International Baccalaureate Diploma

The Sultan's School

"STELLAR NUMBERS" Portfolio

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Contents

Introduction

In this portfolio, I am going to show the expressions of general patterns and numbers for stellar and triangular numbers.

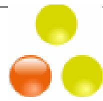
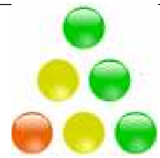
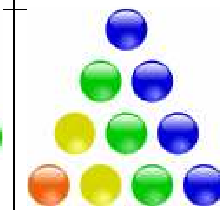
The aim of this portfolio is to “consider geometric shapes which lead to special numbers”.

Triangular shapes were imported from the internet and edited by paint program. 6-stellar diagrams were imported from Google images and edited by paint. 5-stellar shapes were created by paint program.

All the graphs are created by the online program http://functiongrapher.com/.

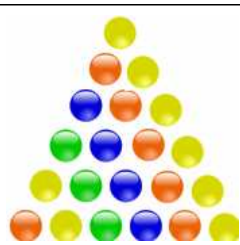
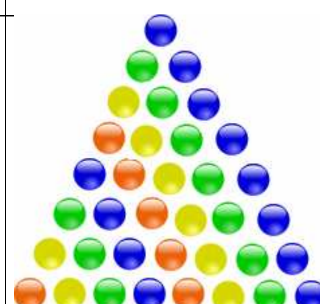
Triangular numbers

The first type of numbers we are considering are triangular numbers. This table shows the first 5 terms of triangular numbers.

n	1	2	3	4	5
Shape					
Number of Dots	1	3	6	10	15

We can calculate the n^{th} term by adding n to the previous number of dots. For example, the value for $n = 6$ equals to 15 +6 which is 21. The same applies for the other values of n.

The next three terms are represented by this diagram:

n	6	7	8
Shape			
Number of dots	21	28	36

When we want to find the expression for this sequence, we have to know which difference is constant.

Seq.: 1, 3, 6, 10, 15…

1st diff.: 2 3 4 5…

2nd diff.: 1 1 1…

We found from the above that the second difference is constant. Therefore the expression will be $\mathrm{an}^2 + \mathrm{bn} + \mathrm{c}$. We can know the coefficient of the n^2 by dividing the 2nd difference by 2 and the result is the coefficient of n^2.

Therefore the first part of the expression is $\frac{1}{2}n^2$

To know the rest, we must list the sequence $\frac{1}{2}\mathrm{n}^2$ and subtract it from the original sequence.

n	1	2	3	4	5	6	7
$\frac{1}{2}n^2$	0.5	2	4.5	8	12.5	18	24.5
Seq.-$\frac{1}{2}n^2$	0.5	1	1.5	2	2.5	3	3.5

The result from subtraction is a linear sequence ($\mathrm{bn} + \mathrm{c}$). We can work out the expression of this sequence and add it to $\frac{1}{2}\mathrm{n}^2$ to get the expression for triangular numbers.

Sequence: 0.5, 1, 1.5, 2, 2.5, 3, 3.5…

1st diff.: 0.5 0.5 0.5 0.5 0.5 0.5

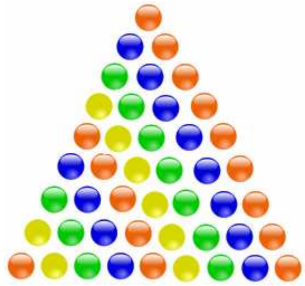
2nd diff.: 0 0 0 0 0

If the general formula of linear sequences is $\mathrm{bn} + \mathrm{c}$, then b will be 0.5 and c will be 0.

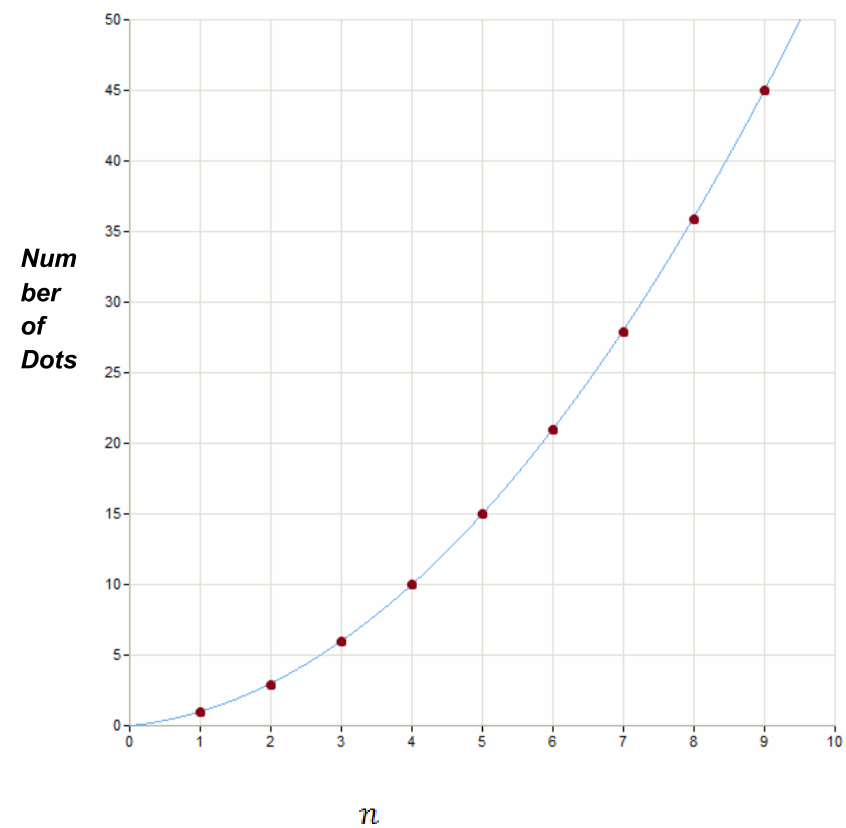
The expression of this sequence will be $\frac{1}{2}n$

Therefore the expression of triangular numbers will be: $\frac{1}{2}n^2 + \frac{1}{2}n$

We can check this by trying the expression with the 9th term. $\frac{1}{2}(9)^2+\frac{1}{2}9=45$. The difference between 45 and 36 is 9, therefore the expression is correct because this difference fits with the previous ones.



The graph below shows that the expression is correct.

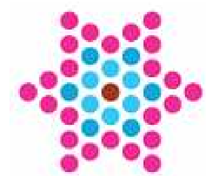


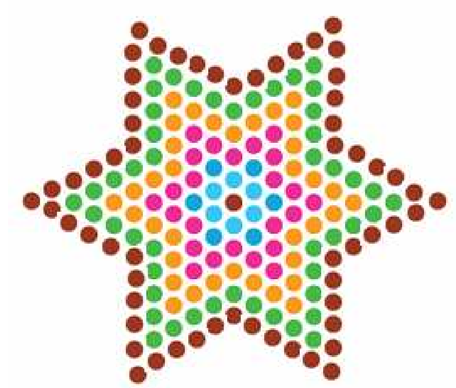
Stellar numbers

The next type of numbers we are considering is stellar numbers. Stellar numbers are sequences of dots. The 1st term is 1 dot. After that, this dot is surrounded by a star in the 2nd term. The previous star is surrounded by a larger star in the following terms. The value of $p$is the number of vertices the star has and this is also called p-stellar number.

6-stellar numbers

These are stellar numbers of stars with 6 vertices.

n	1	2	3	4
Shape				
Number of dots	1	13	37	73

n	5	6
Shape		
Number of dots	121	181

To know the expression of this sequence, we must know which difference is constant.

Sequence: 1, 13, 37, 73, 121, 181

1st diff.: 12 24 36 48 60

2nd diff.: 12 12 12 12

This is a quadratic sequence ($an^2 + bn + c$) because the 2nd difference is constant. a is equal to half the second difference which is 6.

The first part of the expression is $6n^2$

We have to subtract the $6n^2$ sequence from the original sequence to get the linear part of this expression (i.e. $bn + c$).

n	1	2	3	4	5	6
Seq.	1	13	37	73	121	181
$6n^2$	6	24	54	96	150	216
Seq.-$6n^2$	-5	-11	-17	-23	-29	-35

The expression of this sequence will be the final part of the original sequence.

Sequence: -5, -11, -17, -23, -29, -35…

1st diff.: -6, -6, -6, -6, -6

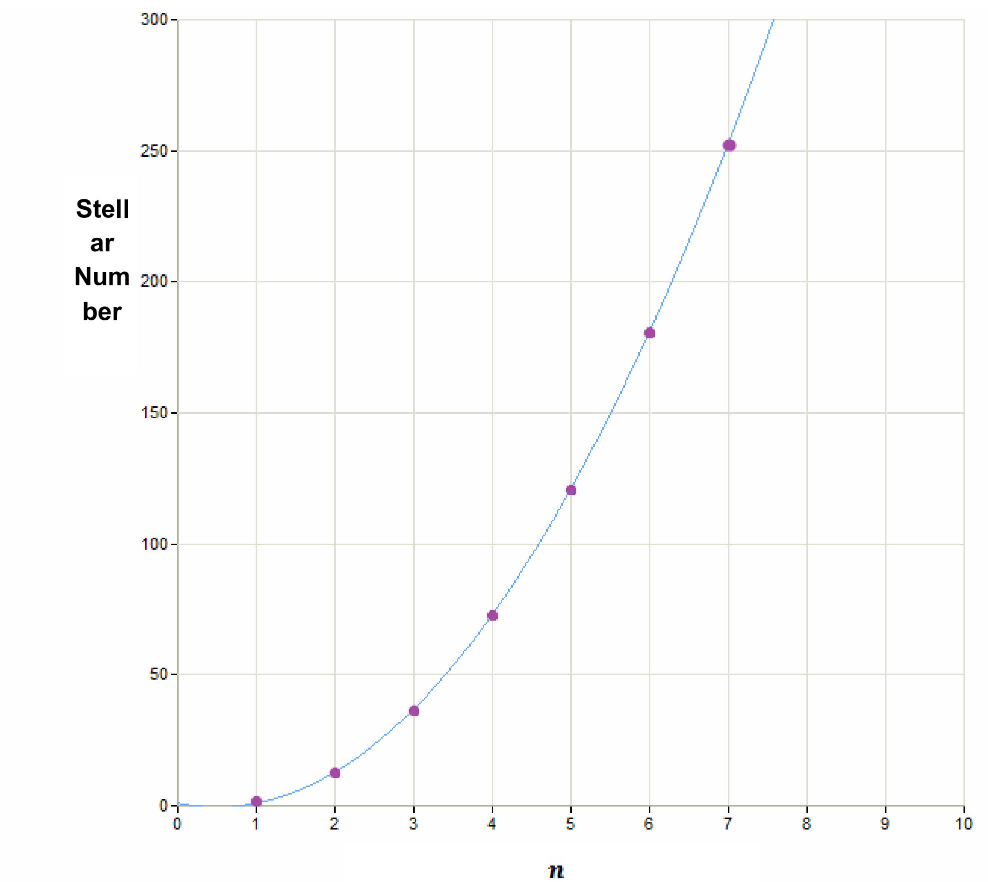
The expression is now $6n^2$-$6n$

However, it is not complete. If we try it on S_2, $6(2)^2$-$6(2)$ it equals 12 not 13. So we will add 1 as c to the expression.

So the expression of S_7 will be $6(7)^2 - 6(7) + 1 = 253$

We know that this is true because the difference between 253 and 181 is 72 which fit with the 1st differences. We can also count the stars in the following diagram.

The following graph shows the accuracy of the expression



These are stellar numbers of stars with 5 vertices.

S_1 S_2 S_3 S_4 S_5 S_6

SN: 1, 11, 31, 61, 101, 151…

Each star consists of a pentagon with a triangle attached to it. The number of dots in all the pentagons is 5. The number of dots in each triangle is 2n-3. I only calculated the last shell and added it to the previous number of dots to get the stellar number.

Seq.: 1, 11, 31, 61, 101, 151…

1^{st} Diff.: 10 20 30 40 50

2^{nd} Diff.: 10 10 10 10

This sequence is quadratic ($an^2 + bn + c$). The first part of the expression is $5n^2$. To know the rest of the expression, we will need to subtract the $5n^2$ sequence from the original sequence.

n	1	2	3	4	5	6
Seq.	1	11	31	61	101	151
$5n^2$	5	20	45	80	125	180
Seq.- $5n^2$	-4	-9	-14	-19	-24	-29

We now have a linear sequence ($bn + c$). This expression will be added to the first part of the stellar number sequence.

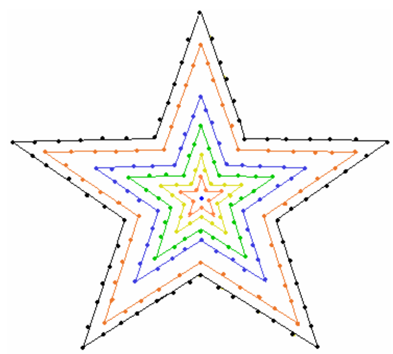
Seq.: -4, -9, -14, -19, -24, -29…

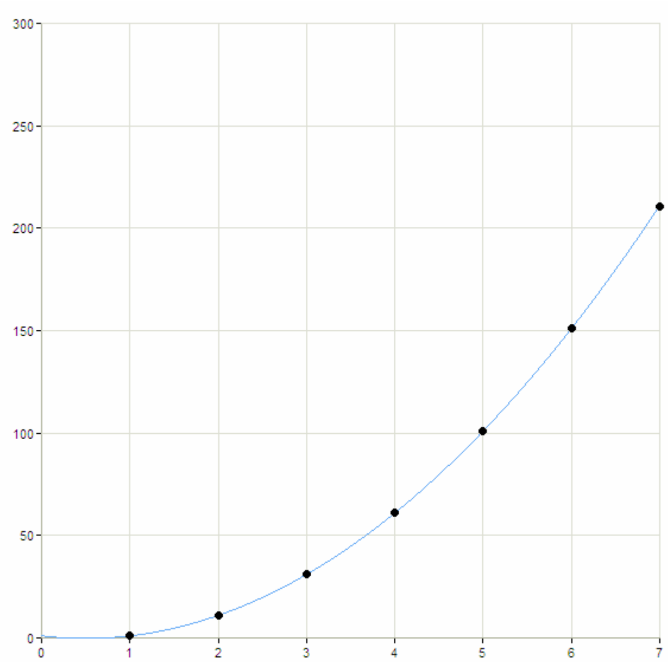
1^{st} Diff.: -5 -5 -5 -5 -5

If we try the expression $5n^2\text{-}5n + 1$ now, it will not work. We have to add +1 to it.

The expression for S_7 is $5(7)^2\text{-}5(7) + 1 = 211$

We can check that this is true by subtracting 151 from 211 and the result should fit in the 1st differences row. The difference is 60 and fits in the 1st differences row. We can also count the stars in the diagram.

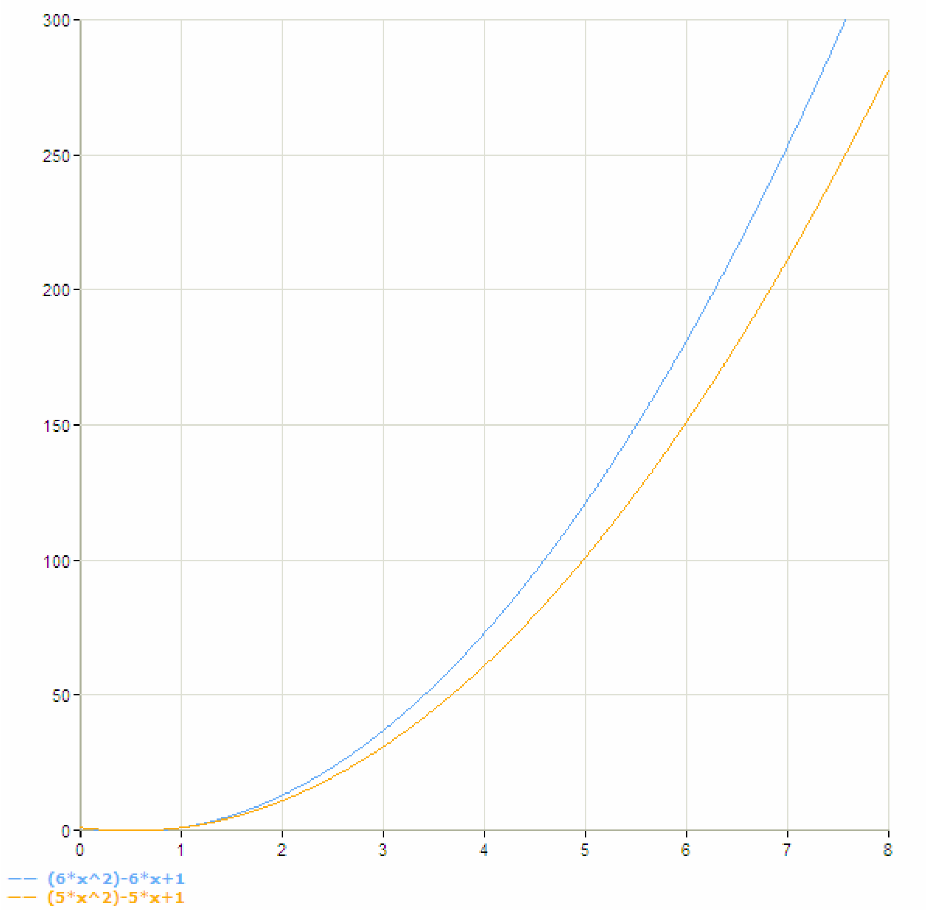




$s_n = 5n^2\text{-}5n + 1$

General Statement:

The functions of these two stellar numbers are very similar as in this



graph.

After knowing the expressions for stellar numbers of 2 different vertices, we found that the common units in the expression are $s_n = n^2 - n + 1$

And we found that the coefficient of n^2 and n is the number of vertices that the star has.

The general statement for every star with p vertex is:

$s_n = pn^2 - pn + 1$

Where p is greater than 2 and n is greater than or equal to 1.

This statement works only for real numbers and positive numbers because you can't have a negative star or half a star.