

Solution for finding the sum of a infinite sequence

The objective of this assignment is to find out the sum of infinite sequences t_n , where

$$t_0 = 1, t_1 = \frac{(x \ln a)^2}{2 \times 1}, t_3 = \frac{(x \ln a)^3}{3 \times 2 \times 1} \dots, t_n = \frac{(x \ln a)^n}{n!} \dots$$

$$t_0 + t_1 + t_2 + t_3 + \dots t_n + \dots$$

In this equation, t is defined by the term number. For example, t_1 is the first term, whereas t_n is the n^{th} term. x and a are various variables. In this equation, $n!$ which is defined by $n! = n(n-1)(n-2) \dots 3 \times 2 \times 1$. Likewise $3!$ is defined by $3! = 3 \times 2 \times 1$, but there is an exception $0! = 1$.

First I will break down the equation so that it will be easier for me to find out the formula. I will examine the S_n defined as the sum of the first t_n . For example, $S_1 = t_0 + t_1$,

$S_3 = t_0 + t_2 + t_3$. I am first going to use this equation $t_n = \frac{(1 \ln 2)^n}{n!}$, where $x = 1$, $a = 2$, and where n is $0 \leq n \leq 10$. So it should look like this:

$$t_0 = 1, t_1 = \frac{(1 \ln 2)^2}{2 \times 1}, t_3 = \frac{(1 \ln 2)^3}{3 \times 2 \times 1}, t_4 = \frac{(1 \ln 2)^4}{4 \times 3 \times 2 \times 1} \dots, t_{10} = \frac{(1 \ln 2)^{10}}{10!}$$

I will find out the S_n for $0 \leq n \leq 10$.

$$S_0, S_1, S_2, S_3, \dots, S_{10}$$

In order to find S_n , I used my TI-84 Plus to figure this out. I will plug in the equation

$t_n = \frac{(x \ln a)^n}{n!}$, where the x is 1, a is 2 and n value is from 0 to 10. The method is shown in the appendix.

I came up with:

$t_0 = 1$	$t_3 \approx 0.055504$
$t_1 \approx 0.693147$	$t_4 \approx 0.009618$
$t_2 \approx 0.240226$	$t_5 \approx 0.001333$

$$t_6 \approx 1.5403530 \times 10^{-4}$$

$$t_9 \approx 1.017809 \times 10^{-7}$$

$$t_7 \approx 1.525273 \times 10^{-5}$$

$$t_{10} \approx 7.054911 \times 10^{-9}$$

$$t_8 \approx 1.321549 \times 10^{-6}$$

In order to check if I got it right, I used Microsoft Excel 2010. The method is shown in the appendix.

A	B
	$t_n = \frac{(1 \ln 2)^n}{n!}$
n	
0	1
1	0.693147
2	0.240227
3	0.055504
4	0.009618
5	0.001333
6	0.000154
7	0.000015
8	0.000001
9	0.000000
10	0.000000

After seeing that my result matches the results in Excel, I decided to then find out the sum. I found out the sum using my TI-84 Plus. The method is shown in the appendix.

This is what I did in order to get S_n for $0 \leq n \leq 10$.

$$S_0 = 1 \leftarrow \text{I got 1 because of } t_0.$$

$S_1 = S_0 + t_1 \leftarrow$ Here S_1 is equal to S_0 plus t_1 because S_0 is the sum of the pervious term. So if I add t_1 which is term number 1, it will give me the S_1 . I will use this formula $S_n = S_{(n-1)} + t_n$ to find out the sum's up to $n = 10$.

$$S_1 = S_0 + t_1 \leftarrow \text{Now, we can sub } S_0 \text{ as 1 and } t_1 \text{ as } 0.693147$$

$$S_1 \approx 1 + 0.693147$$

$$\therefore S_1 \approx 1.693147$$

$$S_n = S_{(n-1)} + t_n$$

Here $n = 2$, so

$$S_2 = S_{(2-1)} + t_2$$

$$S_2 = S_1 + t_2$$

$$S_2 \approx 1.693147 + 0.240226$$

$$S_2 \approx 1.933373$$

I did this for S_n where $n = 0 \leq n \leq 10$

$$S_3 \approx 1.988878$$

$$S_7 \approx 1.999999$$

$$S_4 \approx 1.998496$$

$$S_8 \approx 2.000000$$

$$S_5 \approx 1.999829$$

$$S_9 \approx 2.000000$$

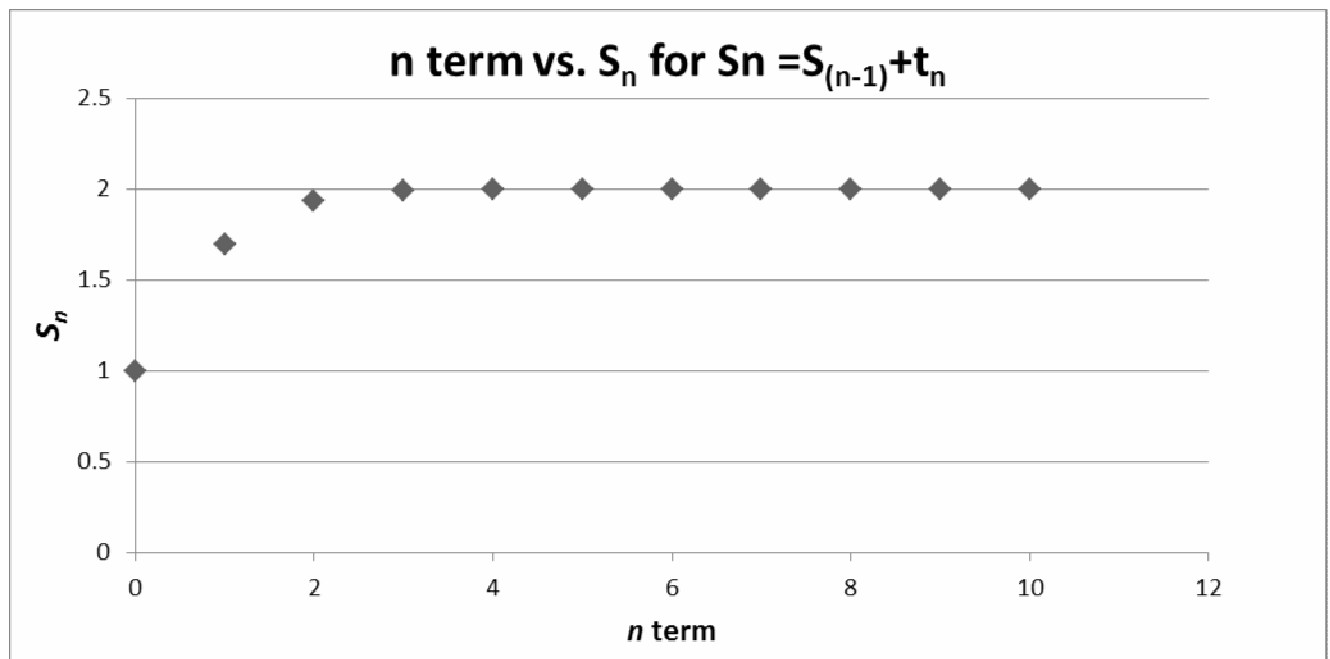
$$S_6 \approx 1.999983$$

$$S_{10} \approx 2.000000$$

On the other hand, I used Microsoft Excel 2010 to do it for me too. I got the same results. The method to do this is shown in the appendix.

	A	B	C
		$t_n = \frac{(1 \ln 2)^n}{n!}$	$S_n = S_{(n-1)} + t_n$
	n		
	0	1	1
	1	0.693147	1.693147
	2	0.240227	1.933374
	3	0.055504	1.988878
	4	0.009618	1.998496
	5	0.001333	1.999829
	6	0.000154	1.999983
	7	0.000015	1.999999
	8	0.000001	2.000000
	9	0.000000	2.000000
	10	0.000000	2.000000

Because I got the same result, I will use Excel to graph. The method is shown in the Appendix.



By looking at this graph, I can say that when n value is past 10, the S_n is 2 and remains as it is. So, as the n value increases, the S_n remains the same.

Now I will examine another sequence, and I will find out the sum of $t_n = \frac{(x \ln a)^n}{n!}$, where $x = 1$, $a = 3$, and n where it goes from 0 to 10 ($0 \leq n \leq 10$). So it should look like this:

$$t_0 = \frac{(1 \ln 3)^0}{0!}, t_1 = \frac{(1 \ln 3)^1}{1!}, t_2 = \frac{(1 \ln 3)^2}{2!}, t_3 = \frac{(1 \ln 3)^3}{3!}, t_4 = \frac{(1 \ln 3)^4}{4!}, \dots, t_{10} = \frac{(1 \ln 3)^{10}}{10!}$$

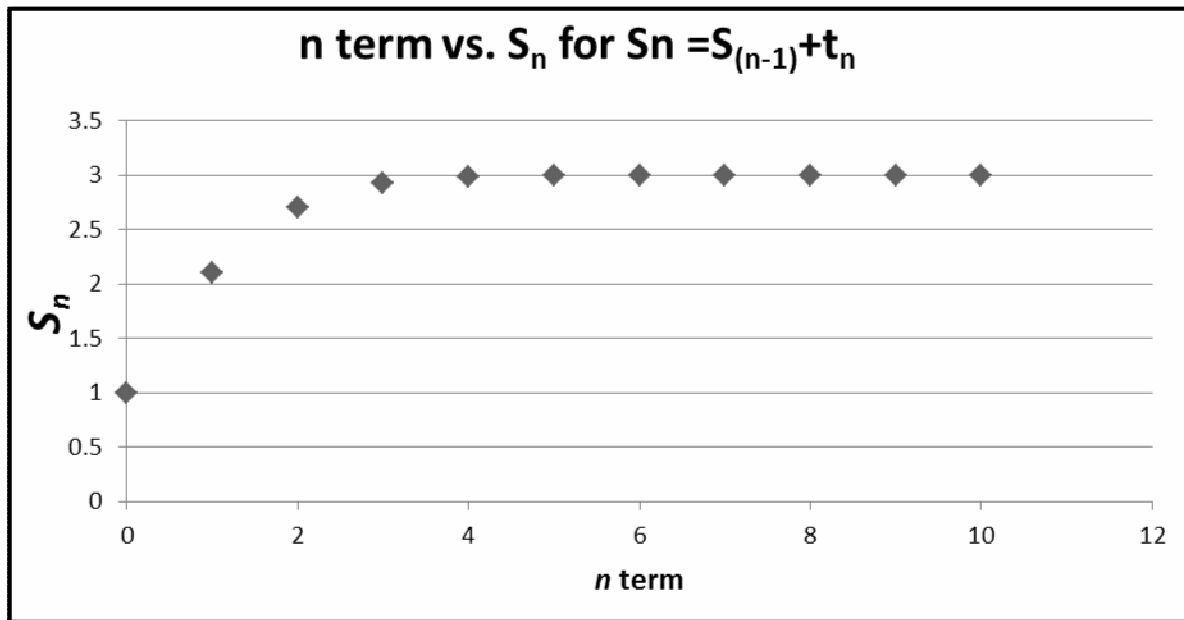
I will find out the S_n for $n = 0 \leq n \leq 10$.

$$S_0, S_1, S_2, S_3, \dots, S_{10}$$

In order to find S_n , I will only use Microsoft Excel 2010 since I know that it gave me the right result before. I will plug in the equation $t_n = \frac{(x \ln a)^n}{n!}$, where the x is 1, a is 3 and n value is from 0 to 10. The method is shown in the appendix.

	A	B	C
1			
2		$t_n = \frac{(1 \ln 3)^n}{n!}$	$S_n = S_{n-1} + t_n$
3	n		
4	0	1	1
5	1	1.098612	2.098612
6	2	0.603474	2.702087
7	3	0.220995	2.923082
8	4	0.060697	2.983779
9	5	0.013336	2.997115
10	6	0.002442	2.999557
11	7	0.000383	2.999940
12	8	0.000053	2.999993
13	9	0.000006	2.999999
14	10	0.000001	3.000000
15			
16			
17			

To find the results, I just subbed in 3 instead if $a = 2$. Now I will graph it using the same process:



By looking at this graph, I can say that when n value is past 10, S_n is 3 and it remains constant. This suggests that when n approaches ∞ (infinity), sum of the infinity (S_∞) value is 3.

After looking at the two sum, when $a = 2$, and when $a = 3$, I think that the general term is

$$S_\infty = a$$

Where S_∞ is the sum of the infinite numbers. I came up with this formula because we know,

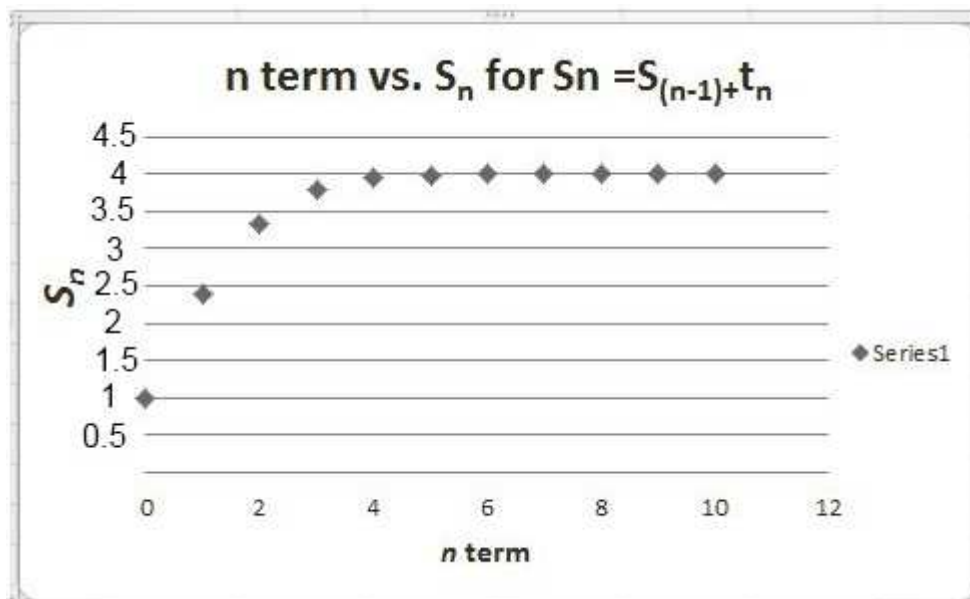
- For $a = 2$, when n kept increasing, the sum of the n value remained the same and that's why $S_n = 2$. We can say that $S_\infty = a$ instead of $S_n = 2$ because n can go up to ∞ and on the other hand a is equivalent to 2.
- For $a = 3$, when n kept increasing, the sum of the n value remained the same and that's why $S_n = 3$. We can say that $S_\infty = a$ instead of $S_n = 3$ because n can go up to ∞ and on the other hand a is equivalent to 3.

$$\text{Thus, } S_\infty = a$$

In order to test this general statement, I am going to let a be 4. So if a is 4, then $S_\infty = 4$. So now I am going to use Microsoft Excel 2010 again to prove this general statement. The method is shown in the appendix.

	A	B	C
1			
2			
3		$t_n = \frac{(1 \ln 4)^n}{n!}$	$S_n = S_{n-1} + t_n$
4	n		
5	0	1	1
6	1	1.386294	2.386294
7	2	0.960906	3.347200
8	3	0.444033	3.791233
9	4	0.153890	3.945123
10	5	0.042667	3.987791
11	6	0.009858	3.997649
12	7	0.001952	3.999601
13	8	0.000338	3.999940
14	9	0.000052	3.999992
15	10	0.000007	3.999999
16			

Now I will graph it so that it is easier to see and check:



As we can see that the graph is increasing but as n value passes 10, we see that S_n remains 4. This proves my general term $S_\infty = a$ as I predicted that $S_\infty = 4$.

Now I will sub different values for both x and a , in the equation

$$t_0 = 1, t_1 = \frac{(x \ln a)^2}{2 \times 1}, t_3 = \frac{(x \ln a)^3}{3 \times 2 \times 1} \dots, t_n = \frac{(x \ln a)^n}{n!} \dots$$

Here, I will let $T_n(a, x)$ be the sum of the first n terms for various values of a and x . For example, $T_9(2, 5)$ be the sum of the first 9 terms when $a = 2$ and $x = 5$.

But before we do anything, I am going to change $(x \ln a)$ to make it look simpler –

$$\begin{aligned} &(x \ln a)^n \\ &= (\ln a^x)^n \end{aligned}$$

I am not sure if this is going to work. So I am going to test it using my TI-84 Plus

Let $x = 3$

Let $a = 2$

Let $n = 2$

$$(x \ln a)^n$$

$$= (3 \ln 2)^2$$

$$\approx 4.324077125$$

Let $x = 3$

Let $a = 2$

Let $n = 2$

$$(\ln a^x)^n$$

$$= (\ln 2^3)^2$$

$$\approx 4.324077125$$

This proves that $(x \ln a)^n = (\ln a^x)^n$

$$\therefore t_0 = 1, t_1 = \frac{(\ln a^x)^2}{2 \times 1}, t_3 = \frac{(\ln a^x)^3}{3 \times 2 \times 1} \dots, t_n = \frac{(\ln a^x)^n}{n!} \dots$$

After looking at this equation, I think the general term will be $T_{\infty} = a^x$. Where T_{∞} is the sum of the infinite numbers when n approaches ∞ . The reason behind this is that, we figured $S_{\infty} = a$ when $t_n = \frac{(1 \ln a)^n}{n!}$ where $x = 1$. But now, we are using $t_n = \frac{(\ln a^x)^n}{n!}$. For example, let $a = 2$ and $x = 2$. This is what it should look like –

$$t_n = \frac{(\ln a^x)^n}{n!}$$

$$t_n = \frac{(\ln 2^2)^n}{n!}$$

$$t_n = \frac{(\ln 4)^n}{n!}$$

Here we can see that $a = 4$, so therefore the result should be

$$S_{\infty} = 4$$

But since we have define $T_n(a, x)$ as the sum, we should rewrite it as $T_{\infty}(2, 2) = 4$.

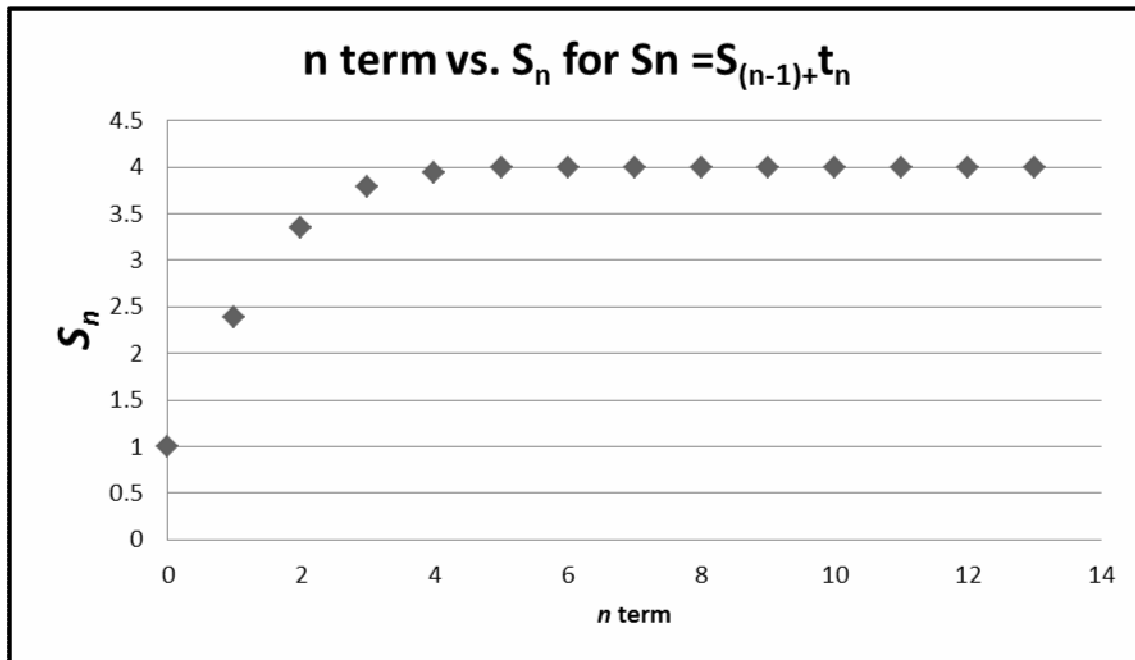
But I have only predicted this equation, in order to test it, I will use Microsoft Excel 2010. The process is shown in the appendix.

$T_n(a, x)$ is the sum of the first n term, for various values of a and x .

$T_n(2, 2)$ is the sum of the first n term, when $a = 2$ and $x = 2$.

1			
2		$t_n = \frac{(1 \ln 4)^n}{n!}$	$S_n = S_{n-1} + t_n$
3	n		
4	0	1	1
5	1	1.386294	2.386294
6	2	0.960906	3.347200
7	3	0.444033	3.791233
8	4	0.153890	3.945123
9	5	0.042667	3.987791
10	6	0.009858	3.997649
11	7	0.001952	3.999601
12	8	0.000338	3.999940
13	9	0.000052	3.999992
14	10	0.000007	3.999999
15	11	0.000001	4.000000
16	12	0.000000	4.000000
17	13	0.000000	4.000000

Here, n is the term number or the n^{th} term, x is 2 and a is 2 from my equation, $\frac{(\ln a^x)^n}{n!}$ is the equation subbing different values of n , in this case is n is defined as $0 \leq n \leq 13$. And $T_n(a, x)$ is defined as the sum of the first n^{th} term. In this case, $T_\infty(2, 2) = 4$, meaning the n value will keep increasing, but the T_n will increase up to 4 unit, and then it will remain constant. It won't go further than 4. I will use a graph and this is explain more –



Now, I am going to predict that when $a = 3$ and $x = 2$ in the equation $T_n = \frac{(\ln a^x)^n}{n!}$, the

$$T_{\infty}(3,2) = 9.$$

	A	B	C
1			
2			
3	n	$t_n = \frac{(\ln 3^2)^n}{n!}$	$S_n = S_{n-1} + t_n$
4	0	1	1
5	1	2.197225	3.197225
6	2	2.413898	5.611122
7	3	1.767959	7.379081
8	4	0.971151	8.350232
9	5	0.426767	8.776999
10	6	0.156284	8.933283
11	7	0.049056	8.982339
12	8	0.013473	8.995812
13	9	0.003289	8.999101
14	10	0.000723	8.999824
15	11	0.000144	8.999968
16	12	0.000026	8.999995
17	13	0.000004	8.999999
18	14	0.000001	9.000000
19	15	0.000000	9.000000
20	16	0.000000	9.000000
21			

So according to my guess, I was totally right. This proves that the general term is $T_{\infty}(a, x) = a^x$.

So now I am going to use my calculator to figure that what happens when $a = 0, a = -2, x = 0, x = -2$.

So, I am going to let $a = 0, x = 2$ and $n = 10$ in $t_n = \frac{(\ln a^x)^n}{n!}$.

I will use my TI-84 Plus in order to find out the situation. It look like this—

$(\ln(0^2))^{10} / (10!)$

After I pressed ENTER key, I got this result –

ERR:DOMAIN
1:Quit
2:Goto

This suggest there is no result when $a = 0$. The reason behind this is that, when we

$(\ln(0^2)) / (10!)$
Error
 $\log(0)$
Error
 $\ln(0)$
Error

punch in $\log 0$ or $\ln 0$ ■ , we get the same error. As we cannot solve it.

Now I am going to let $a = -2, x = 2$ and $n = 10$ in the equation $t_n = \frac{(\ln a^x)^n}{n!}$.

I will use my TI-84 Plus in order to find out the situation. It looks like this –

$(\ln((-2)^2))^{10} / (10!)$
7.2242295E-6

So this means, when I use a as a negative number it will show some results. I will check

if the result shown above different from the equation $t_n = \frac{(\ln 2^2)^{10}}{10!}$.

$(\ln((-2)^2))^{10} / (10!)$
7.2242295E-6
 $(\ln(2^2))^{10} / (10!)$
7.2242295E-6

This means, that when $a = 2$ it is equal to when $a = -2$. But what happens when a is a negative number and x is a odd number. For example, when $(-2)^3 = (-2)(-2)(-2) = -8$.

I will let $a = -2$, $x = 3$ and $n = 10$ in the equation $t_n = \frac{(\ln a^x)^n}{n!}$.

I will again use my TI-84 Plus to find out the result –

`(ln((-2)^3))^10 / 10!`

This is what I get when I hit enter –

ERR:NONREAL ANS
 2nd Quit
 2nd Goto

This simple means that you cannot solve a equation when there is a negative number. For example,

`(ln((-2)^3)) / 10!`
 Error
`log(-10)`
 Error
`ln(-10)`
 Error

In conclusion, when a is a negative number, x had to be an even number to change the negative sign to positive sign.

In short, $x \neq 0$, $a \neq 0$, x can be both positive and negative number, a can also be both negative and positive number but only when x is an even number.