Stopping Distances

Before a vehicle stops, the driver has to think before applying the brakes and the brakes take time to actually stop the vehicle. These two processes vary at different speeds as shown the table below:

Speed (kmh^{-1})	Thinking distance (km)	Braking distance (m)
32	0.006	0.006
48	0.009	0.014
64	0.012	0.024
80	0.015	0.038
96	0.018	0.055
112	0.021	0.075

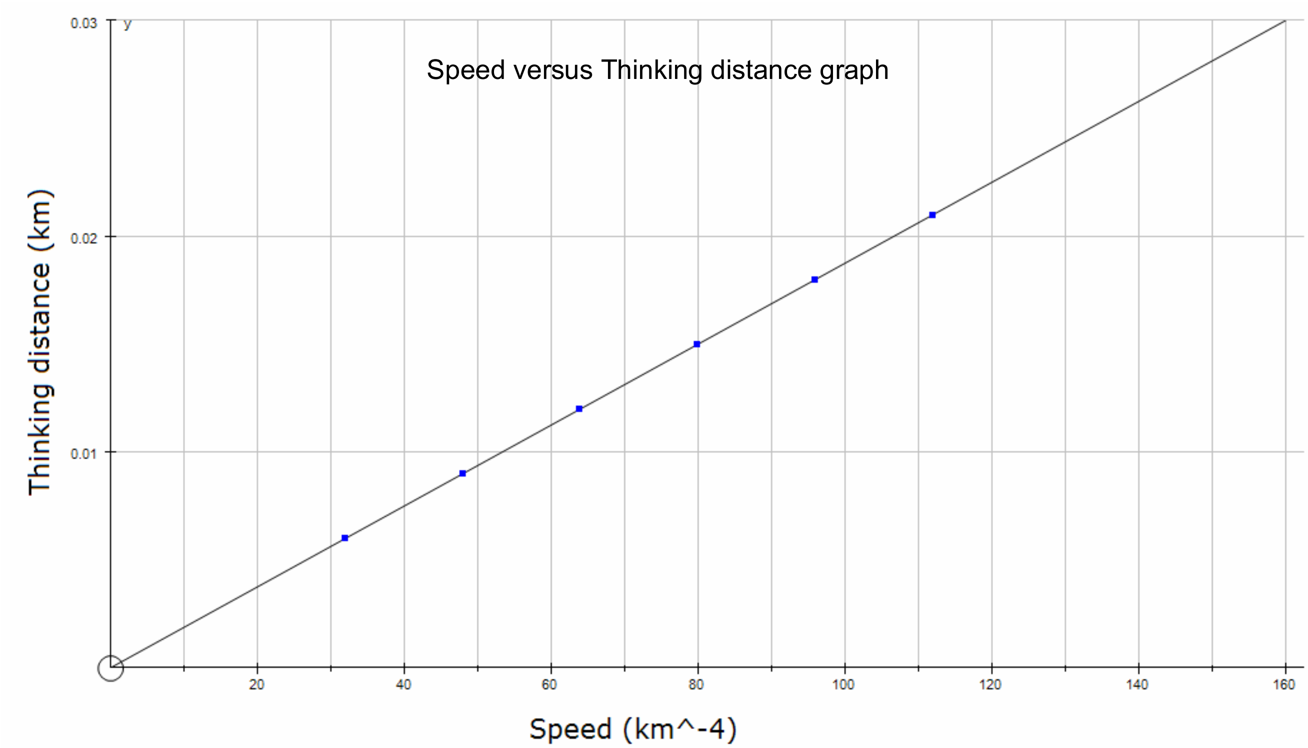
So by adding the thinking distance and braking distance together we are able to find the total average stopping distance:

Stopping distance (km)
0.012
0.023
0.036
0.053
0.073
0.096

Using the records above, our task is to:

- Develop individual functions that model the relationship between speed and thinking distance as well as speed and braking distance
- Develop a model for the relationship between speed and over all stopping distance

1. Using a graphing software I have created two data plots:



The values between the thinking distances have a fixed interval and a set value for the speed making it a straight line with a positive constant gradient of 16/3. The gradual increase of the line suggests that as the speed increases the thinking distance increases proportionally.

Since the graph is a linear, the equation *y=mx+c* can be used to model its behavior where *m* stands for the gradient and *c* is the y-intercept.

Using this linear equation, we have to find *m:*

$$m=\frac{x_2-x_1}{y_2-y_1}$$

by substituting theses values to find the gradient from these two co-ordinate points (32,0.006) and (48,0.009):

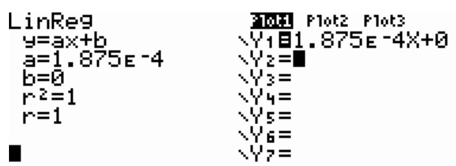
$$m=\frac{0.009-0.006}{48-32}=1.875 \times 10^{-4}$$

the y-intercept of the linear is 0 because as the speed is 0 there is no need to think to brake the vehicle so:

$$c=0$$

the final function we get from using the equation is:

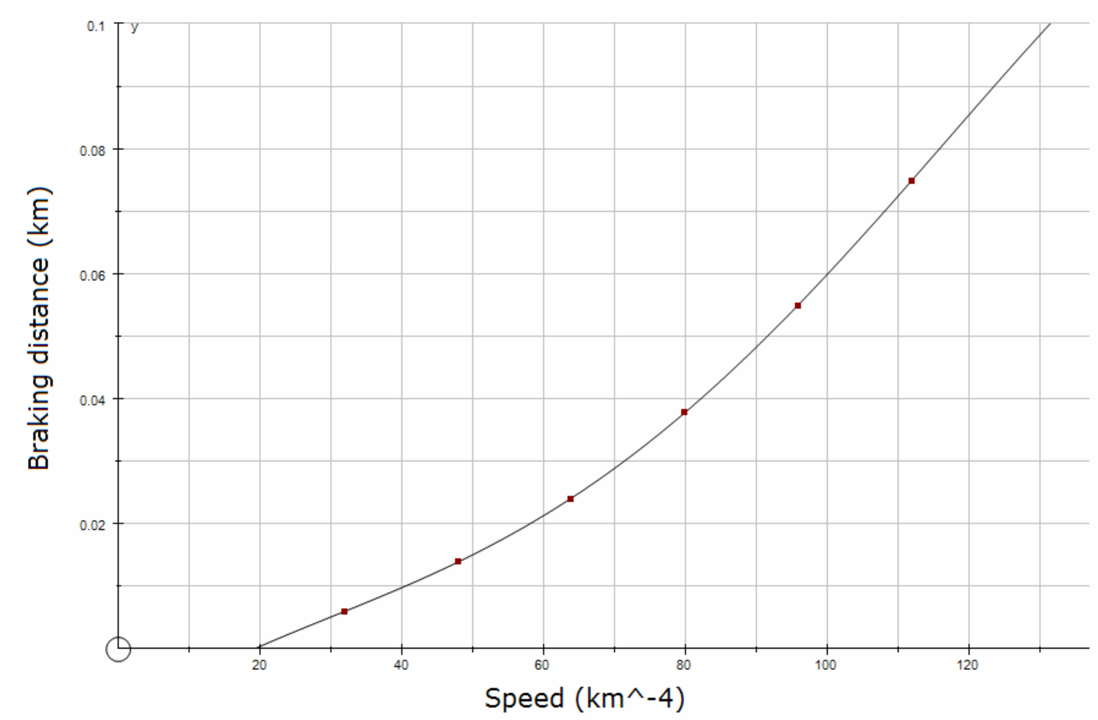
y=1.875*10 ^-4*x*+0



Since there cannot be any negative thinking distances or speed, the equation for the linear of speed versus thinking distance has to be:

y=|1.875*10 ^-4*x*+0|

Speed versus Braking distance



From the values of the braking distance, there isn't a common difference and the braking distance rising exponentially creates a curve suggesting a parabola.

Therefore the quadratic equation, $y=a(x+\alpha)^2$ also known as $y=ax^2+bx+c$ can be applied to model its behavior.

As the speed is 0 the braking distance would be 0 as well therefore the y intercept will be a repeated root, this will mean that $\alpha=0$ given the equation $y=a(x+\alpha)^2$

$$
\begin{aligned}
y&=a(x+\alpha)^2\\
&=a(x+0)^2\\
&=a(x)^2\\
y&=ax^2
\end{aligned}
$$

To solve $y=ax^2$, the co-ordinates (48,0.014) from the graph can be used to be substituted into the graph:

$$
\begin{aligned}
y&=ax^2\\
0.006&=a(32)^2\\
0.006&=a(1024)\\
a&=\frac{0.006}{1024}\\
a&=5.859375*10^{-6}
\end{aligned}
$$

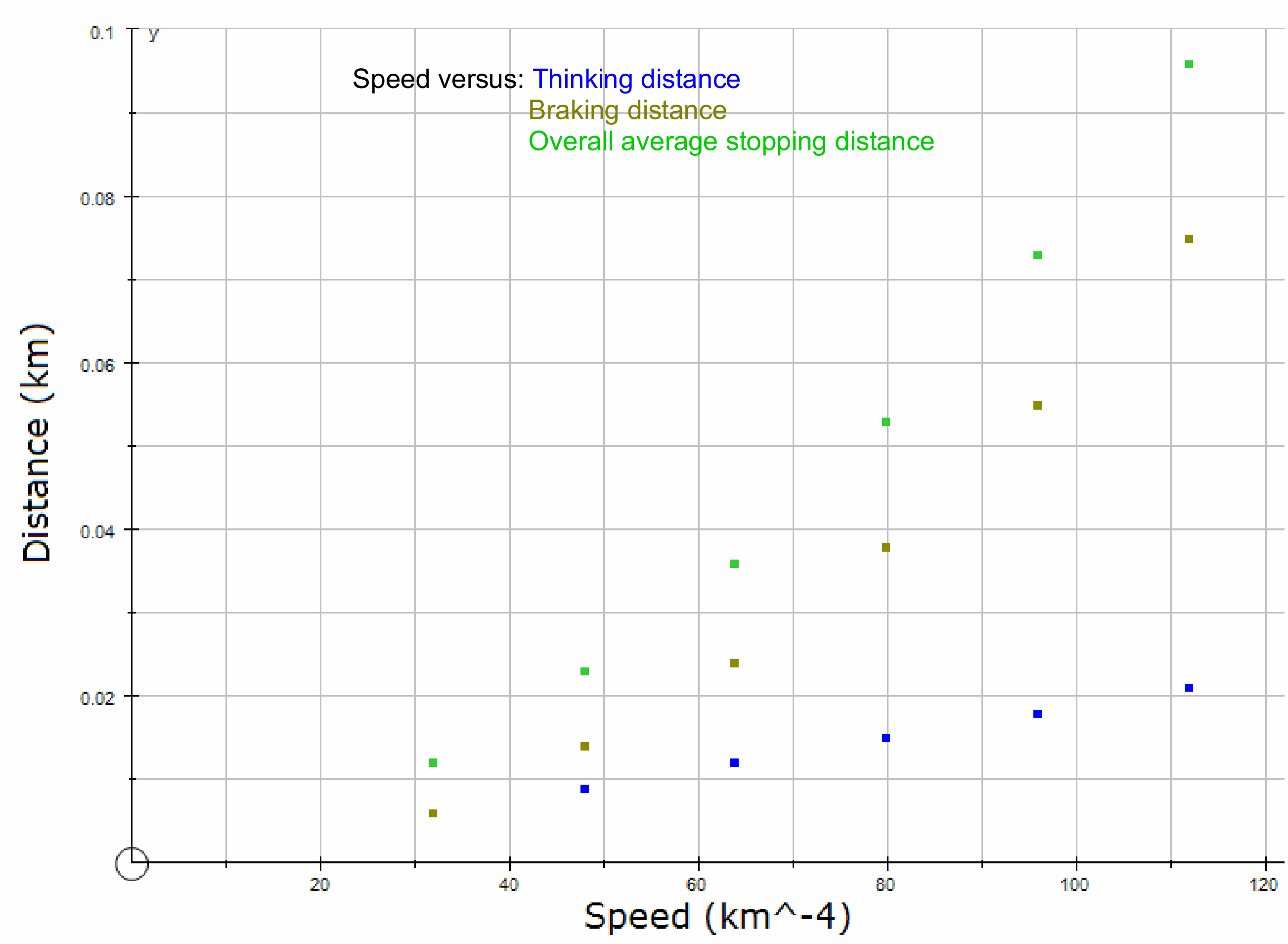
Quadratic equation of Speed versus Braking distance:

$$y=5.859375*10^{-6}x^2$$

Like the linear Speed versus Thinking graph above, there cannot be negative speeds, thinking distances or braking distances therefore the equation has to be:

$$\mathbf{y=|5.859375*10^{-6}x^2|}$$

By adding the thinking and braking distance added together the overall average stopping distance can be obtained. Below is the graph of each of the distances versus the speed:



The overall average stopping distance line, shown above, displays a similar parabolic curve like the braking distance with an exponentially growing y-value. This is in contrast to the linear of the thinking distance which is increasing in distance as well but at a slower rate.

Given that the overall average stopping distance share the same characteristic as the braking distance, the quadratic equation used above can be used here too:

As the speed is 0 the braking distance would be 0 as well therefore the y intercept will be a repeated root, this will mean that a=0 given the equation $y=a(x+a)^2$

$$\begin{aligned} y&=a(x+a)^2 \\ &=a(x+0)^2 \\ &=a(x)^2 \\ y&=ax^2 \end{aligned}$$

The points (80, 0.053) are chosen to substitute into the equation:

$$y=ax^2$$
$$0.053=a(80)^2$$
$$0.053=a(6400)$$
$$a=\frac{0.053}{6400}$$
$$a=8.28125*10^{-6}$$

Therefore the equation for Speed versus Overall Average Stopping distance is:

$$\mathbf{y=8.28125*10^{-6}x^2}$$

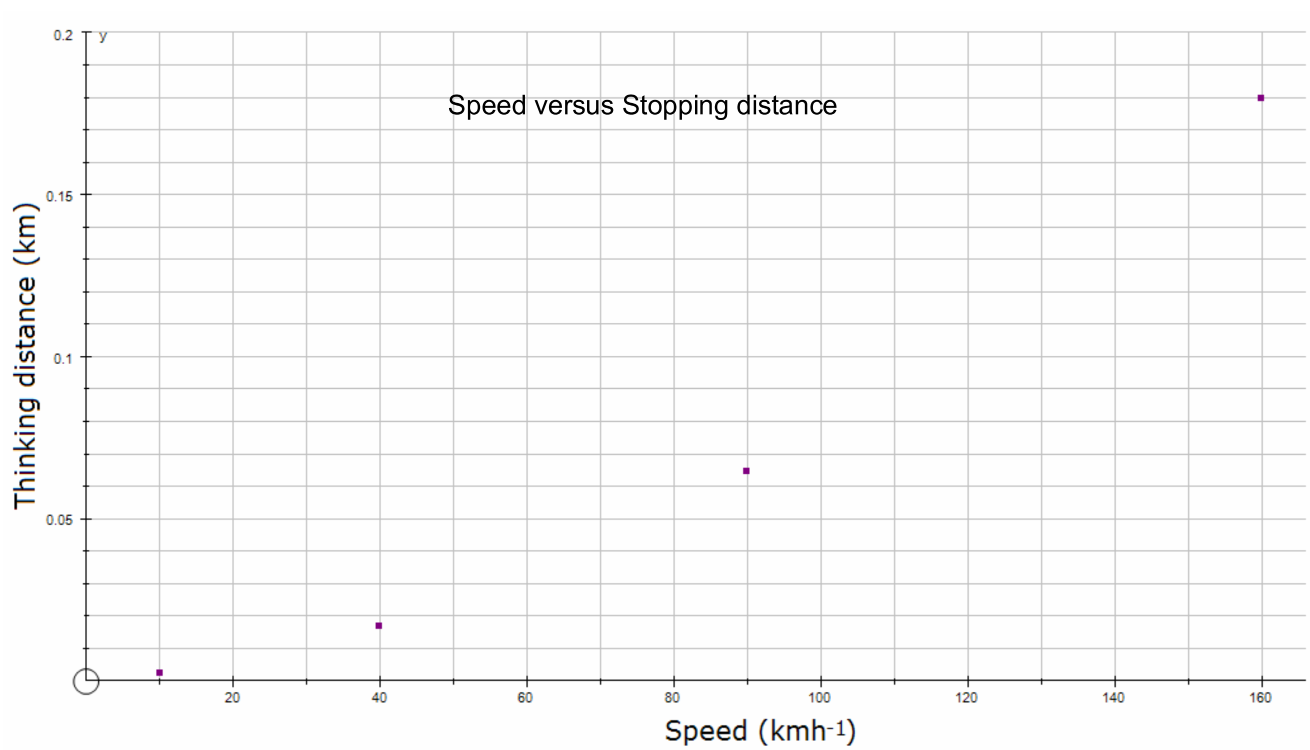
For the reason that the overall average stopping distance can be found by adding the thinking and braking distance together, the functions of the Speed versus Thinking distance and Speed versus Braking distance can be added together to give the equation of the Speed versus the Overall Average Stopping distance.

$$y=\text{function}_1+\text{function}_2$$

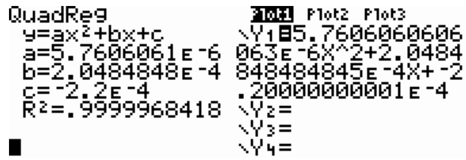
Speed vs. Overall Average Stopping distance= $(|1.875*10^{-4}x+0|) + (|5.859375*10^{-6}x^2|)$

Here is another set of data which can be used to see if the model fits

Speed (kmh^{-1})	Stopping distance (km)
10	0.0025
40	0.017
90	0.065
160	0.18



As we can see, the points are forming a curve which is alike the previous quadratic graphs however, the equation of the graph:



May still be a quadratic equation but it is not exactly the same as the other stopping distance equation because different factors such as the tire friction, the weather/conditions, the driver's age and the road's surface all affect the overall stopping distance. Anomalies have to be taken into account as well as accidents can happen which affect the outcome. With more data, the accuracy of the graph can be improved so that the function can also be modified and become more precise.