-5} x^{1.655859068}}$

Looking at g[illegible] and g[illegible]3 we can see that both functions is quite accurate as they follows the given data plot. Also the power model has a coefficient of determination, r^2, of 0.9993126225 which is very close to 1. However, the model is not as successful as well as the models for Speed versus Thinking distance and Speed versus Braking distance. Since the Stopping stopping distance is obtained by the sum of thinking distance and braking distance, a function can be obtained by adding function$_1$ with function$_2$.

$$\therefore \quad y = function_1 + function_2$$

$$y = (|\ 1.875 \times 10^{-4} x|) + (\ |5.859375 \times 10^{-6} x^2|)$$

Or

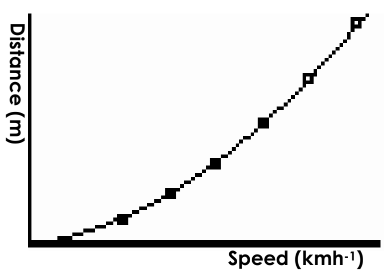
$$y = (|1.875 \times 10^{-4} x|) + (5.7605396088924 \times 10^{-6} x^{2.0075608192828})$$

Using GDC we can compare the quadratic and power model with the above functions.

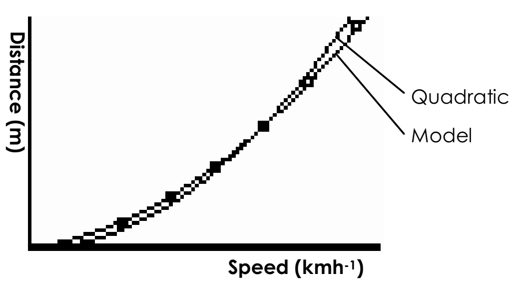
(a) Model A

$$y = (|\ 1.875 \times 10^{-4} x|) + (\ |5.859375 \times 10^{-6} x^2|)$$

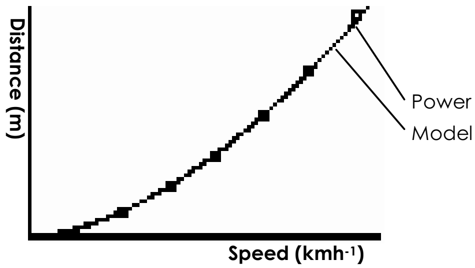
Graph 15. Speed versus Stopping distance using Model A



Graph 16. Model A versus Quadratic Model for Speed versus Stopping distance



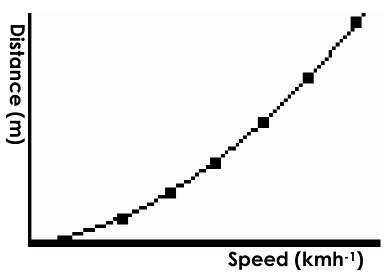
Graph 17. Model A versus Power Model for Speed versus Stopping distance



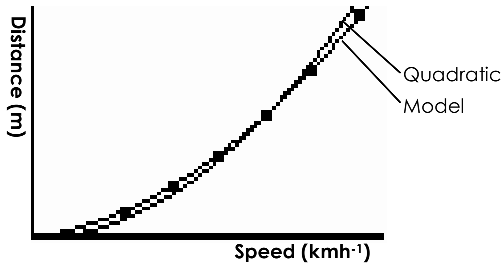
(b) Model B

$$y = (|1.875 \times 10^{-4}x|) + (5.7605396088924 \times 10^{-6}x^{2.0075608192828})$$

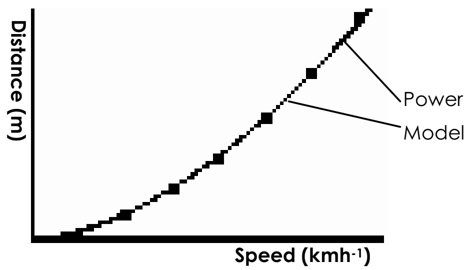
Graph 18. Speed versus Stopping distance using Model B



Graph 19. Model B versus Quadratic Model for Speed versus Stopping distance



Graph 20. Model B versus Power Model for Speed versus Stopping distance



From graph 15 and graph 18 we can see that Model A and Model B are better models compared with the quadratic and power model as we can see in graph 16-17 and graph 19-20. This is because it adds function$_1$ and function$_2$, instead of calculating it algebraically by using equations which depend on the coordinates taken. However, I would say Model B is the best model to represent the data since in graph 18 we can see the function go through every data plot since the data plot is filled by black, whereas in graph 15 the function misses two data plots.

Now that we have a good model to represent the data, it is time to test whether this model would fit further data.

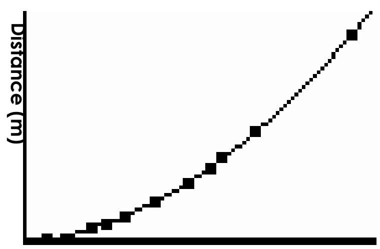
Table 5. New data plots

Speed (kmh^{-1})	Stopping distance (km)
10	0.0025
40	0.017
90	0.065
160	0.18

Table 6. Speed versus Stopping distance with new data

Speed (kmh⁻¹)	Stopping distance (km)
10	0.0025
32	0.012
40	0.017
48	0.023
64	0.036
80	0.053
90	0.065
96	0.073
112	0.096
160	0.18

Graph 21. Speed versus Stopping time using Model B



As we can see the model is a very good fit to represent the data since the function passes through all data plots. However, a more accurate function could be developed by using more data plots.

We also have to consider anomaly results since some things are not accounted for. For example, the friction between the tires and the road, and weather conditions affecting driver's reaction time which may not fit my model accurately.

With more data plots we can make modifications to the function since all the current data plots fit the function.