The English School

IB Mathematics SL
Math Portfolio (Type1)

Stellar Numbers

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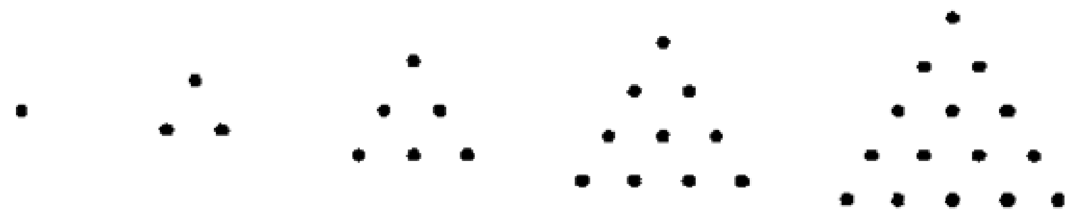
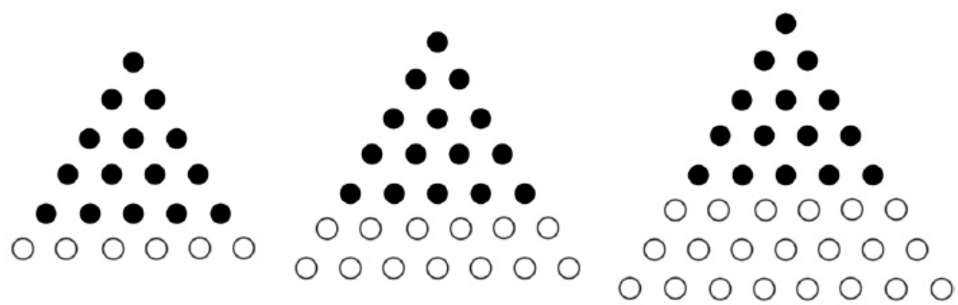
Stellar Numbers

Aim: In this task I will consider geometric shapes, which lead to special numbers

Task 1
Complete the triangular sequence with more than 3 terms.



1 3 6 10 15



21 28 36

Following the triangle sequence:

0+1=1
1+2=3
3+3=6
6+4=10
10+5=15

Therefore,
15+6=21
21+7=28
28+8=36

After looking at the sequence I could realize that these triangular numbers are simply the sum of numbers from 1 to the term number. As the pattern continues the adding number increases arithmetically.

Example: Triangular number 4 is 10 so, 6+4=10; or the Triangular number 7 is 28 so, 21+7= 28

Hence, the general statement that represents the n^{th} triangular number in terms of n is the equation:

$\frac{n(n+1)}{2}$, Were n is the triangular number we want to find

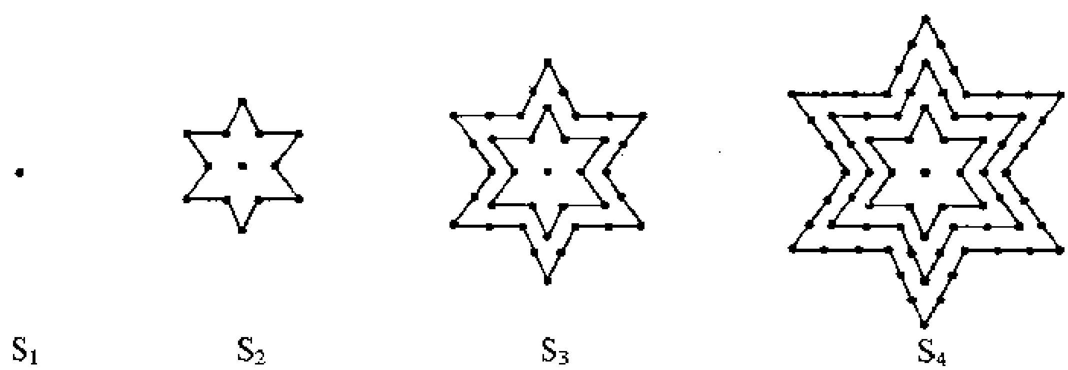
For example, if we want to find the 10th triangular number we replace n with 10

$$\frac{10(10+1)}{2} = \frac{110}{2} = 55$$

This means that the 10th term will have 55 dots in its triangular shape.

Task 2

Find the number of dots (the stellar number) in each stage up to S_6



At S_1 the number of dots is 1, at S_2 the number of dots is 13, at S_3 the number of dots is 37 and in S_4 the number of dots is 73.

The complete pattern in this 6-stellar is at follows:

0+1= 1

1+12=13

13+24=37

37+36=73

Therefore,
S_5 = 73+ 48=121
S_6 = 121 + 60= 181

Similarly to the triangular numbers, the 6-stellar sequences uses the same method but this time the number of dots can be found by preceding term added to by the multiples of 12 (as shown in red). Other way to write the pattern is in this way:

(1+0(12)), (1+1(12)), (1+3(12)) , (1+6(12)) , …

As soon as I wrote it this way I realized that there is clear relationship between the Triangular numbers and the way the 6-stellar numbers n^{th} term.

Task 3
Find an expression for the 6-stellar number at S_7

Now that we know the pattern for the first stages it is easy to recognize the stage S_7

S_7 = S_6 + 12(n-1) (where S_6 =181 and n=7)
= 181+12(7-1)
= 181+12(6) (multiple of 12)
= 181+72
= 253

Task 4
Find the general statement for the 6-stellar number at stage S_n in terms of n

To find the general statement we must look back to the Triangular number equation of $\frac{n(n+1)}{2}$, at this point we need to modify the equation so that it is suitable for the 6-stellar. So, in this case n=12n because of the multiples of 12, the ($n+1$) to $(n-1)$ because each of the numbers multiplied by 12 is one less than the n of the term, and add 1 because the addition of terms starts initially from 1. Following all the rules the general statement should look like this:

$$1 + 12\left(\frac{n(n-1)}{2}\right)$$

Task 5
Repeat the steps above for other values of p

A) Here is the number of dots for the sequence of **5-stellar** number

$S_1 = 0+1 = 1$
$S_2 = 1+10= 11$
$S_3 = 11+20= 31$
$S_4 = 31+30= 61$

Therefore,
$S_5 = 61+40= 101$
$S_6 = 101+50= 151$

Similarly to the triangular numbers, the 5-stellar sequences uses the same method but this time the number of dots can be found by preceding term added to by the multiples of 10 (as shown in red). Other way to write the pattern is in this way:

(1+0(10)), (1+1(10)), (1+3(10)) , (1+6(10)) , …

As soon as I wrote it this way I realized that there is clear relationship between the Triangular numbers and the way the 5-stellar numbers n^{th} term.

Now that we know the pattern for the first stages it is easy to recognize the stage S_7

$S_7 = S_6 + 10(n\text{-}1)$ (where S_6 =151 and n=7)
= 151+10(7-1)
= 151+10(6) (multiple of 10)
= 151+ 60
= 211

As done with the 6-stellar number, to find the general statement we must look back again to the Triangular number equation of $\frac{n(n+1)}{2}$, at this point we need to modify the equation so that it is suitable for the 5-stellar. So, in this case n=10n because of the multiples of 10, the $(n+1)$ to $(n-1)$ because each of the numbers multiplied by 10 is one less than the n of the term, and add 1 because the addition of terms starts initially from 1. Following all the rules the general statement should look like this:

$$1 + 10\left(\frac{n(n-1)}{2}\right)$$

B) Here is the number of dots for the sequence of **8-stellar** number

$S_1 = 0+1 = 1$
$S_2 = 1+16= 17$
$S_3 = 17+32= 49$
$S_4 = 49+48= 97$

Therefore,
$S_5 = 97+64= 161$
$S_6 = 161+80= 241$

Similarly to the triangular numbers, the 8-stellar sequences uses the same method but this time the number of dots can be found by preceding term added to by the multiples of 16 (as shown in red). Other way to write the pattern is in this way:

(1+0(16)), (1+1(16)), (1+3(16)) , (1+6(16)) , …

As soon as I wrote it this way I realized that there is clear relationship between the Triangular numbers and the way the 8-stellar numbers n^{th} term.

Now that we know the pattern for the first stages it is easy to recognize the stage S_7

$S_7 = S_6 + 16(n\text{-}1)$ (where S_6 =241 and n=7)
= 241+16(7-1)
= 241+16(6) (multiple of 16)
= 241+ 96
= 337

As done with the 6 and 5-stellar numbers, to find the general statement we must look back again to the Triangular number equation of $\frac{n(n+1)}{2}$, at this point we need to modify the equation so that it is suitable for the 8-stellar. So, in this case n=16n because of the multiples of 16, the $(n + 1)$ to $(n - 1)$ because each of the numbers multiplied by 16 is one less than the n of the term, and add 1 because the addition of terms starts initially from 1. Following all the rules the general statement should look like this:

$$1 + 16\left(\frac{n(n-1)}{2}\right)$$

Task 7
Produce a general statement; in terms of p and n, that generates the sequence of p-stellar numbers for any of the values of p at stage S_n

Comparing the results of the 5, 6 and 8 stellar numbers I could realize that the pattern of these numbers are identical except for the first number, which varies among sequences. We can also see that all the first term is usually equal to their *p values.* Hence, the general statement for *p* and *n* is:

$$S_n = 1 + (p)\left(\frac{n(n-1)}{2}\right)$$

Task 8
Test the validity of the general statement

Since we now have a general statement we can ensure that it works on larger numbers

For example: if we have $p = 430$ and we want to know S_8, then $n = 8$. Using our general statement we can get a quicker and easier answer:

$S_8 = 1 + 840\left(\frac{8(8-1)}{2}\right)$
$S_8 = 1 + 840(28)$
$S_8 = 1 + 23520$
$S_8 = 23521$ (At S_8 the 420-stellar will have this amount of points)

Task 9
Explain how you arrived at the general statement

In order to arrive at the general statement I did first take the general statement of the triangular numbers on task 1 and then adjusted as a arithmetical sequence so that task 2 could be done. From the general statement of the 6-stellar number, I could now adjust the equation in order to get different values of *p* and its sequences. After doing the examples of 6,7 and 8 stellar numbers, I could therefore, understand the behaviour of the sequences by considering the relation of *p* and *n.* Following this relation, I was able to put either *p* or *n* in the general statement to by then take any data, no matter if it is a small or big value, as well as the geometrical shape of the number.