Miras International School Almaty 2012

INVESTIGATION

Subject: Mathematics SL

Topic: Stellar Numbers

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Stellar Numbers

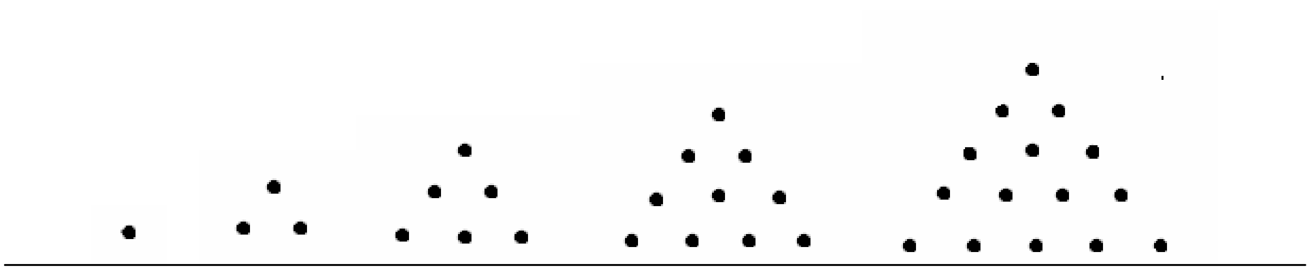
A number sequence is a list of numbers where there is a pattern.[1] We study variations of different sequences and series, such as: geometric, arithmetic and others. We learn different formulas to find the unknown values of the term number, the sum of the terms. And the only reason we do that is to make our live easier and convert huge sequences of numbers into short and exact formulas. In the current investigation I'm going to analyze different kinds of the sequences, which include not only numbers but geometric shapes and stellar numbers as well.

Aim

The aim of the current investigation is to consider different geometric shapes, which lead to specific numbers, to formulate the universal formulas to every specific group of geometric shapes and to test the validity of the gotten general statement .

Procedure

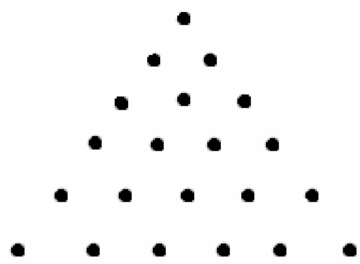
1. The first step is to work out the sequence of triangular pattern of evenly spaced dots. The number of dots in each are examples of triangular numbers (1, 3, 6,…).

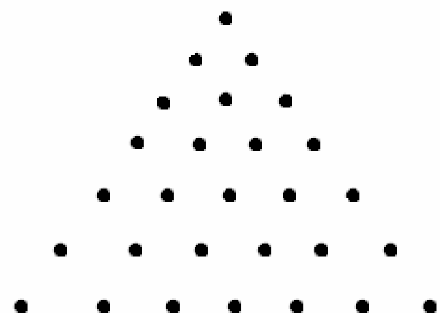


1 **3** **6** **10** **15**

[1] Math SL HaH Text

- To complete the triangular numbers sequence with three more terms.





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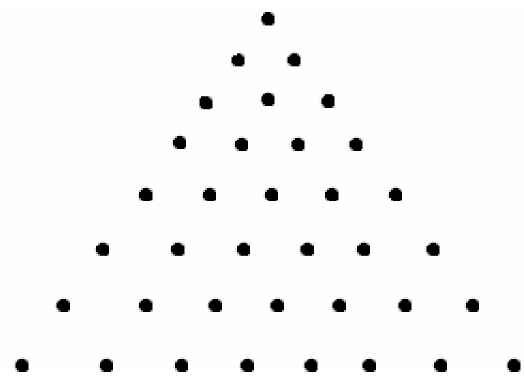

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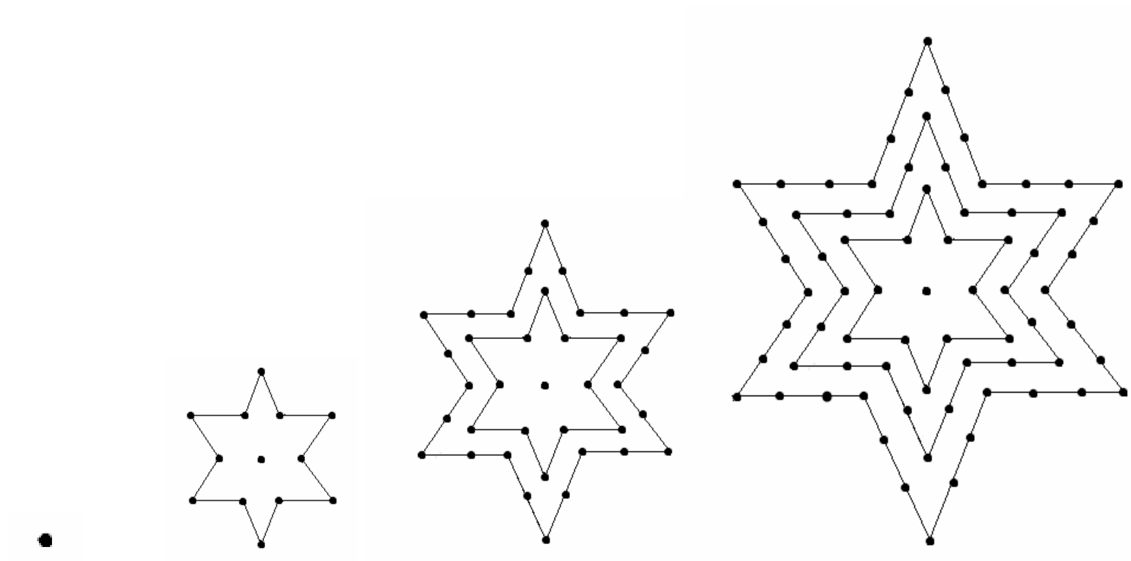
- To find the general statement that represents the ***n***th triangular in terms of ***n***.

Term Number	**Number of Dots**	**nth triangular in terms of n**
n = 1	u□ = 1	u□ = u□ = 1
n = 2	u□ = 3	u□ = u□ + n = 3
n = 3	u□ = 6	u□ = u□ + n = 6
n = 4	u□ = 10	u□ = u□ + n = 10
n = 5	u□ = 15	u□ = u□ + n = 15
n = 6	u□ = 21	u□ = u□ + n = 21
n = 7	u□ = 28	u□ = u□ + n = 28
n = 8	u□ = 36	u□ = u□ + n = 36

The general statement that represents the nth triangular number in terms of n is **$u_n = u_{n-1} + n$**

2. The second task is to consider stellar (star) shapes with p vertices, leading to p - stellar numbers. Stages **S□-S□** represent the first four stages for the stars with **six** vertices.



S□ **S□** **S□**
S□

- <u>To find the number of dots in each stage up to S □.</u>

Term Number	Number of Dots
n = 1	S□ = 1
n = 2	S□ = 2(n-1) 6 + S□ = 12
n = 3	S□ = 2(n-1) 6 + S□ = 72
n = 4	S□ = 2(n-1) 6 + S□ = 120
n = 5	S□ = 2(n-1) 6 + S□ = 180
n = 6	S□ = 2(n-1) 6 + S□ = 252

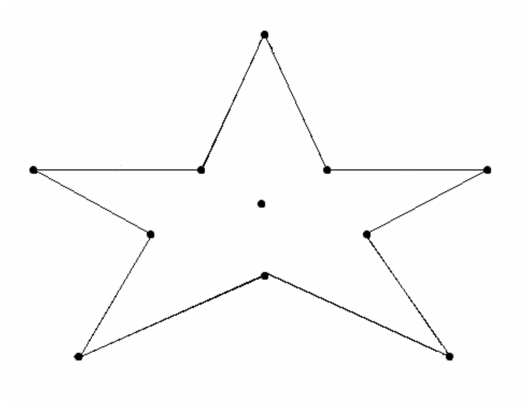
- <u>To find an expression for the 6 -stellar number at stage S □.</u>

S□ = 2(7-1) 6 + S□

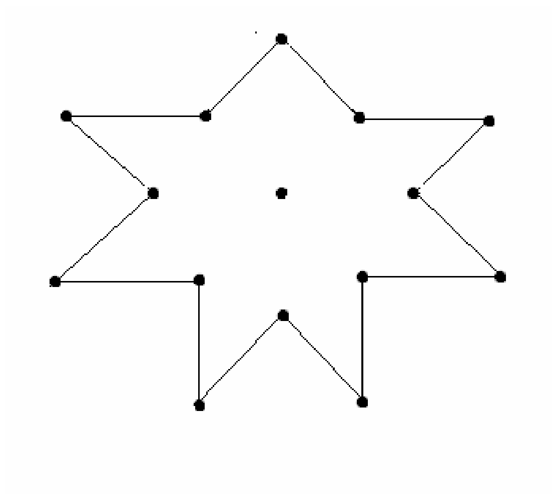
- <u>To find a general statement for the 6-stellar number at stage S in terms of n</u>

S = 2(n-1) 6 + S

- <u>To repeat the steps above for other values of **p** (*p*=5; *p*=7)</u>



P = 5



P = 7

P=5

S = 2(n-1) 5 + S

Term Number	Number of Dots
n = 1	S□ = 1
n = 2	S□ = 2(n-1) 5 + S□ = 11
n = 3	S□ = 2(n-1) 5 + S□ = 31
n = 4	S□ = 2(n-1) 5 + S□ = 61
n = 5	S□ = 2(n-1) 5 + S□ = 101
n = 6	S□ = 2(n-1) 5 + S□ = 151

P=7

S = 2(n-1) 7 + S

Term Number	Number of Dots
n = 1	S□ = 1
n = 2	S□ = 2(n-1) 7 + S□ = 15
n = 3	S□ = 2(n-1) 7 + S□ = 43
n = 4	S□ = 2(n-1) 7 + S□ = 85
n = 5	S□ = 2(n-1) 7 + S□ = 141
n = 6	S□ = 2(n-1) 7 + S□ = 211

- <u>To produce the general statement, in terms of *p* and *n*, that generates the sequence of p-stellar numbers for any value of *p* and stage *S* .</u>

$$S = 2(n-1)\ p + S$$

- <u>To test the validity of the general statement.</u>

  According to the stages of finding **5-stellar, 7 stellar** and different variation of **n** (1-6), described and proved above, we may consume that the general statement **S = 2(n-1) p + S** is valid to be used.

- <u>To discuss the scope or limitations of the general statement.</u>

  The general statement is not universal to use of any number of **n.**

  1) The sequence has to start from 1. We cannot find the first stage using the general formula.
  2) We cannot find the number of dots in terms of huge numbers without using the technology, because each time we use the formula, we have to know the previous term.

- <u>To explain how you arrived at the general statement.</u>

  To find the number of dots in different stellar sequences, we use the general statement
  S = 2(n-1) p + S . We know that each vertex of the star is divided into two rays, therefore **2** in the formula stands for **the number of these rays**. **2** is multiplied by the
  (n-1), which means that the number of dots on the each ray is subtracted by 1, because the internal vertices of the star are divided into two rays as well, so that the number of dots is not surplus. **2(n-1)** is multiplied by the **number of vertices (p)** and the previous stage of the sequenc e **(S)** is added.

<u>Conclusion</u>

The aim of the investigation was to consider different geometric shapes, which lead to specific numbers, to formulate the general statements to every specific group of geometric shapes and to test the validity of the gott en formula. By considering different types of geometric shapes such as triangular numbers, 6-vertices stellar shapes, 5-vertices stellar shapes and 7-stellar shapes, we proved that the use of arithmetic and geometric sequences is not limited by only working with numbers, different geometric forms can be measures as well. The general statement **S = 2(n-1) p + S** was produced, and also tested on validity and limitations.

<u>Bibliography</u>

- Mathematics SL electronic book
- Stellar Numbers Task Sheet. For final assessment in 2011 and 2012