Math Portfolio
Stellar Numbers

Triangles:

Introduction:

Throughout this task, I will first be trying to find a general statement that represents the nth triangular number in terns of n. After doing so, I will be able to complete the triangular numbers sequence by adding three more terms. I will have to go through several trial and error runs as well as numerous steps with the data that has been given in order to find the general statement, which would lead to finding the next couple of terms in the sequence.

Information Given:

T_n = number of dots
n = number of rows

T_n	n
1	1
3	2
6	3
10	4
15	5

To start off, I had to find the positive difference of T_n as well as the positive difference of n, which was simply just one as for the numbers only went up by one every time.

Positive difference of T_n		
1	→	**1**
3=1+2	→	**2**
6=(1+2)+3	→	**3**
10=(1+2+3)+4	→	**4**
15=(1+2+3+4)+5	→	**5**

After using this information and putting it all together, it was clear to see that the polynomial had to be a quadratic equation:
$T_n = an^2 + bn + c$

I then plugged the information given from the very start into the quadratic that I had found:

$T_n = 1$

n = 1

$$1 = a(1)^2 + b(1) + c$$
$$1 = a + b + c$$

$T_n = 3$
n = 2

$$3 = a(2)^2 + b(2) + c$$
$$3 = 4a + 2b + c$$

$T_n = 6$
n = 3

$$6 = a(3)^2 + b(3) + c$$
$$6 = 9a + 3b + c$$

After finding these three different equations, I solved to find the values of a, b and c.

1 = a + b + c   3 = 4a + 2b + c

- 3 = 4a + 2b + c   - 6 = 9a +3b + c
_____   _____

  2 = 3a + b   3 = 5a + b

  2 = 3a + b
- 3 = 5a + b
_____

  1 = 2a

a = 0.5

2 = 3a + b
2 = 3(1/2) + b

b = 0.5

3 = 4a + 2b + c
3 = 4(1/2) + 2(1/2) + c
3 = 2 + 1 + c

c = 0

After finding these three different values, I plugged them back into the quadratic equation that I had found ($T_n = an^2 + bn + c$), which gave me the general statement for this task.

$T_n = 1/2\ n^2 + 1/2\ n$

To verify that my general statement functions correctly, I plugged in any random *n* value which was given at the start of the task into the general statement, and depending on the answer the general statement was either right or wrong. For example, I plugged 5 into the general statement. The T_n value which would have to pop up would be 15 in order to prove my general statement correct.

Examples:

n = 5
$T_n = 1/2\ n^2 + 1/2\ n$
$T_n = 1/2\ (5)^2 + 1/2\ (5)$
$T_n = 15$

n=2
$T_n = 1/2\ n^2 + 1/2\ n$
$T_n = 1/2\ (2)^2 + 1/2\ (2)$
$T_n = 3$

This helps show that the general statement found functions correctly. After finding the general statement, I was able to calculate the next three triangular terms in the sequence.

$T_6 = 1/2\ (6)^2 + 1/2\ (6)$
$T_6 = 18 + 3$
$T_6 = 21$

Drawing done by hand

$T_7 = 1/2\ (7)^2 + 1/2\ (7)$
$T_7 = 24.5 + 3.5$
$T_7 = 28$

Drawing done by hand

$T_8 = 1/2\,(8)^2 + 1/2\,(8)$
$T_8 = 32 + 4$
$\boxed{T_8 = 36}$

Drawing done by hand

Stellar Shapes:

Introduction:

Throughout this task, I will trying to find the number of dots in each stage up to S_6, an expression for the 6-steller number at stage S_7, and a general statement for the 6-stellar number at stage S_n in terms of n. I will, once again, have to go through several trial and error runs as well as numerous steps with the data that has been given in order to find these different things.
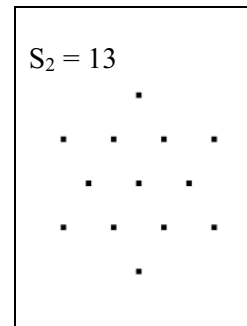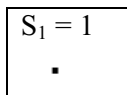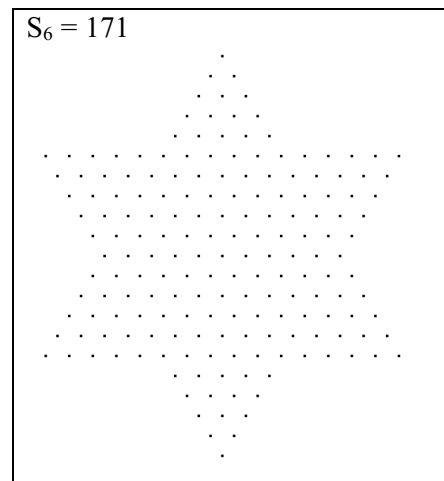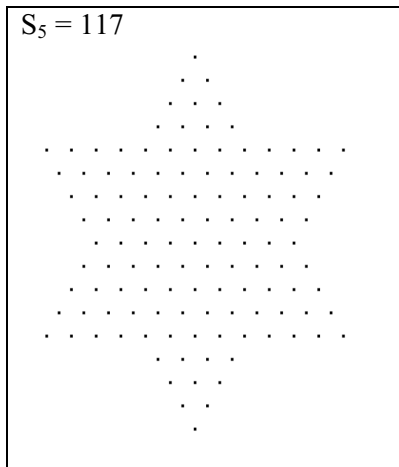
Information Given:

S_n = number of dots in each diagram
n = number of big star around each other, in each diagram

S_n	n
1	1
13	2
37	3
72	4

First of all, I found the number of dots (the stellar number) in each stage up to S6 by drawing each star and counting all the dots in one diagram.

$S_1 = 1$

.

$S_2 = 13$

S₃ = 37

S₄ = 72

S₅ = 117

S₆ = 171
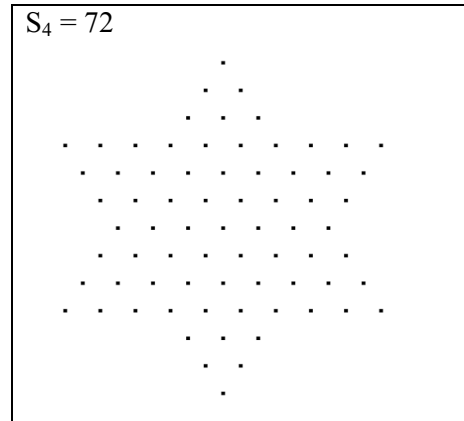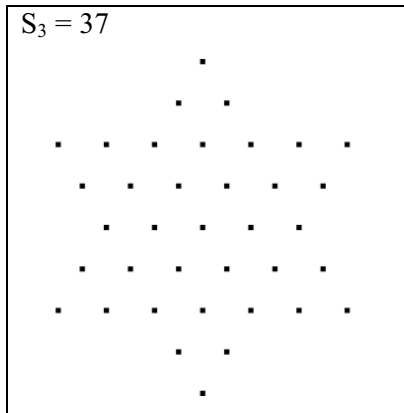
After counting all the dots from each diagram, I was able to put all of the information in a table.

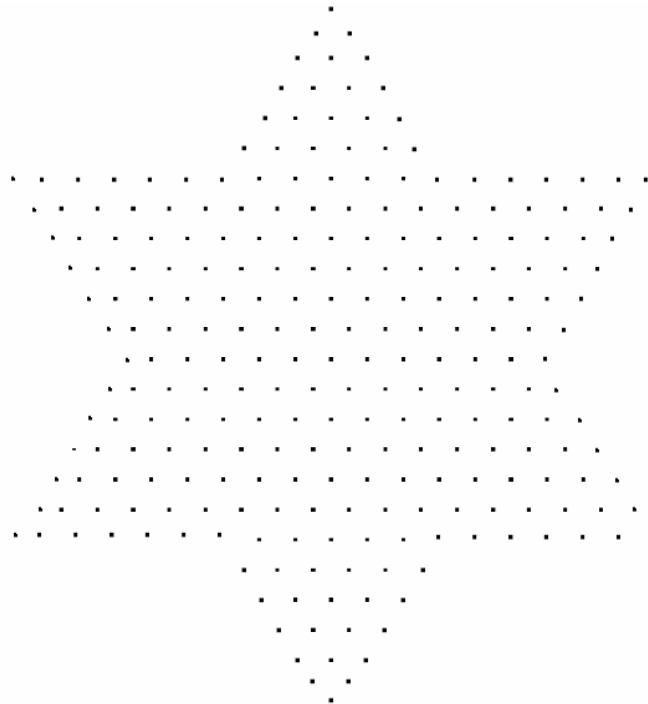| Stellar Shape (n)	Number of Dots	Increase of Dots	Second Difference
1	1	-	-
2	13	12	12
3	37	24	12
4	73	36	12
5	121	48	12
6	181	60	12
7	253	72	12

We can clearly observe that there is an increase of 12 dots for each consecutive stellar shape.

When I apply the pattern that was found above, I was able to find an expression for the 6-stellar number at stage S_7:

$S_7 = 171 + (54 + (9 - 1))$
$S_7 = 171 + 8$
$\boxed{S_7 = 233}$

Following this, I was also able to find a general statement for the 6-stellar number at stage S_n in terms of n. To find it I had to go through several steps.

First, I found the positive difference of S_n:
13-1=**12**
37-13=**24**
72-37=**35**

After doing so, I found the positive differences of 12, 24 and 35 which were 11 and 12.Then I found the positive difference of 11 and 12 which was 1. With the information gathered, I figured out that this had to be cubic:
$T_n = an^3 + bn^2 + cn + d$

I then inserted the information given from the very start into the cubic that I had found:

$S_n = 1$
$n = 1$
$1 = a(1)^3 + b(1)^2 + c(1) + d$
$\boxed{1 = a + b + c + d}$

$S_n = 13$
$n = 2$
$13 = a(2)^3 + b(2)^2 + c(2) + d$
$\boxed{13 = 8a + 4b + 2c + d}$

$S_n = 37$
$n = 3$
$37 = a(3)^3 + b(3)^2 + c(3) + d$
$\boxed{37 = 27a + 9b + 3c + d}$

$S_n = 72$
$n = 4$
$72 = a(4)^3 + b(4)^2 + c(4) + d$
$\boxed{72 = 64a + 16b + 4c + d}$

After finding these three different equations, I solved to find the values of a, b, c and d:

$\begin{aligned} 1 &= a + b + c + d \\ - 13 &= 8a + 4b + 2c + d \\ \hline \end{aligned}$ $\qquad$ $\begin{aligned} 37 &= 27a + 9b + 3c + d \\ - 72 &= 64a + 16b + 4c + d \\ \hline \end{aligned}$

12 = 7a + 3b + c   **35 = 37a + 7b + c**

12 = 7a + 3b + c   1 = a + b + c + d
- 35 = 37a + 7b + c   - 72 = 64a + 16b + 4c + d
_____   _____

23 = 30a + 4b   **71 = 63a + 15b + 3c**

13 = 8a + 4b + 2c + d
- 37 = 27a + 9b + 3c + d
_____

24 = 19a + 5b + c

3(24 = 19a + 5b + c) = **[72 = 57a + 15b + 3c]**

71 = 63a + 15b + 3c
- 72 = 57a + 15b + 3c
_____

1 = -6a

a = -1/6

23 = 30a + 4b
23 = 30(-1/6) + 4b
23 = -5 + 4b
_____

28 = 4b

b = 7

12 = 7a +3b + c
12 = 7(-1/6) + 3 (7) + c
12 = -7/6 + 21 + c

c = -47/6

13 = 8a + 4b + 2c + d
13 = 8(-1/6) + 4(7) + 2(-47/6) + d

d = 2

To be able to find the general statement, I plugged the a, b, c and d values back into the cubic polynomial ($T_n = an^2 + bn + c$) and found the general statement which was:
$S_n = -1/6n^3 + 7n^2 – 47/6n + 2$

To verify that my general statement functions correctly, I plugged in any random n value which was given at the start of the task into the general statement, and depending on the answer the general statement was either right or wrong.

$n = 4$
$S_n = -1/6n^3 + 7n^2 - 47/6n + 2$
$S_4 = -1/16(4)^3 + 7(4)^2 - 47/6(4) + 2$
$S_4 = -1/16(64) + 7(16) - 47/6(4) + 2$
$S_4 = -32/3 + 112 - 94/3 + 2$
$S_4 = 72$

After checking to see if the general statement was correct, I checked the validity of each stage up to S_7 by plugging the n value into the general statement.

S_1:
$S_1 = -1/6(1)^3 + 7(1)^2 - 47/6(1) + 2$
$S_1 = -1/6 + 7 - 47/6 + 2$
$\boxed{S_1 = 1}$

S_2:
$S_2 = -1/6(2)^3 + 7(2)^2 - 47/6(2) + 2$
$S_2 = -4/3 + 28 - 47/3 + 2$
$\boxed{S_2 = 13}$

S_3:
$S_3 = -1/6(3)^3 + 7(3)^2 - 47/6(3) + 2$
$S_3 = -9/2 + 63 - 47/2 + 2$
$\boxed{S_3 = 37}$

S_4:
$S_4 = -1/6(4)^3 + 7(4)^2 - 47/6(4) + 2$
$S_4 = -32/3 + 112 - 94/3 + 2$
$\boxed{S_4 = 72}$

S_5:
$S_5 = -1/6(5)^3 + 7(5)^2 - 47/6(5) + 2$
$S_5 = -125/6 + 175 - 235/6 + 2$
$\boxed{S_5 = 117}$

S_6:
$S_6 = -1/6(6)^3 + 7(6)^2 - 47/6(6) + 2$
$S_6 = -36 + 252 - 47 + 2$
$\boxed{S_6 = 171}$

S_7:
$S_7 = -1/6(7)^3 + 7(7)^2 - 47/6(7) + 2$
$S_7 = -343/6 + 343 - 329/6 + 2$
$\boxed{S_7 = 233}$

The results calculated above were all using 6-stellar stars. I repeated these steps for other values of p but in order to do so I had to identity how different p values would affect the

formula. I discovered that every time that the *p* value would increase by one, the *d* value would increase by 2.

p	*d*
6	2
7	4
8	6
9	8

5-stellar star:
p = 5
n = 2

$S_n = -1/6n^3 + 7n^2 - 47/6n + 0$
$S_2 = -1.33 + 28 - 15.67 + 0$
$S_2 = -17 + 28 + 0$
$S_2 = 11$

Conclusion:

This Stellar Numbers Mathematics Portfolio required finding two general statements as well as different expressions and patterns. The first general statement that I found represented the nth triangular number in terms of *n* ($T_n = 1/2\ n^2 + 1/2\ n$). When using this equation, the number of dots in a triangle can be calculate without having to count the dots individually.

The second general statement that was found was for the 6-stellar number at stage S_n in terms of *n* ($S_n = -1/6n^3 + 7n^2 - 47/6n + 2$). This equation helps calculate the number of dots in each star according to the number of rows in any stellar shape with 6 vertices. I also discovered that the *d* value changes depending on the number of vertices in a star.

There were also several limitations regarding the two different tasks. One was that no values could be below zero, as for that is unrealistic, and this is considered for both tasks. Another limitation regarding the first task (triangles) is that the equation only works for equilateral triangles (triangles with equal sides). Secondly, for the Stellar Star Task, I discovered that the p value has to be at least 4 to be able to be seen in a stellar star shape. Also, the general statement for the Stellar Stars can only be used for 6-Stellar Stars.