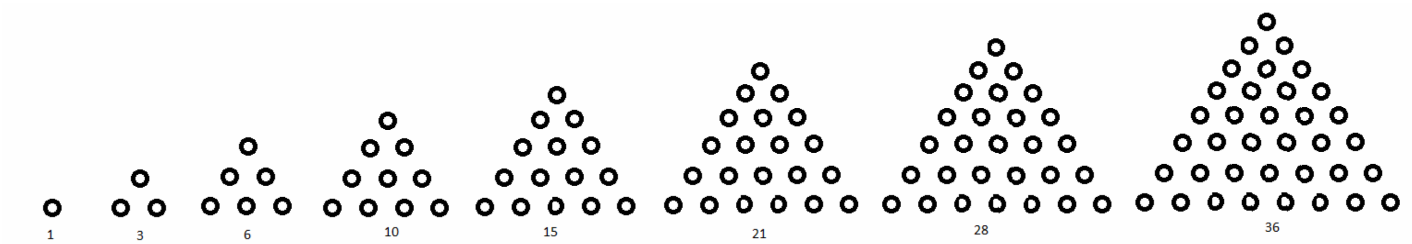
Stellar Numbers

Triangular Numbers:

Diagram 1 : Triangular Numbers



$T_1 = 1$ $T_2 = 3$ $T_3 = 6$ $T_4 = 10$ $T_5 = 15$ $T_6 = 21$

$T_7 = 28$ $T_8 = 36$

First Arithmetic Progression (AP) $= T_2 – T_1$
$= 3 – 1$
$= 2$

Difference from Arithmetic Progression (d) $= (T_3 – T_2) – (T_2 – T_1)$
$= (6 – 3) – (3 – 1)$
$= 3 – 2$
$= 1$

To determine the number of dots in one triangle there are two possible formulas which are possible. The first is that where T_n is equal to that of the first triangle, with the addition of the Sum of n -1. This formula can be simplified after substituting the Arithmetic Progression and difference into the Sum of n – 1. This gives the final formula in which T_n equals one (1) and n-1 divided by two (2) multiplied by four (4) and n minus two (2).

The other formula is that of when T_n is equal to n plus n-1 divided by two (2) multiplied by, two AP plus d of n take two (2). This formula can also be simplified and this gives the result of T_n equal to that of n divided by two, multiplied by that of n with the addition of one (1).

$T_n = T_1 + (Sum_{n-1})$
$T_n = T_1 + [(n-1) \div 2] \times [2 \times AP + (n – 1 -1) d]$
$T_n = 1 + [(n-1) \div 2] \times [2 \times 2 + (n – 2)]$
$T_n = 1 + [(n-1) \div 2] \times [4 + (n – 2)]$

Or

$T_n = n + [(n-1) \div 2] \times [2 \times T_1 + (n – 1 -1) d]$
$T_n = n + [(n-1) \div 2] \times [2 \times 1 + (n – 1 -1)1]$
$T_n = n + [(n-1) \div 2] \times [2 + (n – 2)]$
$T_n = (2n + n^2 –n) \div 2$
$T_n = (n + n^2) \div 2$
$T_n = (n \div 2) (n + 1)$

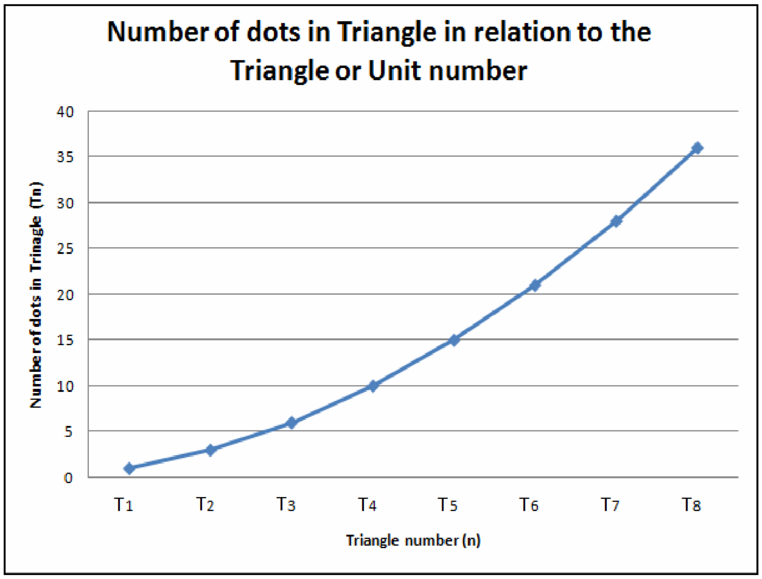
Table 1: Triangular Numbers

Unit (n)	Number of dots in triangle (Tn)	(Tn+1) - (Tn)	Progression
1	1	2	1
2	3	3	1 + 2
3	6	4	1 + 2 + 3
4	10	5	1 + 2 +3 +4
5	15	6	1 + 2 + 3 +4 +5
6	21	7	1 + 2 + 3 + 4 + 5 + 6
7	28	8	1 + 2 + 3 + 4 + 5 + 6 + 7
8	36	9	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8

From this table it is possible to see that each progressive units is equal to the sum of the previous unit plus n. Thus forming the simple formula of $T_n = T_{n-1} + n$. In order to write this as a general formula,

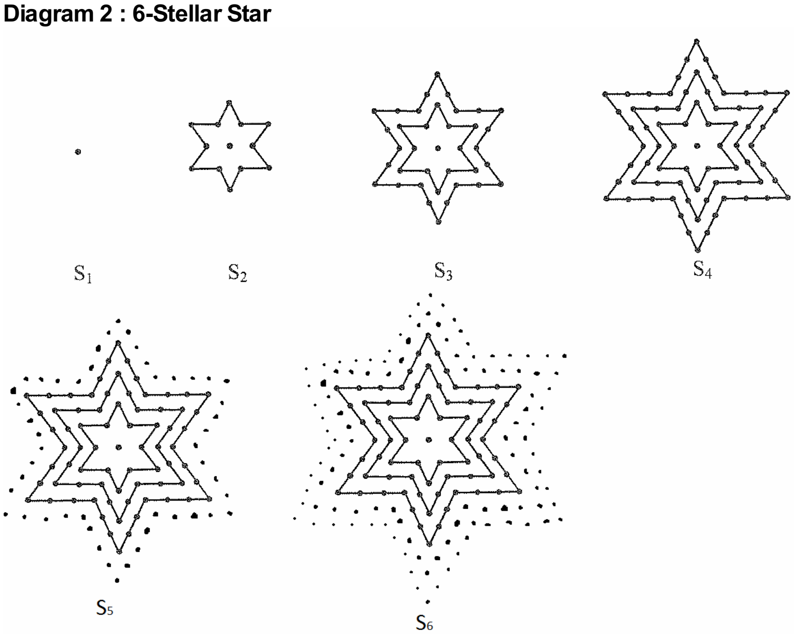
allowing for the calculation of any unit number without the prior knowledge of the previous sum it is necessary to use the formula $T_n = 1 + [(n-1) \div 2] \times [4 + (n - 2)]$ or $T_n = (n \div 2)(n + 1)$

Graph 1: Triangular Numbers- Number of dots in relation to the unit number



Stellar Numbers:

Diagram 2 : 6-Stellar Star



The Arithmetic Progression for the 6-point stellar star is that of twelve (12), or two (2) multiplied by the number of points of the star in such case being that of six (6). Therefore AP is equal to 2p.

Find the expression for the 6-stellar number at stage S_7:

$S_n = S_1 + [(n-1) \div 2] \times [2 \times 2P + (n - 1 - 1) 2P]$
$S_n = 1 + [(n-1) \div 2] \times [2 \times 12 + (n - 2) 12]$
$S_7 = 1 + [(7-1) \div 2] \times [2 \times 12 + (7 - 2) 12]$
$S_7 = 1 + [(6) \div 2] \times [24 + (5) 12]$
$S_7 = 1 + [3] \times [24 + 60]$
$S_7 = 1 + 3 \times 84$
$S_7 = 1 + 252$
$S_7 = 253$

Find a general statement for the 6-stellar number at stage S_n in terms of n:

$S_n = S_1 + [(n-1) \div 2] \times [2 \times 2P + (n - 1 - 1) 2P]$

$S_n = S_1 + [(n-1) \div 2] \times [2 \times 12 + (n - 2) 12]$
$S_n = S_1 + [(n-1) \div 2] \times [24 + (n - 2) 12]$

Test the general statement for the 6-stellar numbers for multiple stages:
General Statement: $S_n = S_1 + [(n-1) \div 2] \times [24 + (n - 2) 12]$

Stage S_3:
$S_3 = S_1 + [(3-1) \div 2] \times [24 + (3 - 2) 12]$
$S_3 = 1 + [2 \div 2] \times [24 + (1) 12]$
$S_3 = 1 + [1] \times [24 + 12]$
$S_3 = 1 + [36]$
$S_3 = 37$
Using the general statement it is possible to calculate that S_3 of the 6- Stellar star has 37 dots in it. From the Diagram of the 6 point Stellar Star it is possible to show the formula is true for S_3.

Stage S_4:
$S_4 = S_1 + [(4-1) \div 2] \times [24 + (4 - 2) 12]$
$S_4 = 1 + [(3) \div 2] \times [24 + (2) 12]$
$S_4 = 1 + [(3) \div 2] \times [24 + 24]$
$S_4 = 1 + [(3) \div 2] \times [48]$
$S_4 = 1 + 72$
$S_4 = 73$
The general statement makes it is possible to calculate that S_4 of the 6- Stellar star has 73 dots in it. The Diagram of the 6 point Stellar Star it is possible to show the formula is true for S_4.

Stage S_5:
$S_5 = S_1 + [(5-1) \div 2] \times [24 + (5 - 2) 12]$
$S_5 = 1 + [(4) \div 2] \times [24 + (3) 12]$
$S_5 = 1 + [2] \times [24 + 36]$
$S_5 = 1 + 2 \times 60$
$S_5 = 1 + 120$
$S_5 = 121$
Using the general statement makes it is possible to calculate that S_5 of the 6- Stellar star has 121 dots. The Diagram of the 6 point Stellar Star it is possible to show the formula is true for S_5.

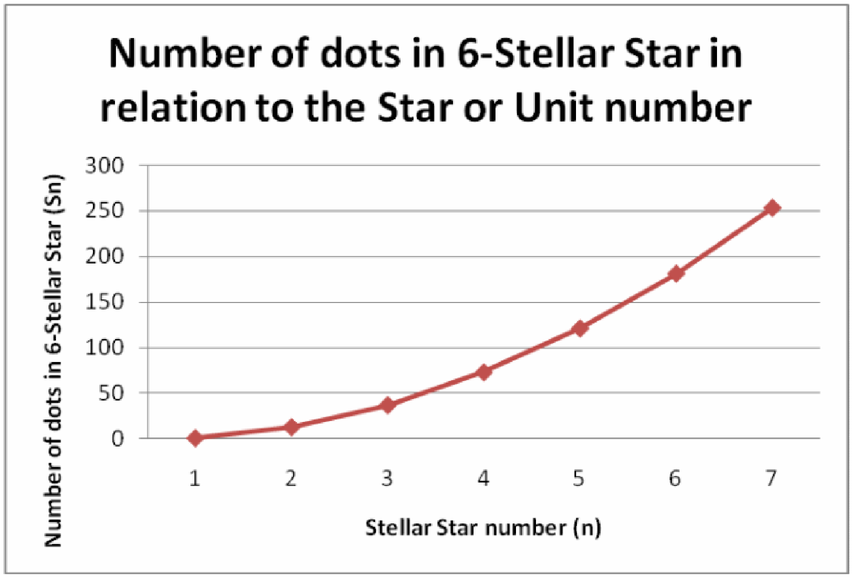
Stage S_6:
$S_6 = S_1 + [(6-1) \div 2] \times [24 + (6 - 2) 12]$
$S_6 = 1 + [(5) \div 2] \times [24 + (4) 12]$
$S_6 = 1 + [(5) \div 2] \times [24 + 48]$
$S_6 = 1 + 180$
$S_6 = 181$
Using the general statement makes it is possible to calculate that S_6 of the 6- Stellar star has 161 dots. The Diagram of the 6 point Stellar Star it is possible to show the formula is true for S_6.

Table 2: 6- Stellar Numbers

Unit (n)	Number of dots in 6-Stellar Star (Sn)	(Sn+1) - (Sn)	Progression
1	1	12	1
2	13	24	1+12
3	37	36	1 + 12 + 24
4	73	48	1 + 12 + 24 +36
5	121	60	1 + 12 + 24 +36 +48
6	181	72	1 + 12 + 24 +36 +48+ 60
7	253	84	1 + 12 + 24 +36 +48 + 60 + 72

From the progression it is possible to visualise the addition of the previous sum of units to two of number of points (in such case 6 x 2) multiple of n – 1.

Graph 2: 6-Stellar Star Numbers in relation to the unit number



In the case when p, the number of points on the star alters a new general statement must be used. This statement involves determining the new arithmetic progression of such series of numbers and then applying it to the new general statement. For example when only a 5-stellar number, the arithmetic progression changes to that of ten (10), which is double the number of points.

Diagram 3 : 5-Stellar Star

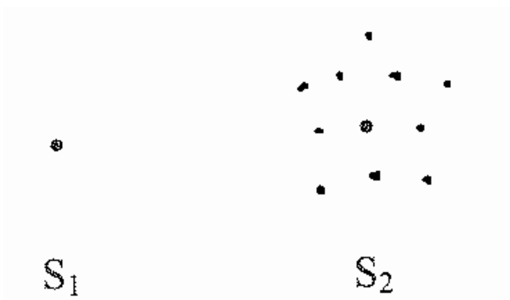


Diagram 4: 4-Stellar Star

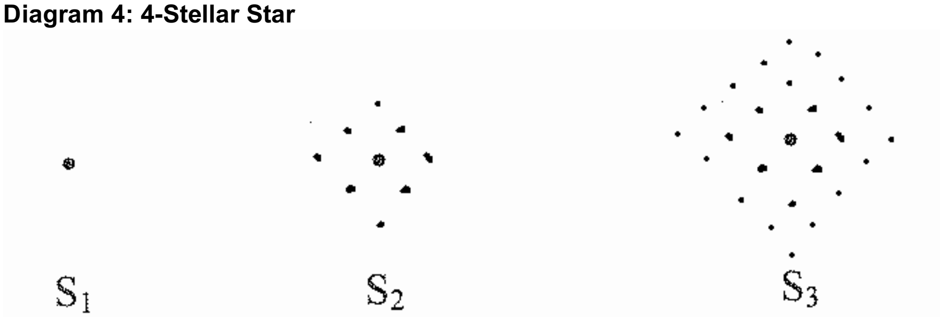
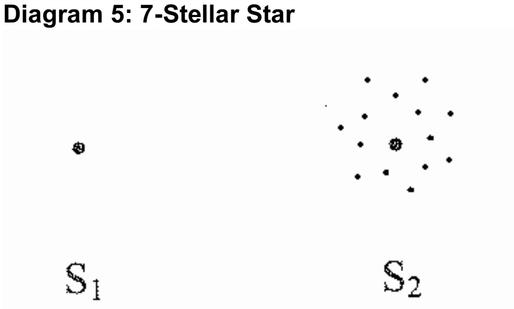


Diagram 5: 7-Stellar Star



Find a general statement for any stellar star with different numbered points:
AP: Arithmetic Progression
$S_n = S_1 + [(n-1) \div 2] \times [2 \times AP + (n - 1 - 1) AP]$
$S_n = S_1 + [(n-1) \div 2] \times [2 \times AP + (n - 2) AP]$

Testing the Validity of this statement
For a five point star:
AP: 10, or (2 multiplied by the number of points)
$S_n = S_1 + [(n-1) \div 2] \times [2 \times AP + (n - 2) AP]$
$S_n = S_1 + [(n-1) \div 2] \times [2 \times 10 + (n - 2) 10]$

Stage S_2:
$S_2 = S_1 + [(n-1) \div 2] \times [2 \times 10 + (n - 2) 10]$
$S_2 = 1 + [(2-1) \div 2] \times [20 + (2 - 2) 10]$
$S_2 = 1 + [1 \div 2] \times [20]$
$S_2 = 1 + [10]$
$S_2 = 11$
Using the general statement it is possible to calculate that S_2 of the 5- Stellar star has 11 dots in it. From the Diagram of the 5 point Stellar Star it is possible to show the formula is true for S_2.

Stage S_3:
$S_3 = S_1 + [(n-1) \div 2] \times [2 \times 10 + (n - 2) 10]$
$S_3 = 1 + [(3-1) \div 2] \times [20 + (3 - 2) 10]$
$S_3 = 1 + [(2) \div 2] \times [20 + 10]$
$S_3 = 1 + [30]$
$S_3 = 31$
Using the general statement it is possible to calculate that S_3 of the 5- Stellar star has 31 dots in it. If the diagram of the 5 Stellar Star is continued it is possible to show that the general formula also applies to S_3.

For a four point star:
AP: 8, or (2 multiplied by the number of points)
$S_n = S_1 + [(n-1) \div 2] \times [2 \times AP + (n - 2) AP]$
$S_n = S_1 + [(n-1) \div 2] \times [2 \times 8 + (n - 2) 8]$

Stage S_2:
$S_2 = S_1 + [(n-1) \div 2] \times [2 \times 8 + (n - 2) 8]$
$S_2 = 1 + [(2-1) \div 2] \times [16 + (2 - 2) 8]$
$S_2 = 1 + [1 \div 2] \times [16]$
$S_2 = 1 + [8]$
$S_2 = 9$
Using the general statement it is possible to calculate that S_2 of the 4- Stellar star has 9 dots in it. Diagram 4 shows that there are nine dots in S_2.

Stage S_3:
$S_3 = S_1 + [(n-1) \div 2] \times [2 \times 8 + (n - 2) 8]$
$S_3 = 1 + [(3-1) \div 2] \times [16 + (3 - 2) 8]$
$S_3 = 1 + [2 \div 2] \times [24]$
$S_3 = 25$
Using the general statement it is possible to calculate that S_3 of the 4- Stellar star has 25 dots in it. The Diagram of the four point star shows that in S_3 has 25 dots in it.

For a seven point star:
AP: 14, or (2 multiplied by the number of points)
$S_n = S_1 + [(n-1) \div 2] \times [2 \times 14 + (n - 2) 14]$
$S_n = S_1 + [(n-1) \div 2] \times [28 + (n - 2) 14]$

Stage S_2:
$S_2 = S_1 + [(n-1) \div 2] \times [28 + (n - 2) 14]$
$S_2 = 1 + [(2-1) \div 2] \times [28 + (2 - 2) 14]$
$S_2 = 1 + [1 \div 2] \times [28]$
$S_2 = 1 + [14]$
$S_2 = 15$
Using the general statement it is possible to calculate that S_2 of the 7- Stellar star has 15 dots in it. Diagram 5 supports this result.

Stage S_3:
$S_3 = S_1 + [(n-1) \div 2] \times [28 + (n - 2) 14]$
$S_3 = 1 + [(3-1) \div 2] \times [28 + (3 - 2) 14]$
$S_3 = 1 + [2 \div 2] \times [42]$

$S_3 = 43$
Using the general statement it is possible to calculate that S_3 of the 7- Stellar star has 43 dots. By continuing diagram 5 it is possible to support this outcome of produced from the general statement.

Limitations:
There are many faults with this general statement. Although it applies to many different numbered points of Stellar Stars, for it to be applied it is necessary to determine that the star has the same formation as that of the star which the general statement was created for. In order to determine this it is necessary to determine that the arithmetic progression is equal to double that of the number of points of the star.

The number of points or P cannot be a negative number as this is not possible to form a negative star shape. The stellar shapes also only work when the number of point or vertices are equal to or greater than three (3). Thus it can be concluded that the formula only works for all possible stellar formation shapes.

This general statement also assumes that the first stellar unit is equal to that of one. Meaning that if the sequence of numbers started at any other numerical value the general formula may not apply. Therefore if your sequence did not include the first few layers it is possible the formula would not work as the S_1.